CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

TERBUKA

COURSE NAME	:	SIGNALS AND SYSTEMS
COURSE CODE	:	BEB 20203
PROGRAMME CODE	:	BEJ
EXAMINATION DATE	:	DECEMBER 2017/ JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A: ANSWER **ALL** QUESTIONS SECTION B: ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **FIFTEEN (15)** PAGES

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SECTION A: ANSWER ALL QUESTIONS

Q1. (a) Given signal $g(t)$ as in **Figure Q1(a)**, sketch a graph for each of the functions below:

(i) $g(2t+2)$
(ii) $g(0.5t+2)$
(iii) $0.5g(2t+2)u(t+1)$

(6 marks)

(b) Based on the system in **Figure Q1(b),** and given that

$$x(t) = 2u(t) - u(t-2) - u(t-4),$$

find the expression for the signal $f(t)$ to produce an output signal

$$y(t) = 4u(t-1) - 2u(t-2) - 2u(t-3.5)$$

Hint : Use graphical method.

(4 marks)

Q2 Given the applied voltage, $v(t) = 100\cos 30t + 80\cos 40t$ V.

(a) Test whether $v(t)$ is periodic or not. If the signal is periodic determine its fundamental period.

(4 marks)

(b) Determine the exponential Fourier series coefficients of $v(t)$.

(3 marks)

(c) Calculate the average power supplied to a network if current is given by

$$i(t) = 12\cos(30t + 65°) + 20\cos(40t + 45°).$$

(3 marks)

Q3 (a) A signal $x(t)$ is given by

$$x(t) = 3\,\mathrm{rect}\left(\frac{t}{4}\right).$$

Find the Fourier transform of

(i) $x(t)$,

(2 marks)

(ii) $y(t) = \frac{d^2}{dt^2}(x(t))$.

(3 marks)

(b) The Fourier transform of $x(t) = \mathrm{rect}\,(t)$ is

$$X(f) = \mathrm{F}[x(t)] = \mathrm{sinc}(f),$$

Determine the inverse Fourier transform of

(i) $X(f) = \mathrm{rect}(f)$

(2 marks)

(ii) $X(f) = e^{-j\pi f}\,\mathrm{sinc}(f)$

(3 marks)

Q4. Given the signal $x(t) = 3e^{-3t}u(t-2)$.

(a) Find the Laplace transform of $x(t)$ using the definition of Laplace transform.

(4 marks)

(b) Sketch the zero-pole plot and region of convergence (if it exists) of the signal $x(t)$.

(2 marks)

(c) Solve $\mathcal{L}[x(t)]$ using the time shifting property of Laplace transform.

(4 marks)

SECTION B: ANSWER THREE (3) QUESTIONS ONLY

Q5 (a) Differentiate between causal and non-causal system.

(2 marks)

(b) Given an impulse response, $h(t)$ of a linear time-invariant (LTI) system as shown in **Figure Q5(b)**.

(i) Identify the stability of the LTI system.

(3 marks)

(ii) If the input signal, $x(t)$ to the LTI system is given by

$$x(t) = e^{-3t}u(t),$$

find the output response of the system, $y(t)$ by using the definition of convolution integral.

(4 marks)

(iii) Determine the causality of the output signal, $y(t)$.

(2 marks)

(c) The impulse response of a linear time invariant system is given by

$$h(t) = \begin{cases} t, & 0 < t < 2T \\ 0, & \text{otherwise,} \end{cases}$$

and the input signal to the system is

$$x(t) = \begin{cases} 1, & 0 < t < T \\ 0, & \text{otherwise.} \end{cases}$$

Using the graphical approach of solving the convolution integral, find the output of the system, $y(t)$.

(9 marks)

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Q6 (a) Given a periodic signal, $x(t)$ as shown in **Figure Q6(a)**.

(i) Analyze the frequency components for the signal $x(t)$ using the Fourier series technique.

(10 marks)

(ii) Calculate its amplitude-phase coefficients (A_n, θ_n) and write the amplitude phase series expression.

(3 marks)

(iii) Plot the amplitude and phase spectrum of $x(t)$.

(2 marks)

(b) Using Parseval's theorem, analyse the percentage of power contained in the first 3 harmonics of $x(t)$ compared to the total power of $x(t)$.

(5 marks)

Q7 (a) Given two different signals

$$x_1(t) = e^{-at}u(t), a > 0, \qquad x_2(t) = e^{-bt}u(t), b > 0$$

where $a \neq b$.
If $x(t)$ is an output of a convolution process between $x_1(t)$ and $x_2(t)$, that is

$$x(t) = x_1(t) * x_2(t),$$

find:

(i) Fourier transform of $x(t)$.

(3 marks)

(ii) Signal $x(t)$ using inverse Fourier transform of signal in Q7(a)(i).

(3 marks)

(b) A basic modulator circuit is shown in **Figure Q7(b).** Modulation is a multiplication between voltage signal, $v(t)$, and a carrier signal, $c(t)$. The process yields a new signal, $m(t)$.

(i) Analyze the Fourier Transform of signal $m(t)$ by using modulation properties.

(4 marks)

(ii) Sketch the spectrum signal of $M(f)$.

(2 marks)

(c) Analyze the output $v_o(t)$ of the electrical circuit shown in **Figure Q7(c)** for

$$v_i(t) = 3e^{-5t}u(t)V.$$

(8 marks)

Q8 (a) **Figure Q8(a)** shows a block diagram of a feedback system.

(i) Analyze the overall system function of the feedback system if

$$H_1(s) = \frac{1}{s+2}, \qquad H_2(s) = \frac{s+2}{s-2}.$$

(6 marks)

(ii) Sketch the zero-pole plot of the system.

(2 marks)

(b) The region of convergence (ROC) of the feedback system in Q8(a) above is unknown. Determine all possible impulse responses of the feedback systems by looking at the stability and causality of the response. Show the region of convergences (ROCs) for all cases.

(12 marks)

-END OF QUESTIONS-

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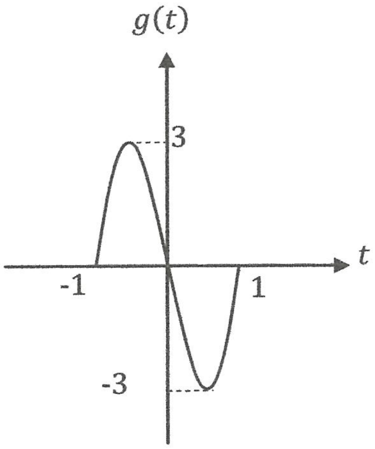

Figure Q1(a)

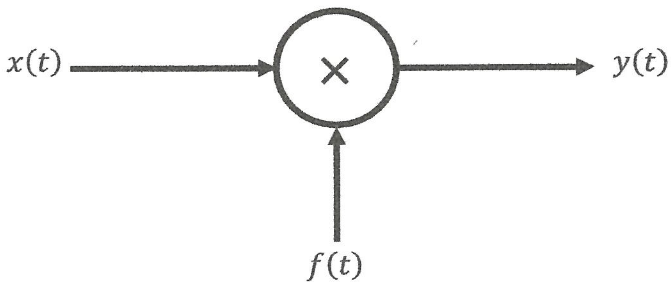

Figure Q1(b)

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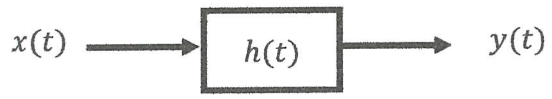

where
$h(t) = e^{6t}\,u(t)$

Figure Q5(b)

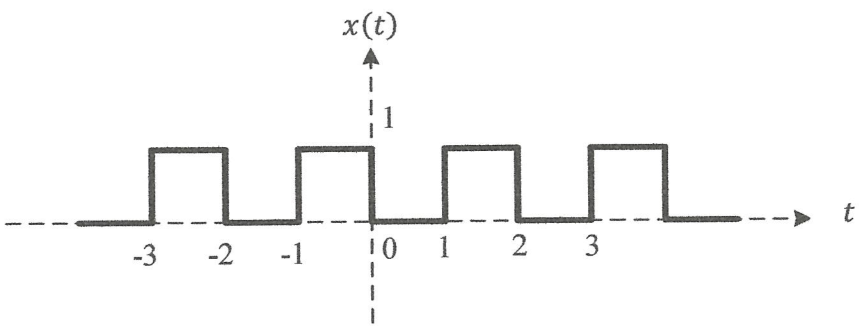

Figure Q6(a)

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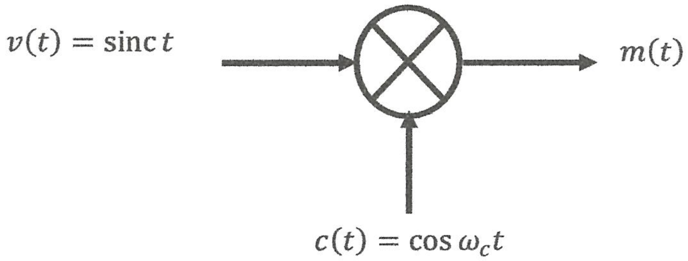

Figure Q7(b)

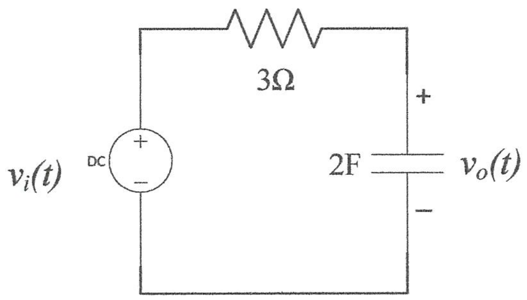

Figure Q7(c)

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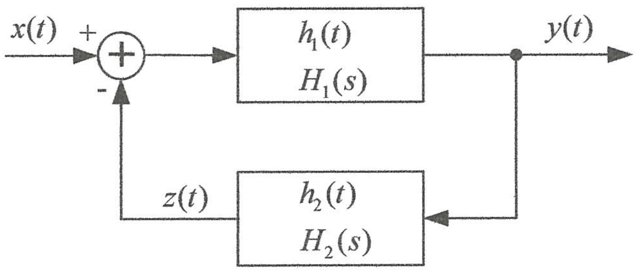

Figure Q8(a)

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TABLE 1: INDEFINITE INTEGRALS

$\int \cos at\,dt = \frac{1}{a}\sin at$	$\int \sin at\,dt = -\frac{1}{a}\cos at$
$\int t\cos at\,dt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at$	$\int t\sin at\,dt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at$
$\int te^{at}dt = \frac{1}{a^2}e^{at}(at-1)$	$\int \frac{1}{(a^2+t^2)}dt = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)$

TABLE 2: EULER'S IDENTITY

$e^{\pm j\pi/2} = \pm j$	$A\angle \pm\theta = Ae^{\pm j\theta}$
$e^{\pm jk\pi} = \cos k\pi$	$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$
$\cos\theta = \frac{1}{2}\left(e^{j\theta}+e^{-j\theta}\right)$	$\sin\theta = \frac{1}{2j}\left(e^{j\theta}-e^{-j\theta}\right)$

TABLE 3: COMPLEX NUMBER

$s = a+jb = \lvert s\rvert\angle\pm\theta = \lvert s\rvert e^{\pm j\theta}$	$\lvert s\rvert = \sqrt[2]{a^2+b^2}$	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

TABLE 4: TRIGONOMETRIC IDENTITIES

$\sin\theta = \cos\left(\theta-\frac{\pi}{2}\right))$	$\cos\theta = \sin(\theta+\frac{\pi}{2})$
$\sin(\propto\pm\beta) = \sin\alpha\cos\beta \pm\cos\alpha\sin\beta$	$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp\sin\alpha\sin\beta$
$\sin^2\alpha+\cos^2\beta = 1$	
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1-2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 5: VALUES OF COSINE, SINE AND EXPONENTIAL FUNCTIONS FOR INTEGRAL MULTIPLE OF π.

Function	Value	Function	Value
$\cos 2n\pi$	1	$e^{j2n\pi}$	1
$\sin 2n\pi$	0	$e^{jn\pi}$	$(-1)^n$
$\cos n\pi$	$(-1)^n$	$e^{\frac{jn\pi}{2}}$	$\begin{cases}(-1)^{\frac{n}{2}} & ,n = \text{even}\\ j(-1)^{\frac{n-1}{2}} & ,n=\text{odd}\end{cases}$
$\sin n\pi$	0		
$\cos\left(\frac{n\pi}{2}\right)$	$\begin{cases}(-1)^{\frac{n}{2}} & ,n=\text{even}\\ 0 & ,n=\text{odd}\end{cases}$	$\sin\left(\frac{n\pi}{2}\right)$	$\begin{cases}(-1)^{\frac{n-1}{2}} & ,n=\text{odd}\\ 0 & ,n=\text{even}\end{cases}$

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TABLE 6: FOURIER SERIES

Exponential	$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$ $x_n = \frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t} dt$
Trigonometric	$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\frac{2\pi}{T}t + b_n \sin n\frac{2\pi}{T}t$ $a_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\cos n\frac{2\pi}{T}t\, dt, \quad n = 0, 1, 2, 3 \ldots$ $b_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\sin n\frac{2\pi}{T}t\, dt, \quad n = 1, 2, 3 \ldots$
Amplitude-phase	$x(t) = X_0 + \sum_{n=1}^{\infty} A_n \cos(n\frac{2\pi}{T}t + \theta_n)$ $A_n = 2\lvert X_n\rvert = \sqrt[2]{a_n^2 + b_n^2} \quad , \quad \theta_n = \angle X_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$
Average Power	$P = V_{dc}I_{dc} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_{V_n} - \theta_{I_n})$

TABLE 7: DEFINITION OF FOURIER AND LAPLACE TRANSFORM

FOURIER TRANSFORM	INVERSE FOURIER TRANSFORM
$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$ $\mathcal{F}[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\, dt$	$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$ $x(t) = \mathcal{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\, df$
LAPLACE TRANSFORM	**INVERSE LAPLACE TRANSFORM**
Bilateral $\mathrm{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$ **Unilateral** $\mathrm{L}[x(t)] = X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$ $s = \sigma + j\omega$	$x(t) = \mathrm{L}^{-1}[X(s)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$ TERBUKA

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TABLE 8: FOURIER TRANSFORM PAIRS

Time domain, $x(t)$	**Frequency domain,** $X(\omega)$	**Frequency domain,** $X(f)$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	$\frac{\delta(f)}{2}+\frac{1}{j2\pi f}$
$u(t+\tau)-u(t-\tau)$	$\frac{2\sin(\omega\tau)}{\omega}=2\tau\,\mathrm{sinc}(\omega\tau)$	$2\tau\,\mathrm{sinc}\,2f\tau$
$\mathrm{rect}(t)$	$\mathrm{sinc}(\frac{\omega}{2})$	$\mathrm{sinc}(f)$
$\lvert t\rvert$	$-\frac{2}{\omega^2}$	$-\frac{2}{(2\pi f)^2}$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$\frac{1}{\alpha+j2\pi f}$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$\frac{1}{\alpha-j2\pi f}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	$\frac{2a}{a^2+4\pi^2f^2}$
$e^{j\omega_o t}$	$2\pi\delta(\omega-\omega_0)$	$\delta(f-f_0)$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$\frac{n!}{(a+j2\pi f)^{n+1}}$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$\frac{\delta(f-f_o)-\delta(f+f_o)}{2j}$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	$\frac{\delta(f-f_o)+\delta(f+f_o)}{2}$
$e^{-at}\sin\omega_0 t\,u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$\frac{2\pi f_0}{(a+j2\pi f)^2+(2\pi f_0)^2}$
$e^{-at}\cos\omega_0 t\,u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$\frac{a+2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$

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TABLE 9: FOURIER TRANSFORM PROPERTIES

Property	**Time domain,** $x(t)$	**Frequency domain,** $X(\omega)$	**Frequency domain,** $X(f)$
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$	$a_1X_1(f)+a_2X_2(f)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
Time shifting	$x(t-t_0)u(t-t_0)$	$e^{-j\omega t_0}X(\omega)$	$e^{-j2\pi f t_0}X(f)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$	$X(f-f_o)$
Modulation	$\cos(\omega_0 t)\,x(t)$ $\sin(\omega_0 t)\,x(t)$	$\frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$ $\frac{1}{2j}[X(\omega-\omega_0)-X(\omega+\omega_0)]$	$\frac{1}{2}[X(f-f_o)+X(f+f_o)]$ $\frac{j}{2}[X(f+f_o)-X(f-f_o)]$
Time differentiation	$\frac{d}{dt}(x(t))$ $\frac{d^n}{dt^n}(x(t))$	$j\omega X(\omega)$ $(j\omega)^nX(\omega)$	$j2\pi fX(f)$ $(j2\pi f)^nX(f)$
Time integration	$\int_{-\infty}^{t}x(t)dt$	$\frac{X(\omega)}{j\omega}+\pi X(\omega)\,\delta(\omega)$	$\frac{X(f)}{j2\pi f}+\frac{1}{2}X(0)\delta(f)$
Frequency differentiation	$t^nx(t)$	$(j)^n\frac{d^n}{d\omega^n}X(\omega)$	$\left(\frac{j}{2\pi}\right)^n\frac{d^n}{df^n}X(f)$
Time Reversal	$x(-t)$	$X(-\omega)$ or $X^*(\omega)$	$X(-f)$
Duality	$X(t)$	$2\pi x(-\omega)$	$X(-f)$
Convolution in t	$x_1(t)*x_2(t)$	$X_1(\omega)\cdot X_2(\omega)$	$X(f)\cdot Y(f)$
Multiplication	$x_1(t)\cdot x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$	$X(f)*Y(f)$
Parseval's Theorem	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2d\omega$	$\int_{-\infty}^{\infty}\lvert X(f)\rvert^2df$

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TABLE 10: LAPLACE TRANSFORM PAIR

Time domain, $x(t)$, $t > 0$	s-domain, $X(s)$	ROC	Time domain, $x(t)$, $t > 0$	s-domain, $X(s)$	ROC
$\delta(t)$	1	All s	$\cos bt$	$\frac{s}{s^2+b^2}$	$Re(s) > 0$
$u(t)$	$\frac{1}{s}$	$Re(s) > 0$	$\sin bt$	$\frac{b}{s^2+b^2}$	$Re(s) > 0$
t	$\frac{1}{s^2}$	$Re(s) > 0$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$Re(s) > -a$
t^n	$\frac{n!}{s^{n+1}}$	$Re(s) > 0$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$Re(s) > -a$
e^{-at}	$\frac{1}{s+a}$	$Re(s) > -a$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$Re(s) > 0$
te^{-at}	$\frac{1}{(s+a)^2}$	$Re(s) > -a$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s) > 0$

TABLE 11: LAPLACE TRANSFORM PROPERTIES

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t), x_2(t)$	$X_1(s), X_2(s)$	R_1, R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s) \cdot X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
		$sX(s) - x(0^+)$ (Unilateral)	R right hand plane
	$\frac{d^n}{dt^n}x(t)$	$s^nX(s) - s^{n-1}x(0^+) - \cdots - sx^{n-2}(0^+) - x^{n-1}(0^+)$	
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{Re(s) > 0\}$

Initial- and Final- Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher order singularities at $t = 0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and has a finite limit as $t \to \infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to\infty} sX(s)$$