Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BEE 21503/ BWM 20403
PROGRAMME CODE	:	BEJ/ BEV
EXAMINATION DATE	:	DECEMBER 2017/ JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWERS **ALL** QUESTIONS

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THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Write the formula of partial derivatives $f_y(x,y)$ definition in terms of limit.

(2 marks)

(b) Suppose that $z = f(x,y)$ is a differentiable function of x and y. State the suitable differentiation technique, either by using partial derivatives, chain rule, or implicit differentiation, for the following functions:

(i) $z^2x + z\cos(x^2y) = 0$ to obtain $\dfrac{\partial z}{\partial y}$.

(ii) $z - 2x^3y^2 + \ln(x + y^{-2}) = 0$ where $x = e^{3m}$ and $y = 9m$ to obtain $\dfrac{dz}{dm}$.

(iii) $-y\cos(x^2y) + x\sin(x^2y) + z = 0$ to obtain $\dfrac{\partial z}{\partial x}$.

(3 marks)

(c) Solve for $\dfrac{\partial w}{\partial \rho}$ and $\dfrac{\partial w}{\partial \phi}$ by using chain rule, given $w = x^2 + y^2 + z^2$, $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$ and $z = \rho\cos\phi$.

(7 marks)

(d) If z is implicitly defined as a function of x and y, which is $\dfrac{y}{y+2} + e^{\ln z} = \ln\dfrac{3(x+4)}{\sqrt{x^2-16}}$, prove that $(x^2-16)\dfrac{\partial z}{dx} + (y+2)^2\dfrac{\partial z}{dy} = -6$.

(13 marks)

Q2 (a) Determine $\int_1^3 \int_0^{\frac{\pi}{4}} 2x\sec^2 y\,dy\,dx$.

(2 marks)

(b) By using double integrals,

(i) find the area of region, R enclosed by curves $y = \dfrac{1}{x^2}$, $y = -x^2$ and lines $x = 1$, $x = 2$.

(3 marks)

(ii) calculate the volume of the the solid enclosed by cylinder $x^2 + y^2 = 9$, planes $y + z = 4$ and $z = 0$.

(7 marks)

(c) By using spherical coordinates, evaluate the volume of the solid bounded below by cone $z = \sqrt{x^2 + y^2}$ and inside sphere $x^2 + y^2 + z^2 = 4z$.

(13 marks)

Q3 (a) With the aid of a suitable diagram,

(i) label the difference between a single integral and a line integral.

(2 marks)

(ii) distinguish the line integral between a scalar field and a vector field.

(3 marks)

(b) Sketch one example for each of the term below, and explain your selections.

(i) A scalar field

(3 marks)

(ii) A vector field

(4 marks)

(c) Analyse the work done by force field $\mathbf{F}(x, y) = x\mathbf{i} + y\mathbf{j}$ on a particle moves from point (0, 0) to (2, 4) along the following curves.

(i) $C_1 : y = 2x$

(5 marks)

(ii) $C_2 : y = x^2$

(5 marks)

(iii) Then, compare your results in (c) (i) and (c) (ii), and investigate the conservative theory of force fields.

(3 marks)

Q4 (a) State the Gauss' theorem and Stokes' theorem.

(2 marks)

(b) Use the Divergence theorem to evaluate the outward flux of vector field, $\mathbf{F}(x, y, z) = x^3\mathbf{i} + x^2y\mathbf{j} + xz\mathbf{k}$ across the surface of the solid bounded by $z = 4 - x^2$, $y + z = 5$, $z = 0$, and $y = 0$.

(9 marks)

(c) In 1831, the physicist Michael Faraday discovered that an electric current can be produced by varying the magnetic flux through a conducting loop. His experiments showed that the electromotive force **E** is related to the magnetic induction **B** by the equation

$$\oint_C \mathbf{E} \bullet dr = -\iint_S \frac{\partial \mathbf{B}}{dt} \bullet \mathbf{n} dS.$$

Use this result to make an interpretation about the relationship between curl **E** and **B** and explain your reasoning.

(2 marks)

(d) Consider the vector field given by $\mathbf{F}(x, y, z)=(x-z)\mathbf{i}+(y-x)\mathbf{j}+(z-xy)\mathbf{k}$. Use Stokes' theorem to find the circulation around the triangle with vertices A(1, 0, 0), B(0, 2, 0) and C(0, 0, 1) oriented counterclockwise looking from the origin toward the first octant.

(12 marks)

– END OF QUESTIONS –

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FORMULAS

Polar coordinate

$x = r\cos\theta,\ \ y = r\sin\theta,\ \ \theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate

$x = r\cos\theta,\ \ y = r\sin\theta,\ \ z = z$ and $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z) r\, dz\, dr\, d\theta$

Spherical coordinate

$x = \rho\sin\phi\cos\theta,\ \ y = \rho\sin\phi\sin\theta,\ \ z = \rho\cos\phi$, then $x^2+y^2+z^2=\rho^2$, for $0\le\theta\le 2\pi,\ 0\le\phi\le\pi$,

and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

$$A = \iint_R dA$$

$m = \iint_R \delta(x,y)dA$, where $\delta(x,y)$ is a density of lamina

$$V = \iint_R f(x,y)\,dA$$

$$V = \iiint_G dV$$

$$m = \iiint_G \delta(x,y,z)dV$$

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If f is a differentiable function of x, y and z, then the

Gradient of f, grad $f(x,y,z) = \nabla f(x,y,z) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

If $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is a vector field in Cartesian coordinate, then the

Divergence of $\mathbf{F}(x,y,z)$, div $\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

Curl of $\mathbf{F}(x,y,z)$, curl $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial Z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial Z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$

$\mathbf{F}$ is conservative vector field if Curl of $\mathbf{F} = 0$.

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Surface Integral

Let S be a surface with equation $z = g(x, y)$ and let R be its projection on the xy-plane.

$$\iint_S f(x,y,z)\,dS = \iint_R f(x,y,g(x,y))\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

Gauss's Theorem

$$\iint_S \mathbf{F}\bullet\mathbf{n}\,dS = \iiint_G \nabla\bullet\mathbf{F}\,dV$$

Stokes' Theorem

$$\iint_S (\nabla\times\mathbf{F})\bullet\mathbf{n}\,dS = \oint_C \mathbf{F}\bullet dr$$

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Identities of Trigonometry and Hyperbolic

Trigonometric Functions

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$

$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$

$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$

$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$

$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$

Hyperbolic Functions

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x\cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2\cosh^2 x - 1$

$= 1 + 2\sinh^2 x$

$1 - \tanh^2 x = \text{sech}^2 x$

$\coth^2 x - 1 = \text{csch}^2 x$

$\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$

$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

$\sinh(x \pm y) = \sinh x\cosh y \pm \cosh x\sinh y$

$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$

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The derivative of f(x) with respect to x

$$f_x(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Indefinite Integrals and Integration of Inverse Functions

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \qquad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x dx = \sinh x + C$$

$$\int \sinh x dx = \cosh x + C$$

$$\int \text{sech}^2 x dx = \tanh x + C$$

$$\int \text{csch}^2 x dx = -\coth x + C$$

$$\int \text{sech} x \tanh x dx = -\text{sech} x + C$$

$$\int \text{csch} x \coth x dx = -\text{csch} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \qquad x^2 < a^2$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C, \qquad x^2 < a^2$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{a^2+x^2} dx = \cot^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{|x|\sqrt{x^2-a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C, \qquad x^2 > a^2$$

$$\int \frac{-1}{|x|\sqrt{x^2-a^2}} dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C, \qquad x^2 > a^2$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \cosh^{-1}\left(\frac{x}{a}\right) + C, \qquad x > a > 0$$

$$\int \frac{-1}{|x|\sqrt{a^2-x^2}} dx = \frac{1}{a}\text{sech}^{-1}\left|\frac{x}{a}\right| + C, \qquad 0 < x < a$$

$$\int \frac{-1}{|x|\sqrt{a^2+x^2}} dx = \frac{1}{a}\text{csch}^{-1}\left|\frac{x}{a}\right| + C, \qquad x \neq 0$$

$$\int \frac{1}{a^2-x^2} dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & x^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & x^2 > a^2 \end{cases}$$