Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	BEE 11303 / BWM 10103
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	DECEMBER 2017 / JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) Evaluate the integration of the following functions:

(i) Using substitution method, find $\int 2x\left(x^2+1\right)^2 dx$

(4 marks)

(ii) Using tabular method, find $\int x^3 \sin x\, dx$

(5 marks)

(iii) Using integral by parts, find $\int x^2 \ln(5x) dx$

(8 marks)

(iv) $$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)} dx$$

(8 marks)

Q2 (a) Evaluate each of the following integrals.

(i) $$\int \frac{\sin^3\theta}{\cos^2\theta} d\theta$$

(4 marks)

(ii) $$\int \sin^2\theta \cos^4\theta\, d\theta$$

(7 marks)

(iii) $$\int \frac{1}{x^2\sqrt{1-x^2}} dx$$ (Hint: Use trigonometric substitution)

(8 marks)

(b) A 25 μF capacitor initially charged to –5 V, is charged by a constant current of 2.5 μA. Find the voltage across the capacitor after 60 seconds.

(Hint: The voltage across the capacitor is given by $v(t_2)-v(t_1)=\frac{1}{C}\int_{t_1}^{t_2} i(t)dt$)

(6 marks)

Q3 (a) Find the derivative of the inverse of each function below.

(i) $f(x)=3x-4$

(3 marks)

(ii) $f(x)=2x^2+3$

(4 marks)

(b) Prove that $\frac{-1}{\sqrt{1-x^2}}$ is the inverse function of $\cos x$. (Hint: Use $\cos^2 x + \sin^2 x = 1$).

(5 marks)

(c) Find the derivative of $y = \sin^{-1} 3x$

(3 marks)

(d) Differentiate the following expressions with respect to x.

(i) $y = \sinh^{-1}(3x+2)$

(5 marks)

(ii) $y = \cosh^{-1}\sqrt{1+x^2}$

(5 marks)

Q4 (a) Find the integration of the following functions with respect to x.

(i) $\int \frac{\cos x}{\sqrt{1+\sin^2 x}} dx$

(4 marks)

(ii) $\int \frac{3}{(x+1)\sqrt{2x^2+4x+6}} dx$

(6 marks)

(b) Evaluate the indicated integrals by trigonometric substitution.

(i) $\int \frac{dx}{x^2\sqrt{x^2+4}}$

(5 marks)

(ii) $\int \sqrt{2-x^2+4x}\, dx$

(5 marks)

(c) Evaluate $\int \frac{\sqrt{x^2-3}}{x^2} dx$ by hyperbolic substitution.

(5 marks)

– END OF QUESTIONS –

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Formulae

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \frac{2t}{1+t^2}$	$\cos x = \frac{1-t^2}{1+t^2}$	$\sin 2x = \frac{2t}{1+t^2}$	$\cos 2x = \frac{1-t^2}{1+t^2}$
$\tan x = \frac{2t}{1-t^2}$	$dx = \frac{2dt}{1+t^2}$	$\tan 2x = \frac{2t}{1-t^2}$	$dx = \frac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions	*Hyperbolic Functions*
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \frac{e^x - e^{-x}}{2}$
$\sin 2x = 2\sin x\cos x$	$\cosh x = \frac{e^x + e^{-x}}{2}$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$= 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x\cosh x$
$= 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$1 + \tan^2 x = \sec^2 x$	$= 2\cosh^2 x - 1$
$1 + \cot^2 x = \csc^2 x$	$= 1 + 2\sinh^2 x$
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$	$\coth^2 x - 1 = \operatorname{csch}^2 x$
$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$	$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$	$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$
$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$	$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$
$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$	$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$
$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$	

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Formulae

Indefinite Integrals	Integration of Inverse Functions
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C$
$\int \cos x \, dx = \sin x + C$	$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \frac{-1}{a^2 + x^2} dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$
$\int \sec^2 x \, dx = \tan x + C$	$\int \frac{1}{\lvert x\rvert\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc^2 x \, dx = -\cot x + C$	$\int \frac{-1}{\lvert x\rvert\sqrt{x^2 - a^2}} dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C$
$\int \sec x \tan x \, dx = \sec x + C$	$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc x \cot x \, dx = -\csc x + C$	$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1}\left(\frac{x}{a}\right) + C$
$\int e^x dx = e^x + C$	$\int \frac{-1}{\lvert x\rvert\sqrt{a^2 - x^2}} dx = \frac{1}{a}\operatorname{sech}^{-1}\left\lvert\frac{x}{a}\right\rvert + C$
$\int \cosh x \, dx = \sinh x + C$	$\int \frac{-1}{\lvert x\rvert\sqrt{a^2 + x^2}} dx = \frac{1}{a}\operatorname{csch}^{-1}\left\lvert\frac{x}{a}\right\rvert + C$
$\int \sinh x \, dx = \cosh x + C$	$\int \frac{1}{a^2 - x^2} dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert > a \end{cases}$
$\int \operatorname{sech}^2 x \, dx = \tanh x + C$	
$\int \operatorname{csch}^2 x \, dx = -\coth x + C$	
$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$	
$\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$	