UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

COURSE NAME	:	DIGITAL SIGNAL PROCESSING
COURSE CODE	:	BEF 35603
PROGRAMME CODE	:	BEV
EXAMINATION DATE	:	DECEMBER 2017 / JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS

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THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

Q1 (a) Briefly explain the process of :

i. Sampling

(2 marks)

ii. Quantization

(2 marks)

(b) Suppose an analog electrocardiogram (ECG) signal is sampled at a rate of 280 samples/s. Determine the highest frequency that can be represented uniquely at this sampling rate and analyze your reasoning for this.

(2 marks)

(c) A continuous voltage signal has the following function:

$$v(t) = 2\sin(80\pi t)\ volt$$

Analyze whether aliasing will occur if the signal is sampled at S = 70 Hz. Formulate the reconstructed signal $v_a(t)$.

(3 marks)

(d) Let the voltage signal $v(t) = 2\sin(80\pi t)$ is sampled with a sampling rate of 200 samples/s and quantized by using rounding technique with quantization interval of 1 volt,

i. Determine the digital frequency and digital period of the sampled signal.

(2 marks)

ii. Calculate the sampled signal $v[n]$ for the first period.

(4 marks)

iii. Calculate the quantized signal $v_q[n]$ for the first period.

(2 marks)

iv. Analyze the actual quantization signal to noise ratio SNR_Q for one period.

(8 marks)

Q2 (a) The Discreter Fourier Transform (DFT) of $x[n]$ is $X_{DFT}[k] = \{1, 2, 3, 4\}$. Determine $x[n]$, using Inverse Discrete Fourier Transform (IDFT).

(7 marks)

(b) Given $X_{DFT}[k]$ as in **Q2(a)**, analyse the Discreter Fourier Transform (DFT) of the following sequences, using the suitable properties of the DFT:

i. $y[n] = x[n+1]$

(5 marks)

ii. $z[n] = x[n] \otimes x[n]$

(3 marks)

(c) Analyze $x[n] \Leftrightarrow X_{DFT}[k] = \{\underset{\Downarrow}{14}, 2-2j, 2, 2+2j\}$, using Decimation in Frequency (DIF) Fast Fourier Transform algorithm (FFT).

Hint: Show that FFT can be used to obtain $x[n]$ from $X_{DFT}[k]$.

(10 marks)

Q3 (a) Determine the z-transform and specify its region of convergence for the following discrete signals:

i. $x[n] = \{2,\ 1,\ \underset{\Downarrow}{3},\ 0,\ 4\}$

(3 marks)

ii. $x[n] = 3^{(n+2)} u[n]$

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(3 marks)

(b) Assume that $x[n]$ represents a right-sided signal. Compute $x[n]$ of the following z-transform using partial fractions.

$$X(z) = \frac{3z^2}{(z^2 - 1.5z + 0.5)(z - 0.25)}$$

(5 marks)

(c) Given a causal system described by the difference equation $y[n] + 4y[n-1] + 4y[n-2] = 2x[n] + 3x[n-1]$, determine the transfer function of the system $H(z)$.

(4 marks)

(d) The transfer function of a system is $H(z) = \dfrac{4z}{(z-0.5)}$. Analyze its response $y[n]$ for the input $x[n] = u[n]$.

(10 marks)

Q4 (a) The analog lowpass filter $H(s)=\dfrac{2}{s^2+2s+2}$ has a cut-off frequency of 1 rad/s. Use this prototype to derive,

i. a low-pass filter with a passband edge of 100 Hz and a sampling frequency of 1 kHz.

(8 marks)

ii. a high-pass filter with a cut-off frequency of 500 Hz and sampling frequency of 2 kHz.

(8 marks)

(b) A notch filter is required to remove 50 Hz interference. Design such a filter, using the bilinear transformation and assuming a bandwidth of 4 Hz and a sampling rate of 300 Hz. (***Hint:*** start with a low-pass analog prototype: $H(s)=\frac{1}{s+1}$)

(9 marks)

– END OF QUESTIONS –

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Table 1 : Some useful identities

$e^{\pm jm\pi} = -1$	$m = 1, 3, 5, \ldots.$
$e^{\pm jm\pi} = 1$	$m = 2, 4, 6, \ldots.$
$e^{jm\pi/2} = j$	$m = 1, 5, 9, \ldots.$
$e^{jm\pi/2} = -j$	$m = 3, 7, 11, \ldots.$
$e^{-jm\pi/2} = -j$	$m = 1, 5, 9, \ldots.$
$e^{-jm\pi/2} = j$	$m = 3, 7, 11, \ldots.$

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Table 2: Properties of the Discrete Fourier Transform (DFT)

Signal x(n)	DFT X(k)
$a_1x_1(n)+a_2x_2(n)$	$a_1X_1(k)+a_2X_2(k)$
$x[n-n_o]$	$X_{DFT}[k]e^{-\frac{j2\pi k n_o}{N}}$
$x[n-0.5N]$	$(-1)^k X_{DFT}[k]$
$x[n]e^{j\frac{2\pi n k_0}{N}}$	$X_{DFT}[k-k_o]$
$(-1)^n x[n]$	$X_{DFT}[k-0.5N]$
$x[-n]$	$X_{DFT}[-k]$
$x[n]y[n]$	$\frac{1}{N}X_{DFT}[k]\otimes Y_{DFT}[k]$
$x[n]\otimes y[n]$	$X_{DFT}[k]Y_{DFT}[k]$
$x[n]\otimes\otimes y[n]$	$X_{DFT}[k]Y^*_{DFT}[k]$
$x[0]=\frac{1}{N}\sum_{k=0}^{N-1}X_{DFT}[k]$ $\quad X_{DFT}[0]=\sum_{n=0}^{N-1}x[n]$	
$x\left[\frac{N}{2}\right]=\frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (N even) $\quad X_{DFT}\left[\frac{N}{2}\right]=\sum_{n=0}^{N-1}(-1)^n X_{DFT}[k]$ (N even)	
$\sum_{n=0}^{N-1}\lvert x[n]\rvert^2=\frac{1}{N}\sum_{k=0}^{N-1}\lvert X_{DFT}[k]\rvert^2$	

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Table 3: Properties of the z- transform

Property	**Signal**	**z-transform**
Linearity	$a_1x_1[n]+a_2x_2[n]$	$a_1X_1(z)+a_2X_2(z)$
Time reversal	$x[-n]$	$X(z^{-1})$
Time shifting	i) $x(n-k)$ ii) $x(n+k)$	i) $z^{-k}X(z)$ ii) $z^{k}X(z)$
Convolution	$x_1(n)*x_2(n)$	$X_1(z)X_2(z)$
Correlation	$r_{x_1x_2}(l)=\sum_{n=-\infty}^{\infty}x_1(n)x_2(n-l)$	$R_{x1x2}(z)=X_1(z)X_2(z^{-l})$
Scaling	$a^nx(n)$	$X(a^{-1}z)$
Differentiation	$nx[n]$	$z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$
Time differentiation	$x[n]-x[n-1]$	$X(z)(1-z^{-1})$
Time integration	$\sum_{k=0}^{\infty}X(k)$	$X(z)=\left(\frac{z}{z-1}\right)$
Initial value theorem	$\lim_{n\to 0}x(n)$	$\lim_{\lvert z\rvert\to\infty}X(z)$
Final value theorem	$\lim_{n\to\infty}x(n)$	$\lim_{\lvert z\rvert\to 1}\left(\frac{z-1}{z}\right)X(z)$

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Table 4: Laplace Transform Pairs

Signal $x(t)$	Laplace Transform $X(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$r(t) = tu(t)$	$\frac{1}{s^2}$
$t^2u(t)$	$\frac{2}{s^3}$
$t^nu(t)$	$\frac{n!}{s^{n+1}}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$
$t^ne^{-\alpha t}u(t)$	$\frac{n!}{(s+\alpha)^{n+1}}$

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Table 5: z-Transform Pairs

Signal $x(t)$	Sequence $x(n)$	z-Transform $X(z)$
$\delta(t)$	$\delta(n)$	1
$\delta(t-k)$	$\delta(n-k)$	z^{-k}
$u(t)$	$u(n)$	$\frac{1}{1-z^{-1}}=\frac{z}{z-1}$
	$-u(-n-1)$	$\frac{1}{1-z^{-1}}=\frac{z}{z-1}$
$r(t)=tu(t)$	$nu(n)$	$\frac{z^{-1}}{(1-z^{-1})^2}=\frac{z}{(z-1)^2}$
	$a^n u(n)$	$\frac{1}{1-az^{-1}}=\frac{z}{z-a}$
	$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}=\frac{z}{z-a}$
	$na^n u(n)$	$\frac{az}{(z-a)^2}$
	$-na^n u(-n-1)$	$\frac{az}{(z-a)^2}$
e^{-at}	e^{-an}	$\frac{1}{1-e^{-a}z^{-1}}=\frac{z}{z-e^{-a}}$
t^2	$n^2u(n)$	$z^{-1}\frac{(1+z^{-1})}{(1-z^{-1})^3}=\frac{z(z+1)}{(z-1)^3}$
te^{-at}	ne^{-an}	$\frac{z^{-1}e^{-a}}{(1-e^{-a}z^{-1})^2}=\frac{ze^{-a}}{(z-e^{-a})^2}$
$\sin\omega_o t$	$\sin\omega_o n$	$\frac{z\sin\omega_o}{z^2-2z\cos\omega_o+1}$
$\cos\omega_o t$	$\cos\omega_o n$	$\frac{z(z-\cos\omega_o)}{z^2-2z\cos\omega_o+1}$

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Table 6: Digital- to- digital Transformations

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-\alpha}{1-\alpha z}$	$\alpha = \frac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\frac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \frac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\frac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \frac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha K}{K+1}, \quad A_2 = \frac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\frac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha}{K+1}, \quad A_2 = \frac{1-K}{1+K}$

Note: The digital lowpass prototype cutoff frequency is Ω_D

All digital frequencies are normalized to $\Omega = 2\pi f/S$

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Table 7: Direct Analog- to- digital Transformations for Bilinear Design

From	Band Edges	Mapping s →	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-1}{C(z+1)}$	$C=\tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\frac{C(z+1)}{z-1}$	$C=\tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1<\Omega_0<\Omega_2$	$\frac{z^2-2\beta z+1}{C(z^2-1)}$	$C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$
Lowpass to bandstop	$\Omega_1<\Omega_0<\Omega_2$	$\frac{C(z^2-1)}{z^2-2\beta z+1}$	$C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$

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