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FINAL EXAMINATION
SEMESTER II
SESSION 2017/2018

COURSE NAME	:	SIGNALS AND SYSTEMS
COURSE CODE	:	BEB 20203
PROGRAMME CODE	:	BEJ
EXAMINATION DATE	:	JUNE/JULY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A: ANSWER **ALL** QUESTIONS SECTION B: ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **FOURTEEN (14)** PAGES

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SECTION A: ANSWER ALL QUESTIONS

Q1. Given signal $x(t)$ as in **Figure Q1(a).** The signal will be used as the input for the system in **Figure Q1(b).** Sketch:

(i) $y(t)$

(4 marks)

(ii) $y_1(t)$

(3 marks)

(iii) $z(t)$

(3 marks)

Q2 A periodic signal $v_i(t)$ is expressed in trigonometric from of Fourier series as the following

$$v_i(t) = \frac{1}{2} + \sum_{\substack{n=1 \\ n=odd}}^{5} \frac{2\pi}{n} \sin(2\pi n(100k)t)$$

(a) Determine fundamental frequency of signal $v_i(t)$, f_0 (Hz)

(1 mark)

(b) Draw the amplitude and the phase spectrum of $v_i(t)$

(6 marks)

(c) If $v_i(t)$ becomes an input to a low pass filter as shown in **Figure Q2(c),** which has the cut-off frequency, $fcut = 350kHz$. Draw the amplitude and phase spectrum of the output, $v_o(t)$.

(3 marks)

Q3 (a) Find the Fourier transform of the signal, $x(t) = e^{-|t|-3}$

(4 marks)

(b) Given the signal, $w(t) = u(t+2) - 3u(t) + 2u(t-2)$

(i) Sketch $f(t) = w(-t)$

(2 marks)

(ii) Determine $F(\omega)$.

(4 marks)

Q4. Given the signal $x(t) = 3e^{-2t}u(t-1)$.

(a) Find the Laplace transform of $x(t)$.

(6 marks)

(b) Sketch the zero-pole plot and region of convergence (if it exists) of the signal $x(t)$.

(4 marks)

SECTION B: ANSWER THREE (3) QUESTIONS ONLY

Q5 An LTI system consist of 3 sub-LTI systems as shown in **Figure Q5**. Impulse responses of sub-LTI systems are expressed as the following:

$$h_1(t) = 2u(t+1)$$
$$h_2(t) = -2u(t-1)$$
$$h_3(t) = u(t+1) - u(t-1)$$

(a) Using the graphical approach of solving the convolution integral, prove that the total response of the system is equal to

$$h(t) = \begin{cases} 0, & t < -2 \\ 2t+4, & -2 \leq t < 0 \\ -2t+4, & 0 \leq t < 2 \\ 0, & t \geq 2 \end{cases}$$

(10 marks)

(b) Draw graph of the total impulse response of the system.

(2 marks)

(c) The input of the system is $x(t) = u(t)$. Using the graphical approach of solving convolution integral, find output of the system, $y(t)$.

(8 marks)

Q6 A periodic signal, $v_i(t)$ is used as an input to a system as shown in **Figure Q6**. Signal $v_i(t)$ is equal to:

$$v_i(t) = 5 + \sin \omega_0 t + 2\cos \omega_0 + 8\cos\left(2\,\omega_0 t + \frac{\pi}{2}\right).$$

(a) Represent v_i (t) in complex exponential Fourier series.

(4 marks)

(b) Determine the voltage across resistor R, $v_R(t)$ in complex exponential Fourier series.

(3 marks)

(c) Sketch the double sided line spectrum (magnitude and phase) of signal $v_R(t)$.

(6 marks)

(d) Find average power, P_{av} for $v_i(t)$.

(3 marks)

(e) Find average power, P_{av} for $v_R(t)$.

(3 marks)

(f) Compare average power for both signal.

(1 mark)

Q7 (a) A second order differential equation is described as

$$\frac{dy^2(t)}{dt} + 4\frac{dy(t)}{dt} + 3y(t) = x(t)$$

By using the differentiation properties of the Fourier transform, calculate the output $y(t)$ if the input signal $x(t) = e^{-t}u(t)$.

(8 marks)

(b) A basic modulator circuit is shown in **Figure Q7(b).** Modulation is a multiplication between message signal, $m(t)$, and a carrier signal, $c(t)$. The process yields a new signal, $v(t)$.

(i) Analyze the Fourier Transform of signal $v(t)$ by using modulation properties.

(5 marks)

(ii) Sketch the spectrum signal of $V(\omega)$.

(2 marks)

(c) The voltage across a 2-Ω resistor of an RC circuit in **Figure Q7(c)** is given by $v_R(t) = 10\, e^{-2t}u(t)\ V$. Determine the total energy dissipated by this resistor using Parseval's Relation.

(5 marks)

Q8 (a) **Figure Q8(a)** shows a block diagram of a feedback system.

(i) Show that the overall system function is

$$H(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)}$$

(3 marks)

(ii) Determine the H(s) if the systems transfer function are

$$H_1(s) = \frac{1}{s+2}, \qquad H_2(s) = \frac{s+2}{s-2}.$$

(2 marks)

(iii) Sketch the zero-pole plot of the system.

(2 marks)

(b) The region of convergence (ROC) of the feedback system in **Q8(a)** above is unknown. Determine all possible impulse responses of the feedback system by looking at the stability and causality of the response. Show the region of convergences (ROCs) for all cases.

(8 marks)

(c) Determine the system output, $y(t)$ if the input, $x(t) = u(t)$.

(5 marks)

-END OF QUESTIONS-

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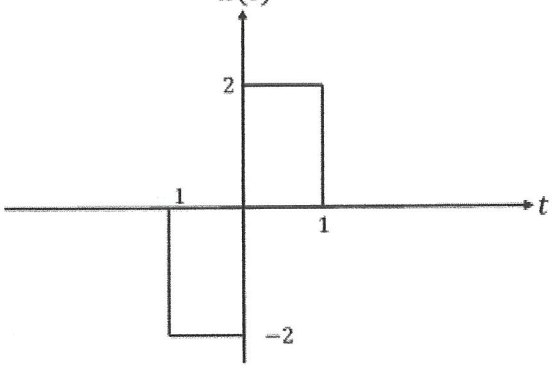

Figure Q1(a)

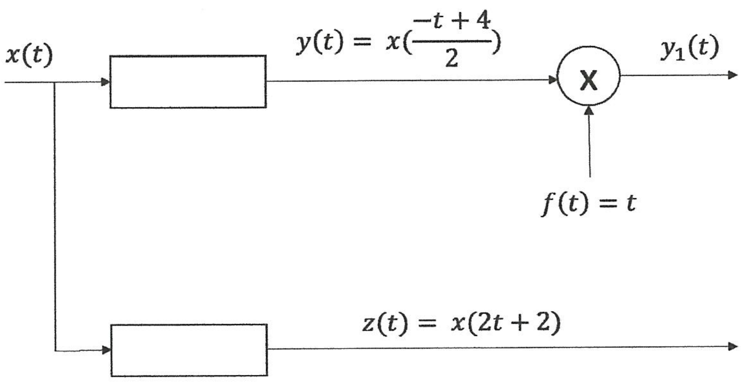

Figure Q1(b)

Low-pass filter

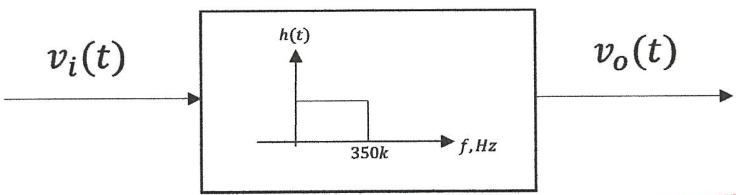

Figure Q2(c)

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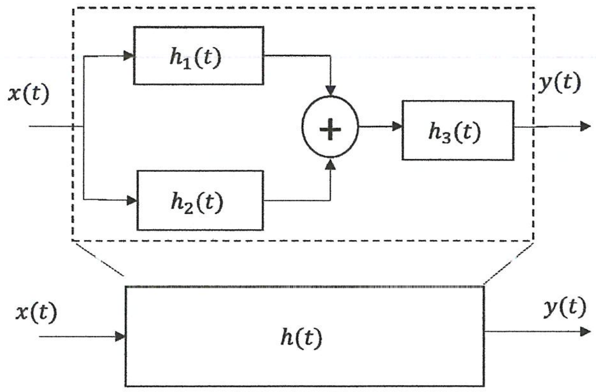

Figure Q5

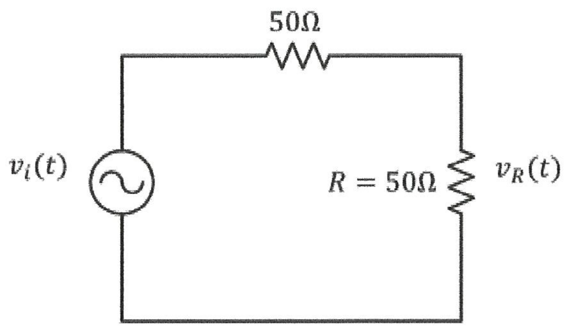

Figure Q6

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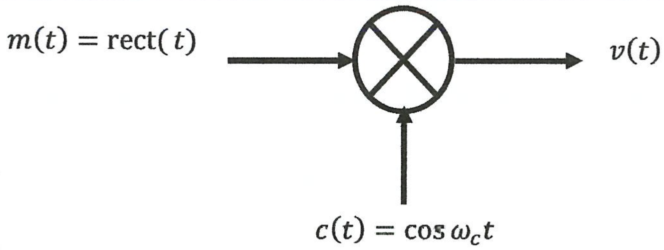

Figure Q7(b)

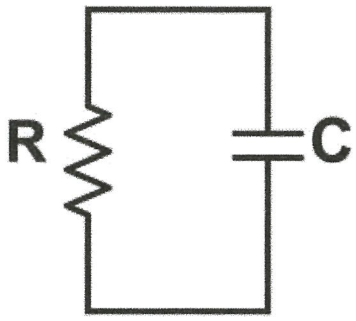

Figure Q7(c)

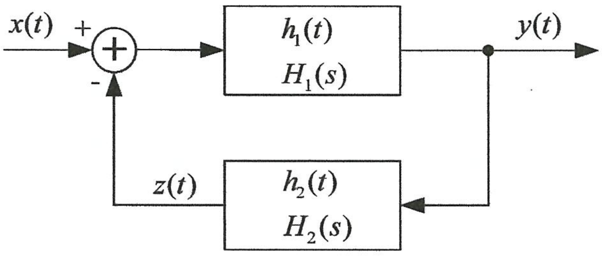

Figure Q8(a)

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TABLE 1: INDEFINITE INTEGRALS

$\int \cos at\, dt = \frac{1}{a}\sin at$	$\int \sin at\, dt = -\frac{1}{a}\cos at$
$\int t\cos at\, dt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at$	$\int t\sin at\, dt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at$
$\int te^{at}dt = \frac{1}{a^2}e^{at}(at-1)$	$\int \frac{1}{(a^2+t^2)}dt = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)$

TABLE 2: EULER'S IDENTITY

$e^{\pm j\pi/2} = \pm j$	$A\angle \pm\theta = Ae^{\pm j\theta}$
$e^{\pm jk\pi} = \cos k\pi$	$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$
$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$	$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$

TABLE 3: COMPLEX NUMBER

$s = a + jb = \|s\|\angle \pm\theta = \|s\|e^{\pm j\theta}$	$\|s\| = \sqrt[2]{a^2+b^2}$	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

TABLE 4: TRIGONOMETRIC IDENTITIES

$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right))$	$\cos\theta = \sin(\theta + \frac{\pi}{2})$
$\sin(\propto \pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin^2\alpha + \cos^2\beta = 1$	
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 5: VALUES OF COSINE, SINE AND EXPONENTIAL FUNCTIONS FOR INTEGRAL MULTIPLE OF π.

Function	Value	Function	Value
$\cos 2n\pi$	1	$e^{j2n\pi}$	1
$\sin 2n\pi$	0	$e^{jn\pi}$	$(-1)^n$
$\cos n\pi$	$(-1)^n$	$e^{\frac{jn\pi}{2}}$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = \text{even} \\ j(-1)^{\frac{n-1}{2}} & ,n = \text{odd} \end{cases}$
$\sin n\pi$	0		
$\cos\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = \text{even} \\ 0 & ,n = \text{odd} \end{cases}$	$\sin\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n-1}{2}} & ,n = \text{odd} \\ 0 & ,n = \text{even} \end{cases}$

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TABLE 6: FOURIER SERIES

Exponential	$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$ $x_n = \frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t} dt$
Trigonometric	$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\frac{2\pi}{T}t + b_n \sin n\frac{2\pi}{T}t$ $a_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t) \cos n\frac{2\pi}{T}t\, dt, \quad n = 0, 1, 2, 3 \ldots$ $b_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t) \sin n\frac{2\pi}{T}t\, dt, \quad n = 1, 2, 3 \ldots$
Amplitude-phase	$x(t) = X_0 + \sum_{n=1}^{\infty} A_n \cos(n\frac{2\pi}{T}t + \theta_n)$ $A_n = 2\lvert X_n \rvert = \sqrt[2]{a_n^2 + b_n^2} \quad , \quad \theta_n = \angle X_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$
Average Power	$P = V_{dc}I_{dc} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_{V_n} - \theta_{I_n})$

TABLE 7: DEFINITION OF FOURIER AND LAPLACE TRANSFORM

FOURIER TRANSFORM	INVERSE FOURIER TRANSFORM
$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$ $\mathcal{F}[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}\, dt$	$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$ $x(t) = \mathcal{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)e^{j2\pi f t}\, df$
LAPLACE TRANSFORM **Bilateral** $\mathrm{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$ **Unilateral** $\mathrm{L}[x(t)] = X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$ $s = \sigma + j\omega$	**INVERSE LAPLACE TRANSFORM** $x(t) = \mathrm{L}^{-1}[X(s)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$

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TABLE 8: FOURIER TRANSFORM PAIRS

Time domain, $x(t)$	**Frequency domain,** $X(\omega)$	**Frequency domain,** $X(f)$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	$\frac{\delta(f)}{2}+\frac{1}{j2\pi f}$
$u(t+\tau)-u(t-\tau)$	$\frac{2\sin(\omega\tau)}{\omega}=2\tau\,\mathrm{sinc}(\omega\tau)$	$2\tau\,\mathrm{sinc}\,2f\tau$
$\mathrm{rect}(t)$	$\mathrm{sinc}(\frac{\omega}{2})$	$\mathrm{sinc}(f)$
$\|t\|$	$-\frac{2}{\omega^2}$	$-\frac{2}{(2\pi f)^2}$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$\frac{1}{a+j2\pi f}$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$\frac{1}{a-j2\pi f}$
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$\frac{2a}{a^2+4\pi^2f^2}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$\delta(f-f_0)$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$\frac{n!}{(a+j2\pi f)^{n+1}}$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$\frac{\delta(f-f_o)-\delta(f+f_o)}{2j}$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	$\frac{\delta(f-f_o)+\delta(f+f_o)}{2}$
$e^{-at}\sin\omega_0 t\,u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$\frac{2\pi f_0}{(a+j2\pi f)^2+(2\pi f_0)^2}$
$e^{-at}\cos\omega_0 t\,u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$\frac{a+2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$

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TABLE 9: FOURIER TRANSFORM PROPERTIES

Property	**Time domain,** $x(t)$	**Frequency domain,** $X(\omega)$	**Frequency domain,** $X(f)$
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$	$a_1X_1(f)+a_2X_2(f)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
Time shifting	$x(t-t_0)u(t-t_0)$	$e^{\ j\omega t_0}X(\omega)$	$e^{\ j2\pi f t_0}X(f)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$	$X(f-f_o)$
Modulation	$\cos(\omega_0 t)\,x(t)$ $\sin(\omega_0 t)\,x(t)$	$\frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$ $\frac{1}{2j}[X(\omega-\omega_0)-X(\omega+\omega_0)]$	$\frac{1}{2}[X(f-f_o)+X(f+f_o)]$ $\frac{j}{2}[X(f+f_o)-X(f-f_o)]$
Time differentiation	$\frac{d}{dt}(x(t))$ $\frac{d^n}{dt^n}(x(t))$	$j\omega X(\omega)$ $(j\omega)^nX(\omega)$	$j2\pi fX(f)$ $(j2\pi f)^nX(f)$
Time integration	$\int_{-\infty}^{t}x(t)dt$	$\frac{X(\omega)}{j\omega}+\pi X(\omega)\,\delta(\omega)$	$\frac{X(f)}{j2\pi f}+\frac{1}{2}X(0)\delta(f)$
Frequency differentiation	$t^nx(t)$	$(j)^n\frac{d^n}{d\omega^n}X(\omega)$	$\left(\frac{j}{2\pi}\right)^n\frac{d^n}{df^n}X(f)$
Time Reversal	$x(-t)$	$X(-\omega)\ \ or\ X^*(\omega)$	$X(-f)$
Duality	$X(t)$	$2\pi x(-\omega)$	$X(-f)$
Convolution in t	$x_1(t)*x_2(t)$	$X_1(\omega)\cdot X_2(\omega)$	$X(f)\cdot Y(f)$
Multiplication	$x_1(t)\cdot x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$	$X(f)*Y(f)$
Parseval's Theorem	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2d\omega$	$\int_{-\infty}^{\infty}\lvert X(f)\rvert^2df$

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TABLE 10: LAPLACE TRANSFORM PAIR

Time domain, $x(t), t > 0$	s-domain, $X(s)$	ROC	Time domain, $x(t), t > 0$	s-domain, $X(s)$	ROC
$\delta(t)$	1	All s	$\cos bt$	$\frac{s}{s^2+b^2}$	$Re(s) > 0$
$u(t)$	$\frac{1}{s}$	$Re(s) > 0$	$\sin bt$	$\frac{b}{s^2+b^2}$	$Re(s) > 0$
t	$\frac{1}{s^2}$	$Re(s) > 0$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$Re(s) > -a$
t^n	$\frac{n!}{s^{n+1}}$	$Re(s) > 0$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$Re(s) > -a$
e^{-at}	$\frac{1}{s+a}$	$Re(s) > -a$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$Re(s) > 0$
te^{-at}	$\frac{1}{(s+a)^2}$	$Re(s) > -a$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s) > 0$

TABLE 11: LAPLACE TRANSFORM PROPERTIES

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t), x_2(t)$	$X_1(s), X_2(s)$	R_1, R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0t}x(t)$	$X(s-s_0)$	Shifted version of R (i.e., s is in the ROC if $s-s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s) \cdot X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
		$sX(s) - x(0^+)$ (Unilateral)	R right hand plane
	$\frac{d^n}{dt^n}x(t)$	$s^nX(s) - s^{n-1}x(0^+) - \cdots - sx^{n-2}(0^+) - x^{n-1}(0^+)$	
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{Re(s) > 0\}$

Initial- and Final- Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher order singularities at $t = 0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and has a finite limit as $t \to \infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to\infty} sX(s)$$