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# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER I
## SESSION 2017/2018

| | |
|---|---|
| COURSE NAME | : DIGITAL SIGNAL PROCESSING |
| COURSE CODE | : BEB 30503 |
| PROGRAMME CODE | : BEJ / BEV |
| EXAMINATION DATE | : DECEMBER 2017 / JANUARY 2018 |
| DURATION | : 3 HOURS |
| INSTRUCTION | : ANSWER **ALL** QUESTIONS |

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THIS QUESTION PAPER CONSISTS OF **THIRTEEN (13)** PAGES

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**Q1** (a) Given a system as shown in **Figure Q1(a)** with the impulse response, $h[n]=\{\overset{\Downarrow}{1},2,1,4\}$ and $g[n]=\{\overset{\Downarrow}{1},3,5\}$.

(i) Derive a mathematical equation of the given system.

(1 mark)

(ii) Analyze the given system for $x[n]=\{\overset{\Downarrow}{1},2,3\}$ to confirm that the system can be simplified as:

$$z[n]=3[x[n]*(h[n]+g[n])]$$

(12 marks)

(b) A signal, $x[n]=\{\overset{\Downarrow}{1},-3,4,-2\}$ is measured by a digital oscilloscope.

(i) Sketch the measured signal.

(2 marks)

(ii) Consider two sets of operations as the followings:

x[n] → [decimate by 2] → x[2n] → [interpolate by 2] → x[n]

x[n] → [interpolate by 2] → x[n/2] → [decimate by 2] → x[n]

Prove that the second set of operations will recovered the $x[n]$ exactly.

(5 marks)

**Q2** (a) With the aid of a diagram, explain the relation between actual and aliased frequency in sampling process.

(3 marks)

(b) A sinusoidal signal, $g(t) = 2.5\sin(800\pi t)$ V is passed to an analog to digital converter (ADC). The sampling frequency of 20% above the Nyquist Rate is used. It will be encoded with 4 bits per sample using offset binary representation and the dynamic range is ±3V. Compare the encoded signal if the sampling frequency of 20% and 25% above the Nyquist Rate using the truncation techniques for $-1 < n < 4$.

(17 marks)

**Q3** (a) Elaborate a difference between even and odd conjugate symmetry of a $N$ samples signal where $N$ is an integer value.

(2 marks)

(b) Analyze the following Discrete Fourier Transform (DFT) by proving that there is a suitable property of DFT to relate with $x[n] = \{\overset{\Downarrow}{4},0,1\}$ and $X[k] = \{\overset{\Downarrow}{5}, 3.5 + j0.866, 3.5 - j0.866\}$.

(i) $Y_{DFT}[k] = \{\overset{\Downarrow}{5}, 3.5 - j0.866, 3.5 + j0.866\}$

(4 marks)

(ii) $G_{DFT}[k] = \{\overset{\Downarrow}{17}, 15.5 + j0.866, 15.5 - j0.866\}$

(4 marks)

Hints :

$$e^{j\frac{2\pi}{3}} = e^{j\frac{8\pi}{3}} = -0.5 + j0.866$$

$$e^{j\frac{4\pi}{3}} = -0.5 - j0.866$$

(c) **Figure Q3(c)** shows twiddle factors and values at intermediate nodes of 4-point ABC algorithm.

(i) State the name of ABC algorithm.

(2 marks)

(ii) Compute the discrete signal $c[n]$ and the DFT signal $C_{DFT}[k]$.

(8 marks)

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**Q4** (a) Based on **Figure Q1(a)**, calculate the signal, $z[n]$ by applying the z-transform approach. The simplification of the system is not required. All the value of $h[n], g[n]$ and $x[n]$ are based on question **Q1**.

(12 marks)

(b) Deterime the inverse z-transform of $X(z) = \frac{z^3}{(z-0.3)(z-0.5)^2}$ using the partial fraction expansion method.

Hints:

$$H(z) = \frac{N(z)}{(z-p_1)(z-p_2)...(z-p_n)} = \frac{A_1}{(z-p_1)} + \frac{A_2}{(z-p_2)} + .. + \frac{A_n}{(z-p_n)}$$

$$A_i = (z-p_i)H(z)\big|_{z=p_i}$$

$$\frac{A_{i1}}{(z-p_i)} + \frac{A_{i2}}{(z-p_i)^2} + .. + \frac{A_{ir}}{(z-p_i)^r}$$

$$A_{ir} = (z-p_i)^r H(z)\big|_{z=p_i}$$

$$A_{i(r-1)} = \frac{d}{dz}(z-p_i)^r H(z)\big|_{z=p_i}$$

$$A_{i(r-k)} = \frac{1}{k!}\frac{d^k}{dz^k}(z-p_i)^r H(z)\big|_{z=p_i}$$



(8 marks)

**Q5** (a) Design a finite impulse response (FIR) highpass filter having a cut-off frequency of 5 kHz with the sample length of 9 for operating at the sampling frequency of 20 kHz using cosinus window.

(13 marks)

(b) Given a differential equation of a filter as:

$$y[n] = 0.5x[n] + 0.5x[n+2] - y[n-1]$$

(i) Prove that this differential equation is referring to an IIR filter

(4 marks)

(ii) Analyze this filter system using an input signal of $x[n] = \{\overset{\Downarrow}{2}, 3, -2\}$ and $y[-1] = 0$.

(3 marks)

**-END OF QUESTIONS -**

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SEMESTER/SESSION : SEM I / 2017/2018 PROGRAMME CODE : BEJ / BEV

COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEB 30503

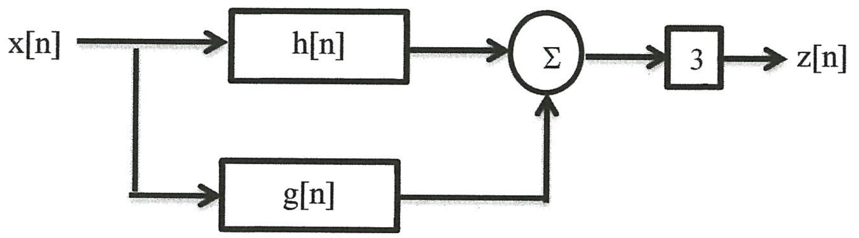


**Figure Q1(a)**



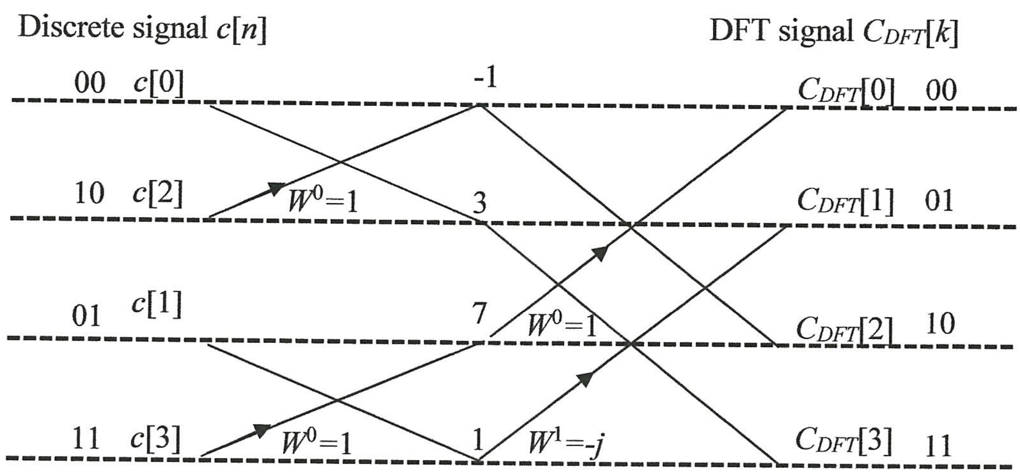


**Figure Q3(c)**

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SEMESTER/SESSION : SEM I / 2017/2018 PROGRAMME CODE : BEJ / BEV
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEB 30503

**Table 1: Properties of the Discrete Fourier Transform (DFT)**

| **Property** | **Signal** | **DFT** | **Remarks** |
|---|---|---|---|
| Shift | $x[n - n_o]$ | $X_{DFT}[k]e^{-j2\pi kn_o/N}$ | No change in magnitude. |
| Shift | $x[n-0.5N]$ | $(-1)^k X_{DFT}[k]$ | Half-period shift for even $N$. |
| Modulation | $x[n]e^{j2\pi nk_o/N}$ | $X_{DFT}[k-k_o]$ | |
| Modulation | $(-1)^n x[n]$ | $X_{DFT}[k-0.5N]$ | Half-period shift for even $N$. |
| Folding | $x[-n]$ | $X_{DFT}[-k]$ | This is circular folding. |
| Product | $x[n]y[n]$ | $\frac{1}{N}X_{DFT}[k]\otimes Y_{DFT}[k]$ | The convolution is periodic. |
| Convolution | $x[n]\otimes y[n]$ | $X_{DFT}[k]Y_{DFT}[k]$ | The convolution is periodic. |
| Correlation | $x[n]\otimes\otimes y[n]$ | $X_{DFT}[k]Y^*_{DFT}[k]$ | The correlation is periodic. |
| Central Ordinates | $x[0]=\frac{1}{N}\sum_{k=0}^{N-1}X_{DFT}[k]$, $X_{DFT}[0]=\sum_{n=0}^{N-1}x[n]$ | | |
| Central Ordinates | $x\left[\frac{N}{2}\right]=\frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ ($N$ even), $X_{DFT}\left[\frac{N}{2}\right]=\sum_{n=0}^{N-1}(-1)^n x[n]$ ($N$ even) | | |
| Parseval's Relation | $\sum_{n=0}^{N-1}\lvert x[n]\rvert^2=\frac{1}{N}\sum_{k=0}^{N-1}\lvert X_{DFT}[k]\rvert^2$ | | |

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SEMESTER/SESSION : SEM I / 2017/2018 PROGRAMME CODE : BEJ / BEV

COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEB 30503

**Table 2: Properties of the z- transform**

| **Property** | **Signal** | **z-transform** |
|---|---|---|
| Linearity | $a_1x_1[n]+a_2x_2[n]$ | $a_1X_1(z)+a_2X_2(z)$ |
| Time reversal | $x[-n]$ | $X(z^{-1})$ |
| Time shifting | i) $x(n-k)$<br>ii) $x(n+k)$ | i) $z^{-k}X(z)$<br>ii) $z^{k}X(z)$ |
| Convolution | $x_1(n)*x_2(n)$ | $X_1(z)X_2(z)$ |
| Correlation | $r_{x_1x_2}(l)=\sum_{n=-\infty}^{\infty}x_1(n)x_2(n-l)$ | $R_{x1x2}(z)=X_1(z)X_2(z^{-l})$ |
| Scaling | $a^nx(n)$ | $X(a^{-1}z)$ |
| Differentiation | $nx[n]$ | $z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$ |
| Time differentiation | $x[n]-x[n-1]$ | $X(z)(1-z^{-1})$ |
| Time integration | $\sum_{k=0}^{\infty}X(k)$ | $X(z)=\left(\frac{z}{z-1}\right)$ |
| Initial value theorem | $\lim_{n\to 0}x(n)$ | $\lim_{\lvert z\rvert\to\infty}X(z)$ |
| Final value theorem | $\lim_{n\to\infty}x(n)$ | $\lim_{\lvert z\rvert\to 1}\left(\frac{z-1}{z}\right)X(z)$ |

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SEMESTER/SESSION : SEM I / 2017/2018 PROGRAMME CODE : BEJ / BEV

COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEB 30503

**Table 3: z-Transform Pairs**

| Signal $x(t)$ | Sequence $x(n)$ | z-Transform $X(z)$ |
|---|---|---|
| $\delta(t)$ | $\delta(n)$ | $1$ |
| $\delta(t-k)$ | $\delta(n-k)$ | $z^{-k}$ |
| $u(t)$ | $u(n)$ | $\frac{1}{1-z^{-1}}=\frac{z}{z-1}$ |
| | $-u(-n-1)$ | $\frac{1}{1-z^{-1}}=\frac{z}{z-1}$ |
| $r(t)=tu(t)$ | $nu(n)$ | $\frac{z^{-1}}{(1-z^{-1})^2}=\frac{z}{(z-1)^2}$ |
| | $a^n u(n)$ | $\frac{1}{1-az^{-1}}=\frac{z}{z-a}$ |
| | $-a^n u(-n-1)$ | $\frac{1}{1-az^{-1}}=\frac{z}{z-a}$ |
| | $na^n u(n)$ | $\frac{az}{(z-a)^2}$ |
| | $-na^n u(-n-1)$ | $\frac{az}{(z-a)^2}$ |
| $e^{-at}$ | $e^{-an}$ | $\frac{1}{1-e^{-a}z^{-1}}=\frac{z}{z-e^{-a}}$ |
| $t^2$ | $n^2u(n)$ | $z^{-1}\frac{(1+z^{-1})}{(1-z^{-1})^3}=\frac{z(z+1)}{(z-1)^3}$ |
| $te^{-at}$ | $ne^{-an}$ | $\frac{z^{-1}e^{-a}}{(1-e^{-a}z^{-1})^2}=\frac{ze^{-a}}{(z-e^{-a})^2}$ |
| $\sin\omega_o t$ | $\sin\omega_o n$ | $\frac{z\sin\omega_o}{z^2-2z\cos\omega_o+1}$ |
| $\cos\omega_o t$ | $\cos\omega_o n$ | $\frac{z(z-\cos\omega_o)}{z^2-2z\cos\omega_o+1}$ |

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**FINAL EXAMINATION**

SEMESTER/SESSION : SEM I / 2017/2018 PROGRAMME CODE : BEJ / BEV

COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEB 30503

**Table 4: Digital- to- digital Transformations**

| Form | Band Edges | Mapping $s \rightarrow$ | Parameters |
|---|---|---|---|
| Lowpass to lowpass | $\Omega_C$ | $\frac{z-\alpha}{1-\alpha z}$ | $\alpha = \frac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$ |
| Lowpass to highpass | $\Omega_C$ | $\frac{-(z+\alpha)}{1+\alpha z}$ | $\alpha = \frac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$ |
| Lowpass to bandpass | $[\Omega_1, \Omega_2]$ | $\frac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$ | $K = \frac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$<br>$\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$<br>$A_1 = \frac{2\alpha K}{K+1}, \; A_2 = \frac{K-1}{K+1}$ |
| Lowpass to bandstop | $[\Omega_1, \Omega_2]$ | $\frac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$ | $K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$<br>$\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$<br>$A_1 = \frac{2\alpha}{K+1}, \; A_2 = \frac{1-K}{1+K}$ |

**FINAL EXAMINATION**

SEMESTER/SESSION : SEM I / 2017/2018 PROGRAMME CODE : BEJ / BEV
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEB 30503

**Table 5: Direct Analog- to- digital Transformations for Bilinear Design**

| From | Band Edges | Mapping s → | Parameters |
|---|---|---|---|
| Lowpass to lowpass | $\Omega_C$ | $\frac{z-1}{C(z+1)}$ | $C=\tan(0.5\Omega_C)$ |
| Lowpass to highpass | $\Omega_C$ | $\frac{C(z+1)}{z-1}$ | $C=\tan(0.5\Omega_C)$ |
| Lowpass to bandpass | $\Omega_1<\Omega_0<\Omega_2$ | $\frac{z^2-2\beta z+1}{C(z^2-1)}$ | $C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$ |
| Lowpass to bandstop | $\Omega_1<\Omega_0<\Omega_2$ | $\frac{C(z^2-1)}{z^2-2\beta z+1}$ | $C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$ |

**FINAL EXAMINATION**

SEMESTER/SESSION : SEM I / 2017/2018 PROGRAMME CODE : BEJ / BEV
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEB 30503

**Table 6: Windows for FIR filter design.**

| **Window** | **Expression** $w_N[n]$, $-0.5(N-1) \le n \le 0.5(N-1)$ |
|---|---|
| Boxcar | 1 |
| Cosine | $\cos\left(\frac{n\pi}{N-1}\right)$ |
| Riemann | $\text{sinc}^L\left(\frac{2n}{N-1}\right), L > 0$ |
| Bartlett | $1-\frac{2\lvert n\rvert}{N-1}$ |
| Von Hann (Hanning) | $0.5+0.5\cos\left(\frac{2n\pi}{N-1}\right)$ |
| Hamming | $0.54+0.46\cos\left(\frac{2n\pi}{N-1}\right)$ |
| Blackman | $0.42+0.5\cos\left(\frac{2n\pi}{N-1}\right)+0.08\cos\left(\frac{4n\pi}{N-1}\right)$ |
| Kaiser | $\frac{I_0\left(\pi\beta\sqrt{1-4\left(\frac{n}{N-1}\right)^2}\right)}{I_0(\pi\beta)}$ |

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**FINAL EXAMINATION**

SEMESTER/SESSION : SEM I / 2017/2018 PROGRAMME CODE : BEJ / BEV
COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEB 30503

**Table 7:Characteristics of the windowed spectrum for various windows.**

| Window | Peak Ripple $\delta_p = \delta_s$ | Passband Attenuation $A_{WP}$(dB) | Peak Sidelobe Attenuation $A_{WS}$ (dB) | Transition Width $F_{WS} \approx C/N$ |
|---|---|---|---|---|
| Boxcar | 0.0897 | 1.5618 | 21.7 | $C = 0.92$ |
| Cosine | 0.0207 | 0.3600 | 33.8 | $C = 2.10$ |
| Riemann | 0.0120 | 0.2087 | 38.5 | $C = 2.50$ |
| von Hann (Hanning) | 0.0063 | 0.1103 | 44.0 | $C = 3.21$ |
| Hamming | 0.0022 | 0.0384 | 53.0 | $C = 3.47$ |
| Blackman | $1.71 \times 10^{-4}$ | $2.97 \times 10^{-3}$ | 75.3 | $C = 5.71$ |

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**FINAL EXAMINATION**

SEMESTER/SESSION : SEM I / 2017/2018 PROGRAMME CODE : BEJ / BEV

COURSE NAME : DIGITAL SIGNAL PROCESSING COURSE CODE : BEB 30503

**Identity**

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

**Finite Summation Formula**

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{n} k\alpha^k = \frac{\alpha\left[1-(n+1)\alpha^n + n\alpha^{n+1}\right]}{(1-\alpha)^2}$$

$$\sum_{k=0}^{n} k^2\alpha^k = \frac{\alpha\left[(1+\alpha)-(n+1)^2\alpha^n + (2n^2+2n-1)\alpha^{n+1} - n^2\alpha^{n+2}\right]}{(1-\alpha)^3}$$

**Infinite Summation Formula**

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \quad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} \alpha^k = \frac{\alpha}{1-\alpha}, \quad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k\alpha^k = \frac{\alpha}{(1-\alpha)^2}, \quad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k^2\alpha^k = \frac{\alpha^2+\alpha}{(1-\alpha)^3}, \quad |\alpha| < 1$$

$$\sum_{k=-\infty}^{\infty} e^{-\alpha|k|} = \frac{1+e^{-\alpha}}{1-e^{-\alpha}}, \quad \alpha > 0$$