CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	SIGNALS & SYSTEMS
COURSE CODE	:	BEB 20203
PROGRAMME	:	BEJ
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A: ANSWER **ALL** QUESTIONS SECTION B: ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **TWELVE** (12) PAGES

SECTION A: ANSWER ALL QUESTIONS

Q1. A continuous time signal is defined as:

$$x(t) = 2t\ ; 0 \leq t \leq 5$$
$$= 0\ \ ; t > 5$$

(a) Illustrate the even part and odd part of the signal.

(5 marks)

(b) Determine the energy and power of the signal, $x(t)$. Then, classify whether $x(t)$ is energy or power signal.

(5 marks)

Q2. Given a periodic signal

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$$x(t) = 2 + \sin\left(\frac{\pi}{3}t + \frac{\pi}{3}\right) + 3\cos(2\pi t + \frac{\pi}{2})$$

(a) Determine the fundamental angular frequency ω_0, of $x(t)$.

(3 marks)

(b) Formulate the corresponding exponential Fourier Series of $x(t)$ and its coefficients.

(5 marks)

(c) Sketch the magnitude and phase plot of, $x(t)$.

(2 marks)

Q3 (a) A signal $x(t)$ is given by

$$x(t) = e^{-2t}u(t)$$

Show that the Fourier transform of the signal, $x(t)$ is

$$X(\omega) = \frac{1}{j\omega + 2}$$

(3 marks)

(b) Determine the impulse response, $h(t)$ of a system given by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

(3 marks)

(c) If the signal, $x(t)$ in **Q3 (a)** is passed through the system, $h(t)$ given in **Q3 (b)**, determine the output, $y(t)$ of the system.

(4 marks)

Q4. (a) Using the definition of Laplace transform, determine the Laplace transform of $x(t)$.

$$x(t) = 3e^{-3t}u(t-2).$$

(4 marks)

(b) Sketch the zero-pole plot and region of convergence, ROC (if it exists) of the signal $x(t)$.

(2 marks)

(c) Solve $\mathcal{L}[x(t)]$ using the time shifting property of Laplace transform.

(4 marks)

SECTION B: ANSWER THREE (3) QUESTIONS ONLY

Q5. (a) Consider the system shown in **Figure Q5 (a).**

(i) Find the overall impulse response of the system, $h(t)$ with impulse responses given below.

$$h_1(t) = 7t\,u(t)$$

$$h_2(t) = 4\,u(t)$$

$$h_3(t) = u(t)$$

(6 marks)

(ii) Identify whether the system is stable or unstable.

(4 marks)

(b) (i) By giving appropriate examples, distinguish causal and non-causal systems.

(6 marks)

(ii) Test the causality of the signal, $s(t)$, given by:

$$s(t) = x(t) + \int_0^t x(\lambda)d\lambda$$

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(4 marks)

Q6. (a) A certain communication system produces an even periodic rectangular pulse train signal, $x(t)$ with amplitude of 1V, fundamental frequency $f_0 = 200$ kHz and duty cycle of 40 percent ($\tau = 0.4$).

(i) Sketch the signal, $x(t)$.

(2 marks)

(ii) Show that the Fourier series coefficients of the signal is given by

$$x_n = \begin{cases} \frac{2}{5}\text{sinc}\left(\frac{2n}{5}\right) & \text{for } n \neq 0, \\ \frac{2}{5} & \text{for } n = 0. \end{cases}$$

(6 marks)

(iii) Sketch the amplitude spectrum of the Fourier series of $x(t)$ for the first **5 (FIVE)** harmonics.

(2 marks)

(b) The signal in **Q6 (a)** is intended for transmission over a transmission media. However, due to limited frequency resource, the channel for the transmission is limited to 1000 kHz. As such, the signal is passed through a simple RC low pass filter with its frequency response given by

$$H(f) = \frac{1}{j2\pi fRC + 1}$$

where RC is the time constant given by

$$RC = \frac{1}{2\pi f_c}$$

and f_c is the cut-off frequency of the filter.

(i) Find the frequency response of the system for $f = 200$ kHz, 400 kHz, 600 kHz, 800 kHz, and 1000 kHz.

(5 marks)

(ii) Evaluate the output of the filter, $y(t)$ for the input signal, $x(t)$ given in **Q6 (a)**.

(5 marks)

Q7 (a) The Fourier transform of a signal, $x(t) = \text{rect}\left(\frac{t}{2}\right)$ is

$$X(\omega) = 2\,\text{sinc}(\omega)$$

Solve the Fourier transform of the following signals.

(i) $2x\left(\frac{t-3}{2}\right)$

(3 marks)

(ii) $x(t)\cos(\omega_0 t)$

(2 marks)

(b) A signal $x(t) = 2u(t)$ is passed through an electrical network having its impulse response, $h(t)$ given by

$$h(t) = \frac{1}{RC} e^{-t/RC}\, u(t)$$

Solve for the output, $y(t)$ of the electrical network for the case of $R = 1\ \Omega$ and $C = 4$ F using the convolution property of Fourier transform.

(15 marks)

Q8 (a) Define region of convergence (ROC).

(2 marks)

(b) Write any two properties of ROC of Laplace transform.

(2 marks)

(c) The output of an LTI system can be easily determined in s-domain using the convolution property of Laplace transform. If a signal

$$x(t) = e^{-2t}\big(u(t) - u(t-3)\big)$$

is an input to a system with the impulse response given by

$$h_1(t) = 3e^{-3t}u(t),$$

(i) Determine the output, $y(t)$ using the Laplace transform convolution property.

(10 marks)

(ii) The system, $h_1(t)$ is cascaded in series to another system, $h_2(t)$ with its transfer function given by

$$H_2(s) = \frac{s-1}{s-2}$$

forming a new system, $h(t)$. Determine the stability of the new system, $h(t)$.

(6 marks)

- END OF QUESTIONS -

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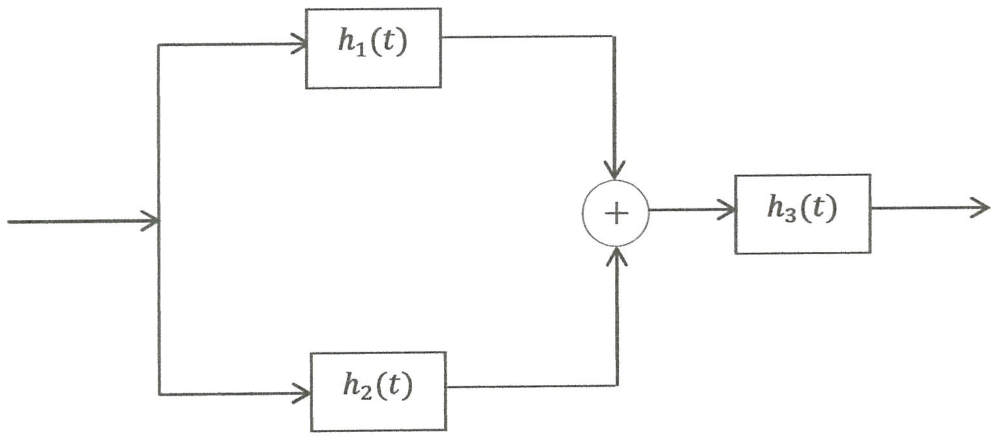

Figure Q5 (a)

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TABLE 1: Trigonometric Identities

Trigonometric identities	
$\sin\alpha = \cos\left(\alpha - \frac{\pi}{2}\right)$	$\cos\alpha = \sin\left(\alpha + \frac{\pi}{2}\right)$
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 2: Values of cosine, sine and exponential functions for integral multiple of π

Function	**Value**
$\cos 2n\pi$	1
$\sin 2n\pi$	0
$\cos n\pi$	$(-1)^n$
$\sin n\pi$	0
$\cos\frac{n\pi}{2}$	$\begin{cases}(-1)^{n/2}, & n = even\\ 0, & n = odd\end{cases}$
$\sin\frac{n\pi}{2}$	$\begin{cases}(-1)^{(n-1)/2}, & n = odd\\ 0, & n = even\end{cases}$
$e^{j2n\pi}$	1
$e^{jn\pi}$	$(-1)^n$
$e^{jn\pi/2}$	$\begin{cases}(-1)^{n/2}, & n = even\\ j(-1)^{(n-1)/2}, & n = odd\end{cases}$

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TABLE 3: Fourier Transform Pairs

Time domain, $f(t)$	Frequency domain, $F(\omega)$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$
$u(t+\tau)-u(t-\tau)$	$2\dfrac{\sin\omega\tau}{\omega}$
$\lvert t\rvert$	$\dfrac{-2}{\omega^2}$
$\mathrm{sgn}(t)$	$\dfrac{2}{j\omega}$
$e^{-at}u(t)$	$\dfrac{1}{\alpha+j\omega}$
$e^{at}u(-t)$	$\dfrac{1}{\alpha-j\omega}$
$t^n e^{-at}u(t)$	$\dfrac{n!}{(\alpha+j\omega)^{n+1}}$
$e^{-a\lvert t\rvert}$	$\dfrac{2\alpha}{\alpha^2+\omega^2}$
$e^{j\omega_o t}$	$2\pi\delta(\omega-\omega_o)$
$\sin\omega_o t$	$j\pi[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]$
$\cos\omega_o t$	$\pi[\delta(\omega+\omega_o)+\delta(\omega-\omega_o)]$
$e^{-at}\sin\omega_o t\,u(t)$	$\dfrac{\omega_o}{(\alpha+j\omega)^2+\omega_o^2}$
$e^{-at}\cos\omega_o t\,u(t)$	$\dfrac{\alpha+j\omega}{(\alpha+j\omega)^2+\omega_o^2}$

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TABLE 4: Fourier Transform Properties

Property	**Time domain, *f(t)***	**Frequency domain, $F(\omega)$**
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a \rvert} F(\frac{\omega}{a})$
Time Shift	$f(t-a)$	$e^{-j\omega a} F(\omega)$
Frequency Shift	$e^{j\omega_0 t} f(t)$	$F(\omega - \omega_0)$
Modulation	$\cos(\omega_0 t) f(t)$	$\frac{1}{2}[F(\omega+\omega_o) + F(\omega-\omega_o)]$
Time Differentiation	$\frac{df}{dt}$ $\frac{d^n f}{dt^n}$	$j\omega F(\omega)$ $(j\omega)^n F(\omega)$
Time Integration	$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$
Frequency Differentiation	$t^n f(t)$	$j^n \frac{d^n}{d\omega^n} F(\omega)$
Reversal	$f(-t)$	$F(-\omega)$ or $F^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Convolution in *t*	$f_1(t) * f_2(t)$	$F_1(\omega) F_2(\omega)$
Convolution in ω	$f_1(t) f_2(t)$	$F_1(\omega) * F_2(\omega)$

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TABLE 5: Laplace Transform

$x(t)$, $t>0$	$X(s)$	ROC
$\delta(t)$	1	$All\,s$
$u(t)$	$\frac{1}{s}$	$Re(s)>0$
t	$\frac{1}{s^2}$	$Re(s)>0$
t^n	$\frac{n!}{s^{n+1}}$	$Re(s)>0$
e^{-at}	$\frac{1}{s+a}$	$Re(s)>-a$
te^{-at}	$\frac{1}{(s+a)^2}$	$Re(s)>-a$
$\cos bt$	$\frac{s}{s^2+b^2}$	$Re(s)>0$
$\sin bt$	$\frac{b}{s^2+b^2}$	$Re(s)>0$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$Re(s)>-a$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$Re(s)>-a$
$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$Re(s)>0$
$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s)>0$
$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s)>0$

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TABLE 6: Laplace Transform Properties

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$ax_1(t)+bx_2(t)$	$aX_1(s)+bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0t}x(t)$	$X(s-s_0)$	Shifted version of R (i.e., s is in the ROC if $s-s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t)*x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathcal{R}e\{s\} > 0\}$

Initial- and Final-Value Theorems

If $x(t)=0$ for $t<0$ and $x(t)$ contains no impulses or higher-order singularities at $t=0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t)=0$ for $t<0$ and $x(t)$ has a finite limit as $t\longrightarrow\infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

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