UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	SIGNALS AND SYSTEMS
COURSE CODE	:	BEB 20203
PROGRAMME	:	BEJ
EXAMINATION DATE	:	DECEMBER 2016/ JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A: ANSWER **ALL** QUESTIONS SECTION B: ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **TWELVE (12)** PAGES

SECTION A: ANSWER ALL QUESTIONS

Q1 (a) Sketch the signal $r(t)$ given by

$$r(t) = \frac{3}{5}\left[1 - \text{rect}\left(\frac{t}{2} - 3\right)\right] u(t).$$

(5 marks)

(b) Given a composite signal

$$y(t) = 2\cos\left(\frac{3\pi}{5}t\right) + \sin\left(\frac{2\pi}{3}t\right),$$

determine the periodicity of $y(t)$.

(5 marks)

Q2 (a) Given a signal

$$x(t) = 4 + \cos\left(3\pi t + \frac{\pi}{3}\right) + 6\sin(2\pi t + \frac{2\pi}{3})$$

(i) Determine the fundamental angular frequency ω_0, of $x(t)$.

(3 marks)

(ii) Synthesis the corresponding exponential Fourier Series of $x(t)$ and its coefficients.

(5 marks)

(b) Using your answer in **Q2(a)(iii)**, determine the average signal power using Parseval's theorem assuming $R = 1\Omega$.

(2 marks)

Q3 (a) Consider the rectangular pulse signal

$$x(t) = \begin{cases} A, & |t| < \frac{\tau}{2} \\ 0, & |t| > \frac{\tau}{2} \end{cases}$$

(i) Derive $X(\omega)$ using the definition of Fourier Transform.

(3 marks)

(ii) Plot the magnitude spectrum of $X(\omega)$.

(4 marks)

(b) Explain the duality property of Fourier Transform with the aid of a simple example.

(3 marks)

Q4 Given the following facts about a real signal $x(t)$ with Laplace transform of $X(s)$.

$X(s)$ has exactly two poles.

$X(s)$ has no zeros in the finite s-plane.

$X(s)$ has a pole at $s = -1 + j$.

$X(0) = 8$.

$e^{2t}x(t)$ is not absolutely integrable.

Determine $X(s)$ and specify its region of convergence.

(10 marks)

SECTION B: ANSWER THREE (3) QUESTIONS ONLY

Q5 (a) $x_{even}(t)$ and $x_{odd}(t)$ are the even and odd components of the signal $x(t)$ respectively. The signals $x_{even}(t)u(-t)$ and $x_{odd}(t)u(-t)$ are given in **Figure Q5(a)(i)** and **Figure Q5(a)(ii)** respectively. Sketch the signal $x(t)$.

(5 marks)

(b) A continuous time signal $x(t)$ in **Figure Q5(b)** is an input to an LTI system of which the impulse response $h(t)$ is given by

$$h(t) = u(t).$$

Determine the output $y(t)$ of the system.

(10 marks)

(c) **Figure Q5(c)** shows a continuous time systems where $x(t)$ is the system input and $y(t)$ is the system output. Deduce the stability of the system $h(t)$ using suitable justification.

(5 marks)

Q6 Given a periodic signal $x(t)$ as shown in **Figure Q6 (a).**

(a) Determine the trigonometric Fourier series coefficients $(a_0, a_n\ and\ b_n)$ of the signal $x(t)$.

(10 marks)

(b) Express the signal $x(t)$ in trigonometric Fourier series by considering only the first 3 harmonics.

(3 marks)

(c) The signal $x(t)$ with the period $T = 10^{-5}$s is passed through an LTI system with frequency response given in **Figure Q6 (c)**. Determine the output of the system.

(7 marks)

Q7 (a) The Fourier transform of a signal $x(t)$ is

$$\mathcal{F}[x(t)] = X(\omega) = \text{sinc}\left(\frac{\omega}{2}\right).$$

Using the properties of Fourier transform, determine the Fourier transform of

$$x\left(\frac{t-1}{2}\right)\cos 2t.$$

(4 marks)

(b) Find the inverse Fourier transform of $X(\omega) = \frac{1}{(4+j\omega)^2}$ by using convolution property.

(6 marks)

(c) Determine $v_0(t)$ of an electrical circuit as shown **Figure Q7(b)** for $v_i(t) = 3e^{-5t}u(t)$.

(10 marks)

Q8 (a) Briefly explain natural response of a system.

(2 marks)

(b) Consider the circuit in **Figure Q8(b)**. Assume that the current $i(t)$ has reached a steady state with the switch at position A. At time $t = 0$, the switch is moved from position A to position B.

(i) Find the differential equation relating $i(t)$ and v_2 for $t > 0^-$. Specify the initial condition for the differential equation in terms of v_1.

(3 marks)

(c) Given that $R = 1\ \Omega$ and $L = 1$ H for the circuit in **Figure Q8(b)**, determine and sketch the current $i(t)$ for each of the following values of v_1 and v_2;

(i) $v_1 = 0$ V, $v_2 = 2$ V

(7 marks)

(ii) $v_1 = 4$ V, $v_2 = 0$ V

(4 marks)

(iii) $v_1 = 4$ V, $v_2 = 2$ V

(4 marks)

- END OF QUESTIONS -

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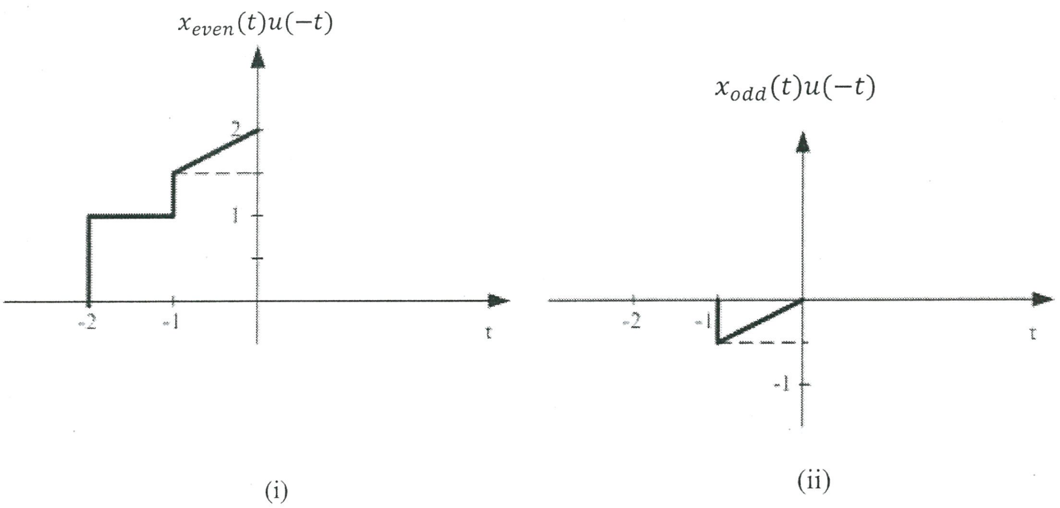

Figure Q5(a)

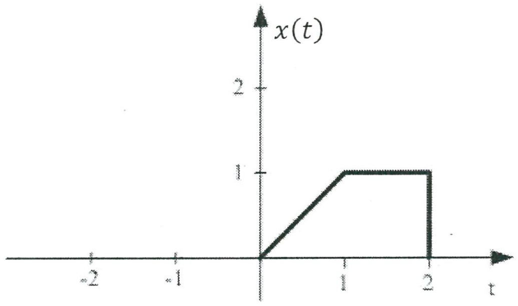

Figure Q5(b)

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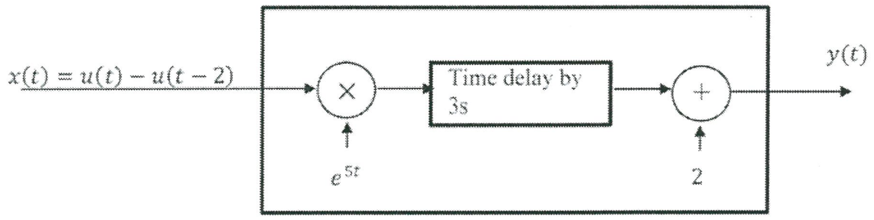

Figure Q5(c)

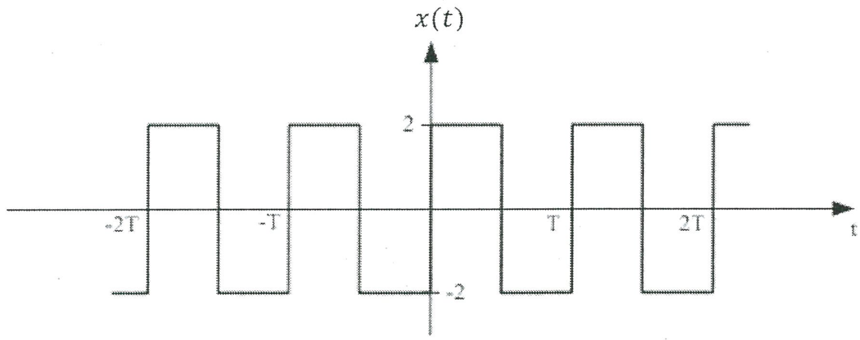

Figure Q6 (a)

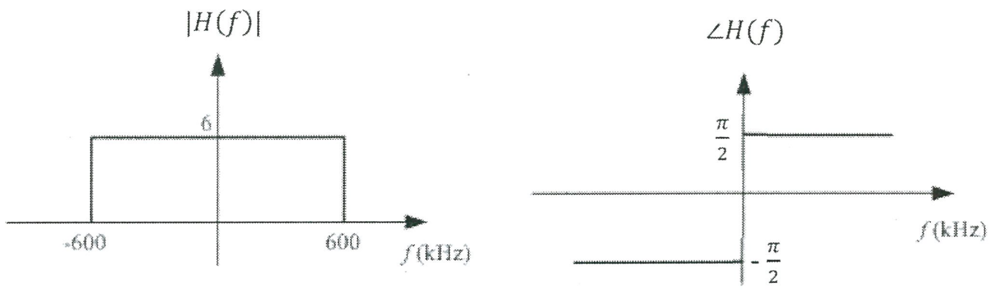

Figure Q6 (c)

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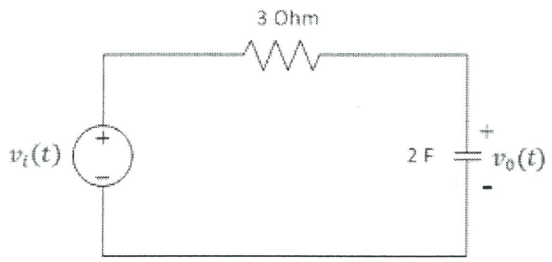

Figure Q7(b)

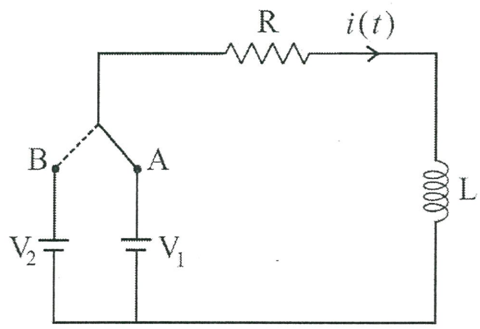

Figure Q8(b)

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TABLE 1: INDEFINITE INTEGRALS

$\int \cos at\, dt = \frac{1}{a}\sin at$	$\int \sin at\, dt = -\frac{1}{a}\cos at$
$\int t\cos at\, dt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at$	$\int t\sin at\, dt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at$
$\int te^{at}dt = \frac{1}{a^2}e^{at}(at-1)$	$\int \frac{1}{(a^2+t^2)}dt = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)$

TABLE 2: EULER'S IDENTITY

$e^{\pm j\pi/2} = \pm j$	$A\angle \pm \theta = Ae^{\pm j\theta}$
$e^{\pm jk\pi} = \cos k\pi$	$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$
$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$	$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$

TABLE 3: COMPLEX NUMBER

$s = a + jb = \lvert s\rvert\angle \pm \theta = \lvert s\rvert e^{\pm j\theta}$	$\lvert s\rvert = \sqrt[2]{a^2+b^2}$	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

TABLE 4: TRIGONOMETRIC IDENTITIES

$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right))$	$\cos\theta = \sin(\theta + \frac{\pi}{2})$
$\sin(\propto \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin^2\alpha + \cos^2\beta = 1$	
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 5: VALUES OF COSINE, SINE AND EXPONENTIAL FUNCTIONS FOR INTEGRAL MULTIPLE OF π.

Function	Value	Function	Value
$\cos 2n\pi$	1	$e^{j2n\pi}$	1
$\sin 2n\pi$	0	$e^{jn\pi}$	$(-1)^n$
$\cos n\pi$	$(-1)^n$	$e^{\frac{jn\pi}{2}}$	$\begin{cases}(-1)^{\frac{n}{2}} & ,n = \text{even}\\ j(-1)^{\frac{n-1}{2}} & ,n = \text{odd}\end{cases}$
$\sin n\pi$	0		
$\cos\left(\frac{n\pi}{2}\right)$	$\begin{cases}(-1)^{\frac{n}{2}} & ,n = \text{even}\\ 0 & ,n = \text{odd}\end{cases}$	$\sin\left(\frac{n\pi}{2}\right)$	$\begin{cases}(-1)^{\frac{n-1}{2}} & ,n = \text{odd}\\ 0 & ,n = \text{even}\end{cases}$

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TABLE 6: FOURIER SERIES

Exponential	$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$ $x_n = \frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t} dt$
Trigonometric	$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\frac{2\pi}{T}t + b_n \sin n\frac{2\pi}{T}t$ $a_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\cos n\frac{2\pi}{T}t\, dt, \quad n = 0, 1, 2, 3 \ldots$ $b_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\sin n\frac{2\pi}{T}t\, dt, \quad n = 1, 2, 3 \ldots$
Amplitude-phase	$x(t) = X_0 + \sum_{n=1}^{\infty} A_n \cos(n\frac{2\pi}{T}t + \theta_n)$ $A_n = 2\lvert X_n \rvert = \sqrt[2]{a_n^2 + b_n^2} \quad , \quad \theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$

TABLE 7: DEFINITION OF FOURIER AND LAPLACE TRANSFORM

FOURIER TRANSFORM	INVERSE FOURIER TRANSFORM
$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$ $\mathcal{F}[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\, dt$	$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$ $x(t) = \mathcal{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\, df$
LAPLACE TRANSFORM **Bilateral** $\mathcal{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$ **Unilateral** $\mathcal{L}[x(t)] = X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$ $s = \sigma + j\omega$	**INVERSE LAPLACE TRANSFORM** $x(t) = \mathcal{L}^{-1}[X(s)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$

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TABLE 8: FOURIER TRANSFORM PAIRS

Time domain, $x(t)$	**Frequency domain,** $X(\omega)$	**Time domain,** $x(t)$	**Frequency domain,** $X(\omega)$
$\delta(t)$	1	1	$2\pi\delta(\omega)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	$u(t+\tau)-u(t-\tau)$	$\frac{2\sin(\omega\tau)}{\omega}$
$\|t\|$	$-\frac{2}{\omega^2}$	$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$e^{j\omega t}$	$2\pi\delta(\omega-\omega_0)$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$		

TABLE 9: FOURIER TRANSFORM PROPERTIES

Property	**Time domain,** $x(t)$	**Frequency domain,** $X(\omega)$
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$
Time scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t-t_0)u(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
Modulation	$\cos(\omega_0 t)\,x(t)$ $\sin(\omega_0 t)\,x(t)$	$\frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$ $\frac{1}{2j}[X(\omega+\omega_0)-X(\omega-\omega_0)]$
Time differentiation	$\frac{d}{dt}(x(t))$ $\frac{d^n}{dt^n}(x(t))$	$j\omega X(\omega)$ $(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{X(\omega)}{j\omega}+\pi X(\omega)\,\delta(\omega)$
Frequency differentiation	$t^n x(t)$	$(j)^n \frac{d^n}{d\omega^n}X(\omega)$
Time Reversal	$x(-t)$	$X(-\omega)$ *or* $X^*(\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Convolution in t	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
Convolution in ω	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$

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TABLE 10: LAPLACE TRANSFORM PAIR

Time domain, $x(t)$, $t > 0$	s-domain, $X(s)$	ROC	Time domain, $x(t)$, $t > 0$	s-domain, $X(s)$	ROC
$\delta(t)$	1	All s	$\cos bt$	$\frac{s}{s^2+b^2}$	$Re(s) > 0$
$u(t)$	$\frac{1}{s}$	$Re(s) > 0$	$\sin bt$	$\frac{b}{s^2+b^2}$	$Re(s) > 0$
t	$\frac{1}{s^2}$	$Re(s) > 0$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$Re(s) > -a$
t^n	$\frac{n!}{s^{n+1}}$	$Re(s) > 0$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$Re(s) > -a$
e^{-at}	$\frac{1}{s+a}$	$Re(s) > -a$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$Re(s) > 0$
te^{-at}	$\frac{1}{(s+a)^2}$	$Re(s) > -a$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s) > 0$

TABLE 11: LAPLACE TRANSFORM PROPERTIES

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t), x_2(t)$	$X_1(s), X_2(s)$	R_1, R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s) \cdot X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
		$sX(s) - x(0^+)$ (Unilateral)	R right hand plane
	$\frac{d^n}{dt^n}x(t)$	$s^nX(s) - s^{n-1}x(0^+) - \cdots - sx^{n-2}(0^+) - x^{n-1}(0^+)$	
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{Re(s) > 0\}$

Initial- and Final- Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher order singularities at $t = 0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and has a finite limit as $t \to \infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to\infty} sX(s)$$