UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BEE21503 / BWM20403
PROGRAMME	:	BEV / BEJ
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS

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THIS QUESTION PAPER CONSISTS OF FIVE **(5) PAGES**

Q1 (a) Relate f_{xy} and f_{yx} for function $f(x,y)=5y^3+3x^2y-4xy+y$.

(7 marks)

(b) Solve for $\frac{\partial z}{\partial m}$ and $\frac{\partial z}{\partial n}$ by using chain rule when $z=\ln(4y^2+4x)$, $x=e^n$ and $y=2m+n$.

(10 marks)

(c) Compare $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $ye^x-5\sin 3z=3z$ and propose a value of y that will produce $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}$.

(8 marks)

Q2 (a) Sketch the surface of the paraboloid $z=5-x^2-y^2$. Then, find the surface area of the portion of the paraboloid $z=5-x^2-y^2$ that lies between the planes z = 1 and z = 4 using polar coordinates.

(10 marks)

(b) Find the volume of the solid that lies below the hemisphere $z=\sqrt{25-x^2-y^2}$, and between the cylinders $x^2+y^2=9$ and $x^2+y^2=16$ using cylindrical coordinates.

(7 marks)

(c) (i) A solid is bounded on top by an upper hemisphere given by $z=\sqrt{12-x^2-y^2}$ and below by a cone, $z=\sqrt{3(x^2+y^2)}$. Its density function is given by $\delta(x,y,z)=x^2+y^2+z^2$. The mass of this solid is given by $m=\int_0^{\sqrt{3}}\int_0^{\sqrt{3-y^2}}\int_{\sqrt{3(x^2+y^2)}}^{\sqrt{12-x^2-y^2}} x^2+y^2+z^2\,dzdxdy$. Convert the m to its associated spherical coordinates.

(2 marks)

(ii) Then calculate the mass of the solid in **Q2 (c)(i)** by using spherical coordinates.

(6 marks)

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Q3 (a) Magnetic field is curling while electric field is diverging. Going uphill at the steepest direction is a gradient. From the mathematical point of view, sketch physical interpretations for :

(i) a gradient of a vector field,

(3 marks)

(ii) a divergence of a vector field, and

(3 marks)

(iii) a curl of a vector field.

(3 marks)

(b) Sketch a condition to prove that the curl of a gradient of a vector is **ZERO (0)**.

(5 marks)

(c) Sketch a condition to prove that the divergence of a curl of a vector is **ZERO (0).**

(5 marks)

(d) If the curl of a vector field is **ZERO (0)** we say that the vector field is irrotational. Explain this in a sketch.

(6 marks)

Note: Sketch hereby means using arrows, dots, or lines representing scalars or vectors in 3D Cartesian.

Q4 (a) Given a force field $\mathbf{F}(x,y)=(3x^2+6xy^2)\mathbf{i}+(6x^2y+4y^2)\mathbf{j}$ acting on a particle moving along curve C from point $A(1,0)$ to point $B(0,1)$.

(i) Show that **F** is conservative.

(ii) Find a potential function $\phi(x,y)$.

(iii) Hence, find the work done by force field $\mathbf{F}$.

(10 marks)

(b) The flux of a vector field $\vec{D}$ across a surface σ is given by $\iint_{\sigma} \vec{D}\cdot\hat{n}\ r\,d\theta\,dz$, determine the flux of $\vec{D}=r^2\cos^2\theta\ \hat{r}+\sin\theta\ \hat{\theta}$ over the closed surface of the cylinder $0\le z\le 1$, $r = 4$ and verify the divergence theorem for this case. (Hint: $\int \cos^2\theta\ d\theta=\frac{\theta}{2}+\frac{1}{4}\sin 2\theta$ and $\nabla\cdot\vec{D}=\frac{1}{r}\left[\frac{\partial(rD_r)}{\partial r}\right]+\frac{1}{r}\frac{\partial D_\theta}{\partial\theta}+\frac{\partial D_z}{\partial z}$)

(15 marks)

\- END OF QUESTIONS -

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FORMULAS

Polar coordinate

$x = r\cos\theta,\ \ y = r\sin\theta,\ \ \theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate

$x = r\cos\theta,\ \ y = r\sin\theta,\ \ z = z$ and $\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$

Spherical coordinate

$x = \rho\sin\varphi\cos\theta,\ \ y = \rho\sin\varphi\sin\theta,\ \ z = \rho\cos\phi$, then $x^2 + y^2 + z^2 = \rho^2$, for $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi$, and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

$$A = \iint_R dA$$

$m = \iint_R \delta(x,y)dA$, where $\delta(x,y)$ is a density of lamina

$$V = \iint_R f(x,y)\,dA$$

$$V = \iiint_G dV$$

$$m = \iiint_G \delta(x,y,z)dV$$

If f is a differentiable function of x, y and z, then the

Gradient of *f*, grad $f(x,y,z) = \nabla f(x,y,z) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

If $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is a vector field in Cartesian coordinate, then the

Divergence of $\mathbf{F}(x,y,z)$, div $\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

Curl of $\mathbf{F}(x,y,z)$, curl $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial Z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial Z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$

F is conservative vector field if Curl of $\boldsymbol{F} = 0$.

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FORMULAS

Surface Integral

Let S be a surface with equation $z = g(x, y)$ and let R be its projection on the xy-plane.

$$\iint_S f(x,y,z)\,dS = \iint_R f(x,y,g(x,y))\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

Gauss's Theorem

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_G \nabla\cdot\mathbf{F}\,dV$$

Stokes' Theorem

$$\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS = \int_C \mathbf{F}\cdot d\mathbf{r}$$

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