Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	BEE 11303 / BWM 10103
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

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THIS PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Solve the questions below **with and without** using L'Hopital's Rule.

(i) $$\lim_{x\to 0}\left(\frac{2x^2-3x+4}{x}+\frac{5x-4}{x}\right)$$

(3 marks)

(ii) $$\lim_{x\to 3^-}\frac{|4x-12|}{x-3}$$

(3 marks)

(b) Suppose the total cost, *C(q)* of producing a quantity, *q* of a product is 5000 plus 5 times *q*. The average cost per unit quantity, *A(q)* is equal to the total cost, *C(q)* divided by the quantity produced, *q*. Calculate the limiting value of the average cost per unit, *A(q)* as *q* tends to infinity.

(4 marks)

(c) Given:

$$f(x)=\begin{cases} x^2-4x-5 & x<a \\ 2x-6 & x\geq a \end{cases}$$

Find possible values of a and decide which value that makes the function *f(x)* continuous everywhere.

(10 marks)

Q2 (a) By using appropriate graph, prove that the derivative of a function *f(x)* is defined by:

$$\frac{dy}{dx}=\lim_{\delta x\to 0}\frac{f(x+\delta x)-f(x)}{\delta x}$$

(6 marks)

(b) Calculate the derivative of $f(x)=4x^3$ by using the first principle as defined in **Q2(a)**.

(5 marks)

(c) The voltage, *V* across an inductor of inductance, *L* is related to the current flow *i(t)* by the following formula:

$$V=L\frac{di}{dt}$$

Given $L=2$ H, and current $i(t)=0.5e^{-2t}$ A.

(i) Determine the rate of change of current *I* at time *t*=0 second. State the unit of measurement.

(3 marks)

(ii) Analyse the time *t* at which the voltage *V* is increasing at a rate of 2 Volts per second.

(6 marks)

Q3 (a) Evaluate:

(i) $\int \frac{1}{\sqrt{x}} \sin \sqrt{x} dx$ using u-substitution method.

(5 marks)

(ii) $\int \frac{2-3x}{3x^2-4x+1} dx$

(5 marks)

(iii) $\int \frac{\sin^7 x}{\cos^4 x} dx$

(7 marks)

(b) A current, *i(t)* of $6\sin 4t$ A flows through a 2 F capacitor. Determine the voltage *v(t)* across the capacitor given that *v(0)* = 1 V. The relation between voltage and current is given by:

$$v(t)=\frac{1}{C}\left[\int_{t_0}^{t} i(t)\right]+v(0)$$

(3 marks)

Q4 (a) Solve the following questions:

(i) $\frac{d}{dx}\left[f^{-1}(x)\right]$ where $f(x)=\sin x$

(4 marks)

(ii) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{1+x}{1-x}\right)\right]$

(5 marks)

(iii) $\int \frac{4}{\sqrt{x^2+16}} dx$. [Hint: Apply hyperbolic substitution]

(5 marks)

(iv) $\int_2^3 \cosh^{-1} x \, dx$

(6 marks)

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Q5 (a) Calculate the following limits:

(i) $$\lim_{x\to 0}\frac{6x-\sin 2x}{2x+3\sin 4x}$$

(3 marks)

(ii) $$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$$

(2 marks)

(b) Given a parametric equation of the curve:

$$x=\frac{1}{1-e^{t}} \text{ and } y=te^{3t}$$

Determine $\frac{dy}{dx}$ in terms of t.

(5 marks)

(c) Determine the following integrals:

(i) $$\int\frac{2x^2}{x^3-4}dx$$

(3 marks)

(ii) $$\int 3^{5x}dx$$

(2 marks)

(d) Solve the differential coefficient of function y. (Leave your answer in its simplest form).

$$y=\sin^{-1}\left\{\frac{x}{a}\right\}$$

(2 marks)

(e) Evaluate the integral of the following hyperbolic function:

$$\int\cosh^2(5x+2)\,dx$$

(3 marks)

– END OF QUESTIONS –

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Formulae

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$ $\tan x = \dfrac{2t}{1-t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{2dt}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$ $\tan 2x = \dfrac{2t}{1-t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$ $dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$$\cos^2 x + \sin^2 x = 1$$
$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$
$$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$$
$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$
$$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$$
$$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$$
$$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$
$$1 - \tanh^2 x = \text{sech}^2 x$$
$$\coth^2 x - 1 = \text{csch}^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$$
$$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$$
$$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$$

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Formulae

Indefinite Integrals	**Integration of Inverse Functions**
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{1}{x}\, dx = \ln\lvert x\rvert + C$	$\int \frac{-1}{\sqrt{a^2 - x^2}}\, dx = \cos^{-1}\left(\frac{x}{a}\right) + C$
$\int \cos x\, dx = \sin x + C$	$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \frac{-1}{a^2 + x^2}\, dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$
$\int \sec^2 x\, dx = \tan x + C$	$\int \frac{1}{\lvert x\rvert\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc^2 x\, dx = -\cot x + C$	$\int \frac{-1}{\lvert x\rvert\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C$
$\int \sec x \tan x\, dx = \sec x + C$	$\int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc x \cot x\, dx = -\csc x + C$	$\int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \cosh^{-1}\left(\frac{x}{a}\right) + C$
$\int e^x dx = e^x + C$	$\int \frac{-1}{\lvert x\rvert\sqrt{a^2 - x^2}}\, dx = \frac{1}{a}\operatorname{sech}^{-1}\left\lvert\frac{x}{a}\right\rvert + C$
$\int \cosh x\, dx = \sinh x + C$	$\int \frac{-1}{\lvert x\rvert\sqrt{a^2 + x^2}}\, dx = \frac{1}{a}\operatorname{csch}^{-1}\left\lvert\frac{x}{a}\right\rvert + C$
$\int \sinh x\, dx = \cosh x + C$	$\int \frac{1}{a^2 - x^2}\, dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert > a \end{cases}$
$\int \operatorname{sech}^2 x\, dx = \tanh x + C$	
$\int \operatorname{csch}^2 x\, dx = -\coth x + C$	
$\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x + C$	
$\int \operatorname{csch} x \coth x\, dx = -\operatorname{csch} x + C$	