Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	DIGITAL SIGNAL PROCESSING
COURSE CODE	:	BEF 35603
PROGRAMME	:	BEV
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL QUESTIONS**

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THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

Q1 (a) Briefly explain the sampling theorem and define the Nyquist rate and the Nyquist interval.

(3 marks)

(b) Given a continuous-time voltage signal and the sampling frequency S used as 20 Hz,

$$v(t) = 2\sin(4\pi t) + \cos(56\pi t)$$

(i) Analyze whether aliasing occur or not for the frequency of each sinusoid components and write down the corresponding signal. Write down the aliased frequency if it occurs.

(3 marks)

(ii) Write down the reconstructed signal $v_a(t)$ and its discrete signal $v_a[n]$.

(2 marks)

(iii) Explain how aliasing can be avoided to recover $v(t)$ from its samples.

(2 marks)

(c) The discrete signal $v_a[n]$ found in **Q1(b)(ii)** is quantized with 4 quantization levels and the dynamic range of 2.5 volts.

(i) Calculate the discrete time signal $v[n]$ obtained for $0 \leq n \leq 3$.

(3 marks)

(ii) Analyze the quantized signal $v_q[n]$ and the quantization error signal $e[n]$ using rounding technique. Show all the working steps.

(8 marks)

(d) Based on a signal $x[n] = \{ \overset{\Downarrow}{-1}, 2, 3, 2\}$ with the sampling interval t_s of 1 second, sketch the reconstructed signal $x(t)$, and what is the interpolated value at $t = 2.4$ s, if

(i) step interpolation is performed

(2 marks)

(ii) linear interpolation is performed

(2 marks)

Q2 (a) Define the N-point Discrete Fourier Transform (DFT) $X_{DFT}[k]$ of an N-sample signal $x[n]$ and the inverse DFT (IDFT).

(2 marks)

(b) Determine the Discrete Fourier Transform (DFT) of the four-point sequence

$$x[n] = \{\overset{\Downarrow}{5},\ 2,\ 1,\ 0\}$$

(6 marks)

(c) Let $x[n] = \left\{\overset{\Downarrow}{1}, 2, 1, 0\right\}$ and $X_{DFT}[k] = \{4, -j2, 0, j2\}$ with $N = 4$. By using suitable properties of discrete Fourier Transform (DFT), calculate $y[n] = x[n]x[n]$ and $Y_{DFT}[k]$

(7 marks)

(d) Analyze $x[n] \Leftrightarrow X_{DFT}[k] = \{\overset{\Downarrow}{2}, -j, 0, j\}$ using Decimation in Time (DIT) Fast Fourier Transform (FFT) Algorithm.

(10 marks)

Q3 (a) Define the Region of Convergence (ROC) in z-transform.

(2 marks)

(b) Determine the z-transform and specify its region of convergence (ROC) of the following signals:

(i) $x[n] = \{\overset{\Downarrow}{1},\ 2,\ -1,\ 0,\ 3\}$

(3 marks)

(ii) $x[n] = (n+1)(3)^n u[n]$

(3 marks)

(c) A causal system is described by the following difference equation:

$$y[n] = 0.5y[n-1] + x[n]$$

(i) Compute the transfer function of the system H(z).

(3 marks)

(ii) Analyze the output response $y[n]$ if the input signal is given by:

$$x[n] = (-1)^n u[n]$$

(10 marks)

(d) The z-transform of $x[n]$ is $X(z) = \dfrac{4z}{(z+0.5)^2}, |z| > 0.5$. Solve the z-transform of following using the z-transform differentiation properties and specify the ROC.

$$y[n] = [n-2]x[n]$$

Hint: Use the property $nx[n] \Leftrightarrow -z\dfrac{dX(z)}{dz}$

(4 marks)

Q4 (a) List **two (2)** types of digital filters and state two comparisons between them.

(4 marks)

(b) Consider an analog filter $H(s) = \dfrac{2}{s+2}$

(i) Convert $H(s)$ to a digital filter $H(z)$, by using impulse invariant transformation at a sampling frequency of $S = 2$ Hz.

(7 marks)

(ii) Convert $H(s)$ to a digital filter $H(z)$, using the mapping based on the forward difference at a sampling rate S. Analyze the conditions for which $H(z)$ will be stable.

(4 marks)

(c) Design a bandpass digital filter with band edges of 1 kHz and 3 kHz using the digital-to-digital frequency transformations technique of IIR filter given the digital low pass filter $H(z) = \dfrac{3(z+1)^2}{31z^2 - 26z + 7}$. This filter has a cutoff frequency of 2 kHz and operates at a sampling frequency of 8 kHz.

(10 marks)

– END OF QUESTIONS –

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Table 1a : Some useful identities

$e^{\pm jm\pi} = -1$	m= 1, 3, 5,....
$e^{\pm jm\pi} = 1$	m= 2, 4, 6,....
$e^{jm\pi/2} = j$	m= 1, 5, 9,....
$e^{jm\pi/2} = -j$	m= 3, 7, 11,....
$e^{-jm\pi/2} = -j$	m= 1, 5, 9,....
$e^{-jm\pi/2} = j$	m= 3, 7, 11,....

Table 1b : Table of Numerical Difference Algorithms

Difference	Numerical Algorithm	Mapping for s
Backward	$y(n) = \dfrac{x(n) - x(n-1)}{t_s}$	$s = \dfrac{z-1}{zt_s}$
Forward	$y(n) = \dfrac{x(n+1) - x(n)}{t_s}$	$s = \dfrac{z-1}{t_s}$
Central	$y(n) = \dfrac{x(n+1) - x(n-1)}{t_s}$	$s = \dfrac{z^2-1}{2zt_s}$

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Table 2: Properties of the Discrete Fourier Transform (DFT)

Signal x(n)	DFT X(k)
$a_1x_1(n)+a_2x_2(n)$	$a_1X_1(k)+a_2X_2(k)$
$x[n-n_o]$	$X_{DFT}[k]e^{-\frac{j2\pi k n_o}{N}}$
$x[n-0.5N]$	$(-1)^k X_{DFT}[k]$
$x[n]e^{j\frac{2\pi n k_0}{N}}$	$X_{DFT}[k-k_o]$
$(-1)^n x[n]$	$X_{DFT}[k-0.5N]$
$x[-n]$	$X_{DFT}[-k]$
$x[n]y[n]$	$\frac{1}{N}X_{DFT}[k]\otimes Y_{DFT}[k]$
$x[n]\otimes y[n]$	$X_{DFT}[k]Y_{DFT}[k]$
$x[n]\otimes\otimes y[n]$	$X_{DFT}[k]Y^{*}_{DFT}[k]$
$x[0]=\frac{1}{N}\sum_{k=0}^{N-1}X_{DFT}[k]$ $\quad X_{DFT}[0]=\sum_{n=0}^{N-1}x[n]$	
$x\left[\frac{N}{2}\right]=\frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (N even) $X_{DFT}\left[\frac{N}{2}\right]=\sum_{n=0}^{N-1}(-1)^n X_{DFT}[k]$ (N even)	
$\sum_{n=0}^{N-1}\lvert x[n]\rvert^2=\frac{1}{N}\sum_{k=0}^{N-1}\lvert X_{DFT}[k]\rvert^2$	

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Table 3: Properties of the z- transform

Property	Signal	z-transform
Linearity	$a_1x_1[n]+a_2x_2[n]$	$a_1X_1(z)+a_2X_2(z)$
Time reversal	$x[-n]$	$X(z^{-1})$
Time shifting	i) $x(n-k)$ ii) $x(n+k)$	i) $z^{-k}X(z)$ ii) $z^{k}X(z)$
Convolution	$x_1(n)*x_2(n)$	$X_1(z)X_2(z)$
Correlation	$r_{x_1x_2}(l)=\sum_{n=-\infty}^{\infty}x_1(n)x_2(n-l)$	$R_{x1x2}(z)=X_1(z)X_2(z^{-l})$
Scaling	$a^n x(n)$	$X(a^{-1}z)$
Differentiation	$nx[n]$	$z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$
Time differentiation	$x[n]-x[n-1]$	$X(z)(1-z^{-1})$
Time integration	$\sum_{k=0}^{\infty}X(k)$	$X(z)=\left(\frac{z}{z-1}\right)$
Initial value theorem	$\lim_{n\to 0}x(n)$	$\lim_{\lvert z\rvert\to\infty}X(z)$
Final value theorem	$\lim_{n\to\infty}x(n)$	$\lim_{\lvert z\rvert\to 1}\left(\frac{z-1}{z}\right)X(z)$

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DR. SHAHNOOR SHANTA

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Table 4: Laplace Transform Pairs

Signal $x(t)$	Laplace Transform $X(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$r(t) = tu(t)$	$\frac{1}{s^2}$
$t^2u(t)$	$\frac{2}{s^3}$
$t^nu(t)$	$\frac{n!}{s^{n+1}}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$
$t^ne^{-\alpha t}u(t)$	$\frac{n!}{(s+\alpha)^{n+1}}$

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Table 5: z-Transform Pairs

Signal $x(t)$	Sequence $x(n)$	z-Transform $X(z)$
$\delta(t)$	$\delta(n)$	1
$\delta(t - k)$	$\delta(n - k)$	z^{-k}
$u(t)$	$u(n)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$
	$-u(-n - 1)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$
$r(t) = tu(t)$	$nu(n)$	$\frac{z^{-1}}{(1-z^{-1})^2} = \frac{z}{(z-1)^2}$
	$a^n u(n)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$
	$-a^n u(-n - 1)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$
	$na^n u(n)$	$\frac{az}{(z-a)^2}$
	$-na^n u(-n - 1)$	$\frac{az}{(z-a)^2}$
e^{-at}	e^{-an}	$\frac{1}{1-e^{-a}z^{-1}} = \frac{z}{z-e^{-a}}$
t^2	$n^2 u(n)$	$z^{-1}\frac{(1+z^{-1})}{(1-z^{-1})^3} = \frac{z(z+1)}{(z-1)^3}$
te^{-at}	ne^{-an}	$\frac{z^{-1}e^{-a}}{(1-e^{-a}z^{-1})^2} = \frac{ze^{-a}}{(z-e^{-a})^2}$
$\sin\omega_o t$	$\sin\omega_o n$	$\frac{z\sin\omega_o}{z^2 - 2z\cos\omega_o + 1}$
$\cos\omega_o t$	$\cos\omega_o n$	$\frac{z(z-\cos\omega_o)}{z^2 - 2z\cos\omega_o + 1}$

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Table 6: Digital- to- digital Transformations

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-\alpha}{1-\alpha z}$	$\alpha = \frac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\frac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \frac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\frac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \frac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha K}{K+1}, \; A_2 = \frac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\frac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha}{K+1}, \; A_2 = \frac{1-K}{1+K}$

Note: The digital lowpass prototype cutoff frequency is Ω_D

All digital frequencies are normalized to $\Omega = 2\pi f/S$

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Table 7: Direct Analog- to- digital Transformations for Bilinear Design

From	Band Edges	Mapping s →	Parameters
Lowpass to lowpass	Ω_C	$\dfrac{z-1}{C(z+1)}$	$C=\tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\dfrac{C(z+1)}{z-1}$	$C=\tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1<\Omega_0<\Omega_2$	$\dfrac{z^2-2\beta z+1}{C(z^2-1)}$	$C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\dfrac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$
Lowpass to bandstop	$\Omega_1<\Omega_0<\Omega_2$	$\dfrac{C(z^2-1)}{z^2-2\beta z+1}$	$C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\dfrac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$