CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

COURSE NAME	:	DIGITAL SIGNAL PROCESSING
COURSE CODE	:	BEB 30503
PROGRAMME	:	BEJ
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL QUESTIONS FROM SECTION A** AND ANSWER **ONE (1) QUESTION ONLY FROM SECTION B**

TERBUKA

THIS QUESTION PAPER CONSISTS OF **SIXTEEN (16)** PAGES

CONFIDENTIAL

SECTION A

Q1 (a) Given the odd symmetry signal

$$x_o[n] = \{1.5, A, \underset{\Downarrow}{B}, 3.5, C\}$$

Based on your understanding, solve for the values of A, B and C.

(3 marks)

(b) From question **Q1(a),** solve the numeric sequence of $x[n]$ if $x_{\text{average}} = 2$, and the starting index of $x[n]$ is 0.

(6 marks)

(c) The impulse response of a linear time-invariant system is

$$h[n] = \{1, \underset{\Downarrow}{2}, 1, -1\}$$

Analyze the response of the system to the input signal

$$x[n] = \{\underset{\Downarrow}{1}, 2, 3, 1\}$$

(7 marks)

(d) Suppose we have a computer program that performs convolution. Suggest a method such that we can use the same computer program to perform cross-correlation.

(4 marks)

TERBUKA

Q2 (a) A signal $g[n]$ is represented by sum of the following discrete signals.

Type of discrete signal	Amplitude or multiplier	Starting index, n
Unit impulse $\delta[n]$	-4	-1
Unit ramp $r[n]$	-1	-2
Unit step $u[n]$	5	-1
Unit ramp $r[n]$	+1	+3

Analyze the numeric sequence of $g[n]$.

(5 marks)

(b) Based on the answer obtained from **Q2 (a)**, solve for

$$y[n] = g[2n + 0.75]$$

assuming linear interpolation if needed.

(5 marks)

(c) Consider an analog signal

$$x(t) = 3\cos 100\,\pi t$$

(i) Determine the minimum sampling rate required to avoid aliasing.

(2 marks)

(ii) Suppose that the signal is sampled at the rate $F_s = 200\text{Hz}$. Construct the discrete-time signal after sampling.

(2 marks)

(iii) Suppose that the signal is sampled at the rate $F_s = 75\text{Hz}$. Construct the discrete-time signal after sampling.

(2 marks)

(iv) Deduce the frequency $0 < F < F_s/2$ of a sinusoid that yields samples identical to those samples in **Q2(c)(iii)**.

(4 marks)

Q3 (a) Consider a system in which the impulse response is known to be

$$h(n) = u(n) \text{ for } n \geq 0$$

Analyze the output $y(n)$ for the input $x(n) = (0.5)^n + (0.6)^{n+1}$

(4 marks)

(b) Given an input signal $x[n] = 0.2^n u[n]$ into a linear time-invariant system with the impulse response $h[n] = 0.4^n u[n]$.
Analyze the cross-correlation $r_{xh}[n]$ for $n \geq 0$

(4 marks)

(c) Proof that the highest rate of oscillation in a discrete-time sinusoid is obtained when the angular frequency is $\omega = \pi$ rad/sample or equivalently the frequency is $f = 1/2$ cycles/sample.
Utilize $x(n) = \cos \omega n$ as the example.

(4 marks)

(d) Consider an analog signal

$$x(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$$

(i) Determine the Nyquist rate for this signal

(2 marks)

(ii) Assume now that we sample this signal using a sampling rate $F_s = 5000$ samples/s. Construct the discrete-time signal obtained after sampling.

(4 marks)

(iii) Deduce what the output analog signal will be if we reconstruct from the samples utilizing ideal interpolation.

(2 marks)

Q4 (a) List **TWO (2)** types of Fast Fourier Transform (FFT) algorithm.

(2 marks)

(b) Given the Discrete Fourier Transform (DFT) pair;

$$x[n] = \{\overset{\Downarrow}{A}, B, 1, -4\} \Leftrightarrow X_{DFT}[k] = \{\overset{\Downarrow}{-1}, 1 - j4, 7, 1 + j4\}$$

Calculate the following signal using DFT properties;

(i) value of A and B

(4 marks)

(ii) $Y_{DFT}[k]$ if $y[n] = x[n] \otimes x[n]$

(3 marks)

(iii) $G_{DFT}[k]$ if $g[n] = x[-n]$

(2 marks)

(c) Prove the DFT pair given in **Q4(b)** using Fast Fourier Transform (FFT) algorithm as in **Figure Q4**.

(9 marks)

SECTION B

Q5 (a) Consider the linear time invariance (LTI) system for which the input $x[n]$ and output $y[n]$ satisfy the linear constant-coefficient difference equation

$$y[n]-\frac{1}{2}y[n-1]=x[n]+\frac{1}{3}x[n-1]$$

(i) Compute the impulse response $h[n]$ for $|z|>1$.

(5 marks)

(ii) Determine the stability and causality of the system.

(3 marks)

(b) Digital filter for processing speech signals has the following specification call for a passband of 5 kHz and a stopband of 4 kHz. The sampling frequency is 10 kHz.

(i) State the filter type.

(2 marks)

(ii) Design FIR filter using suitable window with $N=5$.

(10 marks)

Q6 (a) Consider a rectangular signal

$$x[n] = \begin{cases} 1, & 0 \le n \le 5 \\ 0, & \text{otherwise} \end{cases}$$

Let $g[n] = x[n] - x[n-1]$,

(i) Compute the signal $g[n]$, $G(z)$ and its region of convergence (ROC).

(5 marks)

(ii) Note that $y[n] = 2^n g[n]$.

Determine the z-transform of $y[n]$.

(3 marks)

(b) Digital filter for processing speech signals has the following specification call for a passband of 4 kHz and a stopband of 5 kHz. The sampling frequency is 10 kHz.

(i) State the filter type.

(2 marks)

(ii) Design Infinite Impulse Response (IIR) Butterworth using the impulse invariance and bilinear transformation to meet this specifications, with $C = 4$.

(10 marks)

-END OF QUESTIONS-

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2016/2017 PROGRAMME: BEJ

COURSE NAME: DIGITAL SIGNAL PROCESSING COURSE CODE: BEB 30503

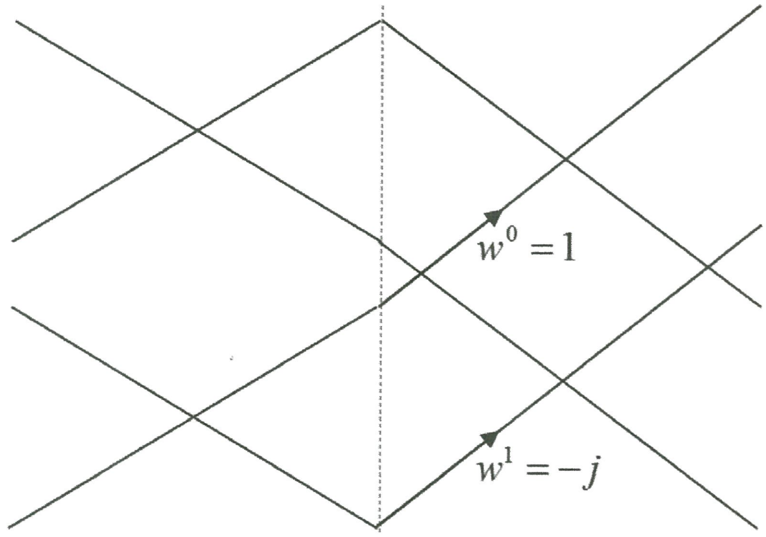

FIGURE Q4

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2016/2017 PROGRAMME: BEJ

COURSE NAME:DIGITAL SIGNAL PROCESSING COURSE CODE: BEB 30503

Table 1: Properties of the Discrete Fourier Transform (DFT)

Property	Signal	DFT	Remarks
Shift	$x[n - n_o]$	$X_{DFT}[k] = e^{-j2\pi kn_o/N}$	No change in magnitude.
Shift	$x[n-0.5N]$	$(-1)^k X_{DFT}[k]$	Half-period shift for even N.
Modulation	$x[n]e^{j2\pi nk_o/N}$	$X_{DFT}[k-k_o]$	
Modulation	$(-1)^n x[n]$	$X_{DFT}[k-0.5N]$	Half-period shift for even N.
Folding	$x[-n]$	$X_{DFT}[-k]$	This is circular folding.
Product	$x[n]y[n]$	$\frac{1}{N}X_{DFT}[k] \otimes Y_{DFT}[k]$	The convolution is periodic.
Convolution	$x[n] \otimes y[n]$	$X_{DFT}[k]Y_{DFT}[k]$	The convolution is periodic.
Correlation	$x[n] \otimes \otimes y[n]$	$X_{DFT}[k]Y^*_{DFT}[k]$	The correlation is periodic.
Central Ordinates	$x[0] = \frac{1}{N}\sum_{k=0}^{N-1} X_{DFT}[k], \quad X_{DFT}[0] = \sum_{n=0}^{N-1} x[n]$		
Central Ordinates	$x\left[\frac{N}{2}\right] = \frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (N even), $X_{DFT}\left[\frac{N}{2}\right] = \sum_{n=0}^{N-1}(-1)^n x[n]$ (N even)		
Parseval's Relation	$\sum_{n=0}^{N-1} \lvert x[n] \rvert^2 = \frac{1}{N}\sum_{k=0}^{N-1} \lvert X_{DFT}[k] \rvert^2$		

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2016/2017 PROGRAMME: BEJ

COURSE NAME:DIGITAL SIGNAL PROCESSING COURSE CODE: BEB 30503

Table 2: Properties of the z- transform

Property	**Signal**	**z-transform**
Linearity	$a_1x_1[n]+a_2x_2[n]$	$a_1X_1(z)+a_2X_2(z)$
Time reversal	$x[-n]$	$X(z^{-1})$
Time shifting	i) $x(n-k)$ ii) $x(n+k)$	i) $z^{-k}X(z)$ ii) $z^{k}X(z)$
Convolution	$x_1(n)*x_2(n)$	$X_1(z)X_2(z)$
Correlation	$r_{x_1x_2}(l)=\sum_{n=-\infty}^{\infty}x_1(n)x_2(n-l)$	$R_{x1x2}(z)=X_1(z)X_2(z^{-l})$
Scaling	$a^nx(n)$	$X(a^{-1}z)$
Differentiation	$nx[n]$	$z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$
Time differentiation	$x[n]-x[n-1]$	$X(z)(1-z^{-1})$
Time integration	$\sum_{k=0}^{\infty}X(k)$	$X(z)=\left(\frac{z}{z-1}\right)$
Initial value theorem	$\lim_{n\to 0}x(n)$	$\lim_{\lvert z\rvert\to\infty}X(z)$
Final value theorem	$\lim_{n\to\infty}x(n)$	$\lim_{\lvert z\rvert\to 1}\left(\frac{z-1}{z}\right)X(z)$

TERBUKA

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2016/2017 PROGRAMME: BEJ

COURSE NAME:DIGITAL SIGNAL PROCESSING COURSE CODE: BEB 30503

Table 3: z-Transform Pairs

Signal $x(t)$	Sequence $x(n)$	z-Transform $X(z)$
$\delta(t)$	$\delta(n)$	1
$\delta(t - k)$	$\delta(n - k)$	z^{-k}
$u(t)$	$u(n)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$
	$-u(-n - 1)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$
$r(t) = tu(t)$	$nu(n)$	$\frac{z^{-1}}{(1-z^{-1})^2} = \frac{z}{(z-1)^2}$
	$a^n u(n)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$
	$-a^n u(-n - 1)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$
	$na^n u(n)$	$\frac{az}{(z-a)^2}$
	$-na^n u(-n - 1)$	$\frac{az}{(z-a)^2}$
e^{-at}	e^{-an}	$\frac{1}{1-e^{-a}z^{-1}} = \frac{z}{z-e^{-a}}$
t^2	$n^2 u(n)$	$z^{-1}\frac{(1+z^{-1})}{(1-z^{-1})^3} = \frac{z(z+1)}{(z-1)^3}$
te^{-at}	ne^{-an}	$\frac{z^{-1}e^{-a}}{(1-e^{-a}z^{-1})^2} = \frac{ze^{-a}}{(z-e^{-a})^2}$
$\sin\omega_o t$	$\sin\omega_o n$	$\frac{z\sin\omega_o}{z^2 - 2z\cos\omega_o + 1}$
$\cos\omega_o t$	$\cos\omega_o n$	$\frac{z(z-\cos\omega_o)}{z^2 - 2z\cos\omega_o + 1}$

TERBUKA

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2016/2017 PROGRAMME: BEJ

COURSE NAME:DIGITAL SIGNAL PROCESSING COURSE CODE: BEB 30503

Table 4: Digital- to- digital Transformations

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-\alpha}{1-\alpha z}$	$\alpha = \frac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\frac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \frac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\frac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \frac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha K}{K+1}$, $A_2 = \frac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\frac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha}{K+1}$, $A_2 = \frac{1-K}{1+K}$

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2016/2017 PROGRAMME: BEJ

COURSE NAME: DIGITAL SIGNAL PROCESSING COURSE CODE: BEB 30503

Table 5: Direct Analog- to- digital Transformations for Bilinear Design

From	Band Edges	Mapping s →	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-1}{C(z+1)}$	$C = \tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\frac{C(z+1)}{z-1}$	$C = \tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1 < \Omega_0 < \Omega_2$	$\frac{z^2-2\beta z+1}{C(z^2-1)}$	$C = \tan[0.5(\Omega_2-\Omega_1)], \beta = \cos\Omega_0$ or $\beta = \frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$
Lowpass to bandstop	$\Omega_1 < \Omega_0 < \Omega_2$	$\frac{C(z^2-1)}{z^2-2\beta z+1}$	$C = \tan[0.5(\Omega_2-\Omega_1)], \beta = \cos\Omega_0$ or $\beta = \frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$

Table 6: Windows for FIR filter design.

Window	Expression $w_N[n]$, $-0.5(N-1) \leq n \leq 0.5(N-1)$
Boxcar	1
Cosine	$\cos\left(\frac{n\pi}{N-1}\right)$
Riemann	$\text{sinc}^L\left(\frac{2n}{N-1}\right), L > 0$
Bartlett	$1-\frac{2\lvert n\rvert}{N-1}$
Von Hann (Hanning)	$0.5+0.5\cos\left(\frac{2n\pi}{N-1}\right)$
Hamming	$0.54+0.46\cos\left(\frac{2n\pi}{N-1}\right)$
Blackman	$0.42+0.5\cos\left(\frac{2n\pi}{N-1}\right)+0.08\cos\left(\frac{4n\pi}{N-1}\right)$
Kaiser	$\frac{I_0\left(\pi\beta\sqrt{1-4\left(\frac{n}{N-1}\right)^2}\right)}{I_0(\pi\beta)}$

Table 7:Characteristics of the windowed spectrum for various windows.

Window	**Peak Ripple** $\delta_p = \delta_s$	**Passband Attenuation** A_{WP}(dB)	**Peak Sidelobe Attenuation** A_{WS} (dB)	**Transition Width** $F_{WS} \approx C/N$
Boxcar	0.0897	1.5618	21.7	$C = 0.92$
Cosine	0.0207	0.3600	33.8	$C = 2.10$
Riemann	0.0120	0.2087	38.5	$C = 2.50$
von Hann (Hanning)	0.0063	0.1103	44.0	$C = 3.21$
Hamming	0.0022	0.0384	53.0	$C = 3.47$
Blackman	1.71×10^{-4}	2.97×10^{-3}	75.3	$C = 5.71$

FINAL EXAMINATION

SEMESTER/SESSION: SEM I/2016/2017 PROGRAMME: BEJ

COURSE NAME:DIGITAL SIGNAL PROCESSING COURSE CODE: BEB 30503

Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

Finite Summation Formula

$$\sum_{k=0}^{n} \alpha = \frac{n(n+1)}{2}$$

$$\sum_{k=0}^{n} \alpha^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=0}^{n} \alpha^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{n} k\alpha^k = \frac{\alpha\left[1-(n+1)\alpha^n + n\alpha^{n+1}\right]}{(1-\alpha)^2}$$

$$\sum_{k=0}^{n} k^2\alpha^k = \frac{\alpha\left[(1+\alpha)-(n+1)^2\alpha^n + (2n^2+2n-1)\alpha^{n+1} - n^2\alpha^{n+2}\right]}{(1-\alpha)^3}$$

Infinite Summation Formula

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \quad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} \alpha^k = \frac{\alpha}{1-\alpha}, \quad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k\alpha^k = \frac{\alpha}{(1-\alpha)^2}, \quad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k^2\alpha^k = \frac{\alpha^2+\alpha}{(1-\alpha)^3}, \quad |\alpha| < 1$$

$$\sum_{k=-\infty}^{\infty} e^{-\alpha|k|} = \frac{1+e^{-\alpha}}{1-e^{-\alpha}}, \quad \alpha > 0$$