Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	: DIGITAL SIGNAL PROCESSING
COURSE CODE	: BEB 30503
PROGRAMME	: BEJ
EXAMINATION DATE	: JUNE/JULY 2016
DURATION	: 3 HOURS
INSTRUCTION	: ANSWER **ALL QUESTIONS FROM SECTION A** AND ANSWER **ONE (1) QUESTION ONLY FROM SECTION B**

THIS QUESTION PAPER CONSISTS OF **FOURTEEN (14)** PAGES

SECTION A

Q1 (a) Consider a periodic signal

$$x[n] = 3rect\left(\frac{n+1}{4}\right) - 3\delta[n+3] + 2tri\left(\frac{n+A}{2}\right)$$

Which has period $N = 5$.

(i) Based on the period of $x[n]$, examine the possibilities of A if $A \geq 1$.

(5 marks)

(ii) From **Q1 (a)(i)**, find the numeric sequence of $x[n]$ for ALL possibilities.

(2 marks)

(b) Choose the most appropriate $x[n]$ if the average power of this signal is 12 watt.

(3 marks)

(c) Given a signal $x[n] = 0.4^n\, u[n-3]$ and $h[n] = 0.2^{n-1}u[n]$. Calculate the convolution signal, $y[n] = x[n] * h[n]$.

(8 marks)

(d) Briefly explain the process of periodic convolution by using cyclic method.

(2 marks)

Q2 A group of students are assigned to digitalize an audio signal, $w(t) = 3\sin(1200\pi t) + \sin(900\pi t)$ over $0 \le t \le 3\ ms$ using system '**A**'. Below are the requirements to complete the task.

Task requirements

The audio signal is sampled at 25% above the Nyquist rate. Then the signal is quantized with the dynamic range that varies ± 10 volts and the resolution of 2.5 volts.

(a) Illustrate the diagram of system '**A**'.

(3 marks)

(b) Analyse the quantized signal, $W_{Q1}[n]$ and digital signal, $W_{C1}[n]$ using truncation technique.

(11 marks)

(c) An input signal is written as:

$$x[n] = \{\overset{\Downarrow}{3}, 7, 4, -1\} \Leftrightarrow X_{DFT}[k] = \{\overset{\Downarrow}{3}, -1-8j, 1, -1+8j\}.$$

Using suitable properties, calculate $r[n] = x[n]x[n]$ and $R_{DFT}[k]$.

(6 marks)

Q3 (a) Define the Region of Convergence (ROC) in z-transform.

(3 marks)

(b) Calculate the z-transforms and ROC of the following:

(i) $x[n]=(n+1)(2)^{n}u[n]$

(3 marks)

(ii) $x[n]=(n-1)(2)^{n+2}u[n]$

(3 marks)

(c) The z-transform of $x[n]$ is $X(z)=\dfrac{4z}{(z+0.5)^2}, |z|>0.5$. Solve the z-transform of the following using properties and specify the ROC.

(i) $y[n]=(2)^{n}n\,x[n]$

(3 marks)

(ii) $h[n]=x[n]-x[n-1]$

(3 marks)

(d) Assume that $x[n]$ as a right-sight signal. Produce the $x[n]$ of the following z-transform using partial fractions.

$$X(z)=\frac{3z^3}{(z^2-1.5z+0.5)(z-0.25)}$$

(5 marks)

Q4 (a) State two comparisons betwen the Infinite Impulse Response (IIR) filter and Finite Impulse Response (FIR) filter.

(4 marks)

(b) Explain why the digital filter is better than analog filter.

(5 marks)

(c) Design a bandpass digital filter with band edges of 1 kHz and 3 kHz using the digital-to-digital frequency transformations technique of IIR filter with the digital Low Pass Filter $H(z)=\dfrac{z+1}{z^2-z+0.2}$. This filter has a cutoff frequency of 0.5 kHz and operates at a sampling frequency of 10 kHz.

(11 marks)

SECTION B

Q5 (a) Find the periodic convolution between $x_p[n]=\{-2,\overset{\Downarrow}{0},1,-4\}$ and $h_p[n]=\{-6,0,\overset{\Downarrow}{2},1\}$ using appropriate method.

(6 marks)

(b) Analyse the discrete signal, $x[n]$ in terms of unit ramps and steps.

$$x[n] = 4\delta[n-2] + 2\delta[n] + 3\delta[n+1] + \delta[n+2]$$

(10 marks)

(c) A signal $x(t) = \sin(20\pi t)$ is ideally sampled at 15 Hz. Estimate the signal y(t) that is recovered if sampled signal is passed through an ideal filter with cutoff frequency, $f_c = 8\,Hz$.

(4 marks)

Q6 (a) Find $y[n] = x[2n - 0.75]$ using linear interpolation if a numeric sequence is $x[n] = \{2, \overset{\Downarrow}{7}, -3\}$.

(6 marks)

(b) Analyse $x[n] \Leftrightarrow X_{DFT}[k] = \{\overset{\Downarrow}{4}, -2j, 0, 2j\}$ using the Fast Fourier Transform (FFT) approach.

(10 marks)

(c) Sequences $x_1[n]$ and $x_2[n]$ are given below;

$$x_1[n] = 3\delta[n] + 2\delta[n-1]$$
$$x_2[n] = 2\delta[n] - \delta[n-1]$$

Express the z-transform of their convolution using properties.

(4 marks)

-END OF QUESTIONS-

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Table 1: Properties of the Discrete Fourier Transform (DFT)

Property	**Signal**	**DFT**	**Remarks**
Shift	$x[n - n_o]$	$X_{DFT}[k] = e^{-j2\pi k n_o/N}$	No change in magnitude.
Shift	$x[n - 0.5N]$	$(-1)^k X_{DFT}[k]$	Half-period shift for even N.
Modulation	$x[n]e^{j2\pi n k_o/N}$	$X_{DFT}[k - k_o]$	
Modulation	$(-1)^n x[n]$	$X_{DFT}[k - 0.5N]$	Half-period shift for even N.
Folding	$x[-n]$	$X_{DFT}[-k]$	This is circular folding.
Product	$x[n]y[n]$	$\frac{1}{N} X_{DFT}[k] \otimes Y_{DFT}[k]$	The convolution is periodic.
Convolution	$x[n] \otimes y[n]$	$X_{DFT}[k] Y_{DFT}[k]$	The convolution is periodic.
Correlation	$x[n] \otimes \otimes y[n]$	$X_{DFT}[k] Y^{*}_{DFT}[k]$	The correlation is periodic.
Central Ordinates	$x[0] = \frac{1}{N}\sum_{k=0}^{N-1} X_{DFT}[k]$, $X_{DFT}[0] = \sum_{n=0}^{N-1} x[n]$		
Central Ordinates	$x\left[\frac{N}{2}\right] = \frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (N even), $X_{DFT}\left[\frac{N}{2}\right] = \sum_{n=0}^{N-1}(-1)^n x[n]$ (N even)		
Parseval's Relation	$\sum_{n=0}^{N-1}\left\|x[n]\right\|^2 = \frac{1}{N}\sum_{k=0}^{N-1}\left\|X_{DFT}[k]\right\|^2$		

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Table 2: Properties of the z- transform

Property	**Signal**	**z-transform**
Linearity	$a_1x_1[n]+a_2x_2[n]$	$a_1X_1(z)+a_2X_2(z)$
Time reversal	$x[-n]$	$X(z^{-1})$
Time shifting	i) $x(n-k)$ ii) $x(n+k)$	i) $z^{-k}X(z)$ ii) $z^{k}X(z)$
Convolution	$x_1(n)*x_2(n)$	$X_1(z)X_2(z)$
Correlation	$r_{x_1x_2}(l)=\sum_{n=-\infty}^{\infty}x_1(n)x_2(n-l)$	$R_{x1x2}(z)=X_1(z)X_2(z^{-l})$
Scaling	$a^nx(n)$	$X(a^{-1}z)$
Differentiation	$nx[n]$	$z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$
Time differentiation	$x[n]-x[n-1]$	$X(z)(1-z^{-1})$
Time integration	$\sum_{k=0}^{\infty}X(k)$	$X(z)=\left(\frac{z}{z-1}\right)$
Initial value theorem	$\lim_{n\to 0}x(n)$	$\lim_{\lvert z\rvert\to\infty}X(z)$
Final value theorem	$\lim_{n\to\infty}x(n)$	$\lim_{\lvert z\rvert\to 1}\left(\frac{z-1}{z}\right)X(z)$

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Table 3: z-Transform Pairs

Signal $x(t)$	Sequence $x(n)$	z-Transform $X(z)$
$\delta(t)$	$\delta(n)$	1
$\delta(t - k)$	$\delta(n - k)$	z^{-k}
$u(t)$	$u(n)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$
	$-u(-n - 1)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$
$r(t) = tu(t)$	$nu(n)$	$\frac{z^{-1}}{(1-z^{-1})^2} = \frac{z}{(z-1)^2}$
	$a^n u(n)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$
	$-a^n u(-n - 1)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$
	$na^n u(n)$	$\frac{az}{(z-a)^2}$
	$-na^n u(-n - 1)$	$\frac{az}{(z-a)^2}$
e^{-at}	e^{-an}	$\frac{1}{1-e^{-a}z^{-1}} = \frac{z}{z-e^{-a}}$
t^2	$n^2 u(n)$	$z^{-1}\frac{(1+z^{-1})}{(1-z^{-1})^3} = \frac{z(z+1)}{(z-1)^3}$
te^{-at}	ne^{-an}	$\frac{z^{-1}e^{-a}}{(1-e^{-a}z^{-1})^2} = \frac{ze^{-a}}{(z-e^{-a})^2}$
$\sin\omega_o t$	$\sin\omega_o n$	$\frac{z\sin\omega_o}{z^2-2z\cos\omega_o+1}$
$\cos\omega_o t$	$\cos\omega_o n$	$\frac{z(z-\cos\omega_o)}{z^2-2z\cos\omega_o+1}$

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Table 4: Digital- to- digital Transformations

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-\alpha}{1-\alpha z}$	$\alpha = \frac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\frac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \frac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\frac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \frac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha K}{K+1}, \quad A_2 = \frac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\frac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha}{K+1}, \quad A_2 = \frac{1-K}{1+K}$

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Table 5: Direct Analog- to- digital Transformations for Bilinear Design

From	Band Edges	Mapping s →	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-1}{C(z+1)}$	$C=\tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\frac{C(z+1)}{z-1}$	$C=\tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1<\Omega_0<\Omega_2$	$\frac{z^2-2\beta z+1}{C(z^2-1)}$	$C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$
Lowpass to bandstop	$\Omega_1<\Omega_0<\Omega_2$	$\frac{C(z^2-1)}{z^2-2\beta z+1}$	$C=\tan[0.5(\Omega_2-\Omega_1)], \beta=\cos\Omega_0$ or $\beta=\frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$

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Table 6: Windows for FIR filter design.

Window	**Expression** $w_N[n]$, $-0.5(N - 1) \leq n \leq 0.5(N - 1)$
Boxcar	1
Cosine	$\cos\left(\frac{n\pi}{N-1}\right)$
Riemann	$\text{sinc}^L\left(\frac{2n}{N-1}\right),\ L>0$
Bartlett	$1-\frac{2\lvert n\rvert}{N-1}$
Von Hann (Hanning)	$0.5+0.5\cos\left(\frac{2n\pi}{N-1}\right)$
Hamming	$0.54+0.46\cos\left(\frac{2n\pi}{N-1}\right)$
Blackman	$0.42+0.5\cos\left(\frac{2n\pi}{N-1}\right)+0.08\cos\left(\frac{4n\pi}{N-1}\right)$
Kaiser	$\frac{I_0\left(\pi\beta\sqrt{1-4\left(\frac{n}{N-1}\right)^2}\right)}{I_0(\pi\beta)}$

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Table 7: Characteristics of the windowed spectrum for various windows.

Window	Peak Ripple $\delta_p = \delta_s$	Passband Attenuation A_{WP} (dB)	Peak Sidelobe Attenuation A_{WS} (dB)	Transition Width $F_{WS} \approx C/N$
Boxcar	0.0897	1.5618	21.7	$C = 0.92$
Cosine	0.0207	0.3600	33.8	$C = 2.10$
Riemann	0.0120	0.2087	38.5	$C = 2.50$
von Hann (Hanning)	0.0063	0.1103	44.0	$C = 3.21$
Hamming	0.0022	0.0384	53.0	$C = 3.47$
Blackman	1.71×10^{-4}	2.97×10^{-3}	75.3	$C = 5.71$

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Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

Finite Summation Formula

$$\sum_{k=0}^{n} \alpha = \frac{n(n+1)}{2}$$

$$\sum_{k=0}^{n} \alpha^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=0}^{n} \alpha^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{n} k\alpha^k = \frac{\alpha\left[1-(n+1)\alpha^n + n\alpha^{n+1}\right]}{(1-\alpha)^2}$$

$$\sum_{k=0}^{n} k^2\alpha^k = \frac{\alpha\left[(1+\alpha)-(n+1)^2\alpha^n + (2n^2+2n-1)\alpha^{n+1} - n^2\alpha^{n+2}\right]}{(1-\alpha)^3}$$

Infinite Summation Formula

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \quad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} \alpha^k = \frac{\alpha}{1-\alpha}, \quad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k\alpha^k = \frac{\alpha}{(1-\alpha)^2}, \quad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k^2\alpha^k = \frac{\alpha^2+\alpha}{(1-\alpha)^3}, \quad |\alpha| < 1$$

$$\sum_{k=-\infty}^{\infty} e^{-\alpha|k|} = \frac{1+e^{-\alpha}}{1-e^{-\alpha}}, \quad \alpha > 0$$