CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER 2
SESSION 2015/2016

COURSE NAME	:	SIGNALS & SYSTEMS
COURSE CODE	:	BEB 20203
PROGRAMME	:	BEJ
EXAMINATION DATE	:	JUNE/JULY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **TEN** (10) PAGES

ANSWER ALL QUESTIONS

Q1. (a) A continuous time signal is defined as:

$$x(t) = 2[u(t) - u(t-2)]$$

(i) Illustrate the even part and odd part of the signal.

(5 marks)

(ii) Sketch the output $y(t) = x(2t-3)$.

(5 marks)

(b) Differentiate energy and power signals.

(3 marks)

(c) Given that:

$$r(t) = \sqrt{2}\sin 4\pi t + \frac{5}{3}\cos 8\pi t$$

Verify whether $r(t)$ is periodic. If periodic, determine the fundamental period.

(7 marks)

Q2. (a) Define causal system by giving one appropriate example.

(5 marks)

(b) Test the causality of the signal, $s(t)$, given by:

$$s(t) = x(t) + \int_0^t x(\lambda)d\lambda$$

(4 marks)

(c) By using convolution integral, find the overall impulse response $H(t)$ for two cascaded systems with the system impulse responses, $h_1(t)$ and $h_2(t)$ expressed as:

$$h_1(t) = 5e^{-3t}\,;$$
$$h_2(t) = 2e^{-7t}$$

(5 marks)

(d) State the commutative property of convolution. Then, show that the answer obtained in **Q2 (c)** satisfies the commutative property of convolution.

(6 marks)

Q3. (a) State any TWO (2) properties of Fourier Series. (4 marks)

(b) The following signals, $p(t)$and $q(t)$ are the experimental output signals from a signal generator. With the aid of a diagram, plot the magnitude and phase spectrums of both signals.

(i) $p(t) = 2 + 4\cos\left(50t + \frac{\pi}{2}\right) + 12\cos(100t - \frac{\pi}{3})$ (4 marks)

(ii) $q(t) = 4\cos(2\pi(1000t)\cos(2\pi 750000t)$ (4 marks)

(c) The output of an RLC circuit is shown in **Figure Q3 (c).** Find the first 5 components of the trigonometric Fourier series for the waveform. Assume that $\omega = 1$.

(8 marks)

Q4. (a) Explain how Fourier transform is obtained from Fourier Series. (4 marks)

(b) Given two signals as shown in **Figure Q4 (b)(i)** and **Figure Q4 (b)(ii)**, determine the Fourier transform and sketch the magnitude spectrum of each signal.

(10 marks)

(c) Frequency domain analysis and Fourier transform are cornerstone of signal and system analysis. Let $\mathcal{F}_1(\omega)$ is the Fourier transform of $f_1(t)$ and $\mathcal{F}_2(\omega)$ is the Fourier transform of $f_2(t)$. Show that multiplication in the time domain, corresponds to convolution in the frequency domain divided by the constant 1/2π, as shown in the mathematical equation below.

$$f_1(t)f_2(t) \Leftrightarrow \frac{1}{2\pi}\mathcal{F}_1(\omega) * \mathcal{F}_2(\omega)$$

(6 marks)

Q5. (a) Define region of convergence (R.O.C) and write any two properties of R.O.C of Laplace transform.

(4 marks)

(b) The output of an LTI system can be easily determined in s-domain using the convolution property of Laplace transform. If a signal

$$x(t) = e^{-2t}\big(u(t) - u(t-3)\big)$$

is an input to a system with the impulse response given by

$$h_1(t) = 3e^{-3t}u(t),$$

(i) Determine the output $y(t)$ using the Laplace transform convolution property.

(10 marks)

(ii) The system $h_1(t)$ is cascaded in series to another system $h_2(t)$ with its transfer function given by

$$H_2(s) = \frac{s-1}{s-2}$$

forming a new system $h(t)$. Determine the total response of the new system, $h(t)$ if the system is stable.

(6 marks)

- END OF QUESTIONS -

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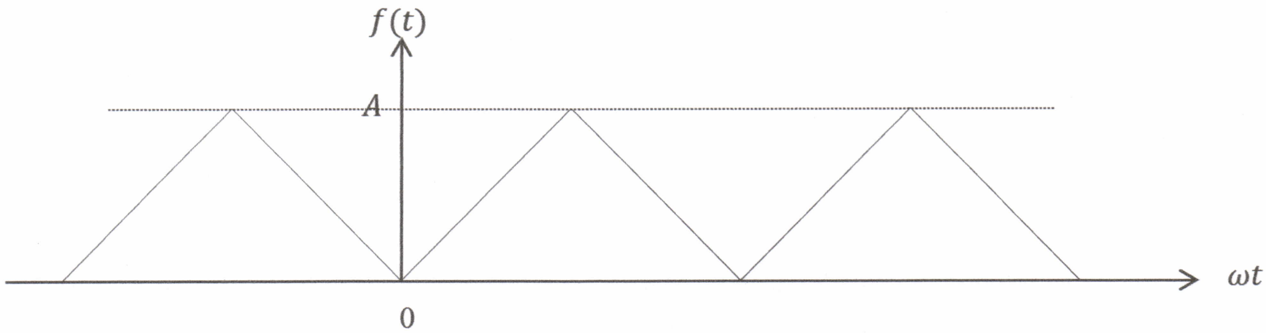

Figure Q3 (c)

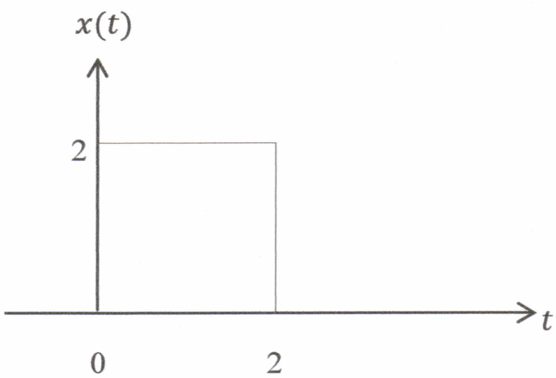

Figure Q4 (b)(i)

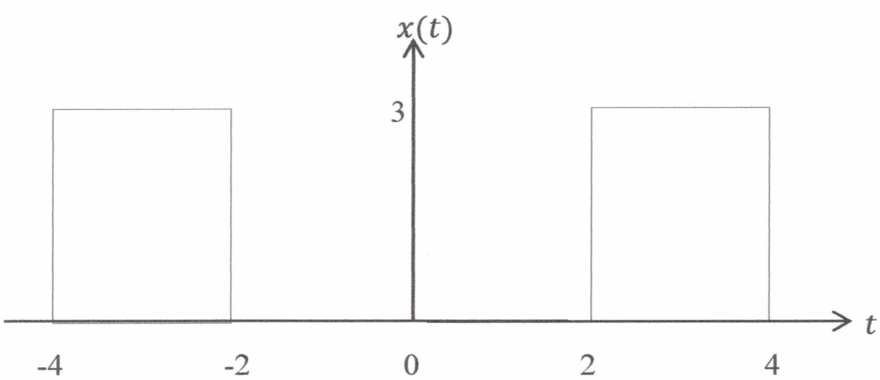

Figure Q4 (b)(ii)

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TABLE 1: INDEFINITE INTEGRALS

$\int \cos at\, dt = \frac{1}{a} \sin at$	$\int \sin at\, dt = -\frac{1}{a} \cos at$
$\int t \cos at\, dt = \frac{1}{a^2} \cos at + \frac{1}{a} t \sin at$	$\int t \sin at\, dt = \frac{1}{a^2} \sin at - \frac{1}{a} t \cos at$

TABLE 2: EULER'S IDENTITY

$e^{\pm \frac{j\pi}{2}} = \pm j$	$A\angle \pm \theta = Ae^{\pm j\theta}$
$e^{\pm jn\pi} = \cos(n\pi)$	$e^{\pm j\theta} = \cos\theta \pm j \sin\theta$
$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$	$\sin\theta = \frac{1}{2}(e^{j\theta} - e^{-j\theta})$

TABLE 3: TRIGONOMETRIC IDENTITIES

$\sin\alpha = \cos\left(\alpha - \frac{\pi}{2}\right)$	$\cos\alpha = \sin\left(\alpha + \frac{\pi}{2}\right)$
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 4: VALUES OF COSINE, SINE AND EXPONENTIAL FUNCTIONS FOR INTEGRAL MULTIPLE OF π

Function	Value	Function	Value
$\cos(2n\pi)$	1	$e^{\frac{jn\pi}{2}}$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = even \\ (-1)^{\frac{n-1}{2}} & ,n = odd \end{cases}$
$\sin(2n\pi)$	0		
$\cos(n\pi)$	$(-1)^n$	$\cos\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = even \\ 0 & ,n = odd \end{cases}$
$\sin(n\pi)$	0		
$e^{j2n\pi}$	1	$\sin\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n-1}{2}} & ,n = even \\ (-1)^{\frac{n+1}{2}} & ,n = odd \end{cases}$
$e^{jn\pi}$	$(-1)^n$		

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TABLE 5: FOURIER SERIES

Exponential	$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$, $x_n = \frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t}$
Trigonometric	$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\frac{2\pi}{T}t + b_n \sin n\frac{2\pi}{T}t$ $a_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\cos n\frac{2\pi}{T}t\ dt$ $b = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\sin n\frac{2\pi}{T}t\, dt$
Amplitude-phase	$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2\pi n}{T}t + \angle\phi_n\right)$

FOURIER TRANSFORM

$$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

INVERSE FOURIER TRANSFORM

$$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$$

LAPLACE TRANSFORM

$$\mathcal{L}[x(t)] = X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$$

INVERSE LAPLACE TRANSFORM

$$x(t) = \mathcal{L}^{-1}[X(s)] = \frac{1}{j2\pi}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$$

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TABLE 6: FOURIER TRANSFORM PAIRS

Time domain, $x(t)$	Frequency domain, $X(\omega)$	Time domain, $x(t)$	Frequency domain, $X(\omega)$
$\delta(t)$	1	$t^n e^{-at} u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$
1	$2\pi\delta(\omega)$	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	$e^{-j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$u(t-\tau)-u(t+\tau)$	$2\frac{\sin\omega\tau}{\omega}$	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$
$\|t\|$	$\frac{-2}{\omega^2}$	$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$		

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TABLE 7: FOURIER TRANSFORM PROPERTIES

Property	**Time domain, $\boldsymbol{x(t)}$**	**Frequency domain, $\boldsymbol{X(\omega)}$**
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t-t_0)u(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$
Modulation	$\cos(\omega_0 t)\,x(t)$	$\frac{1}{2}[X(\omega+\omega_0) + X(\omega-\omega_0)]$
Time differentiation	$\frac{d}{dt}(x(t))$ $\frac{d^n}{dt^n}(x(t))$	$j\omega X(\omega)$ $(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{F(\omega)}{j\omega} + \pi\,F(0)\delta(\omega)$
Frequency differentiation	$t^n x(t)$	$(j)^n \frac{d^n}{d\omega^n}X(\omega)$
Time Reversal	$x(-t)$	$X(-\omega)\ or\ X^*(\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Convolution in t	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Convolution in ω	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$

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TABLE 8: LAPLACE TRANSFORM

$x(t),\ t > 0$	$X(s)$	$x(t),\ t > 0$	$X(s)$
$\delta(t)$	1	$\cos bt$	$\frac{s}{s^2+b^2}$
$u(t)$	$\frac{1}{s}$	$\sin bt$	$\frac{b}{s^2+b^2}$
t	$\frac{1}{s^2}$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
t^n	$\frac{n!}{s^{n+1}}$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
e^{-at}	$\frac{1}{s+a}$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$

TABLE 9: LAPLACE TRANSFORM PROPERTIES

Name	Operation in Time Domain	Operation in Frequency Domain
1. Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$
2. Differentiation	$\frac{d^nx(t)}{dt^n}$	$s^nX(s) - s^{n-1}x(0^-) - \cdots - x^{(n-1)}(0^-)$
3. Integration	$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\frac{X(s)}{s} + \frac{x^{(-1)}(0^-)}{s}$
4. s-shift	$x(t)\exp(-\alpha t)$	$X(s+\alpha)$
5. Delay	$x(t-t_0)u(t-t_0)$	$X(s)\exp(-st_0)$
6. Convolution	$x_1(t) * x_2(t) = \int_0^{\infty} x_1(\lambda)x_2(t-\lambda)\, d\lambda$	$X_1(s)X_2(s)$
7. Product	$x_1(t)x_2(t)$	$\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X_1(s-\lambda)X_2(\lambda)\, d\lambda$
8. Initial value (provided limits exist)	$\lim_{t\to 0^+} x(t)$	$\lim_{s\to\infty} sX(s)$
9. Final value (provided limits exist)	$\lim_{t\to\infty} x(t)$	$\lim_{s\to 0} sX(s)$
10. Time scaling	$x(at),\quad a > 0$	$a^{-1}X\left(\frac{s}{a}\right)$