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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION SEMESTER II SESSION 2015/2016

COURSE NAME : ENGINEERING MATHEMATICS V
COURSE CODE : BEE 31702 / BWM 20502
PROGRAMME : BEJ/BEV
EXAMINATION DATE : JUNE/JULY 2016
DURATION : 2 HOURS 30 MINUTES
INSTRUCTION : ANSWER ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF TWELVE (12) PAGES

UNIVERSITI TUN HUSSEIN ONN MALAYSIA
FACULTY OF ELECTRICAL ENGINEERING
DEPARTMENT OF ELECTRICAL ENGINEERING
JALAN TUN HUSSEIN ONN
80100 SKUDAI, JOHORE BHARU, MALAYSIA
TEL: 071-2560000
FAX: 071-2560001
WWW.UTHM.MY

Q1 A special dangerous mutated virus has spread across the globe, which causes a zombie apocalypse. This has demanded an anti-virus vaccine to cure the sickness. The leading pharmaceutical company, Umbrella Cooperation, has discovered that females are more susceptible to the attack and are less likely to accept the cure. In order to verify this hypothesis, the study surveyed a random sample of 425 people and each person is injected with the vaccine with various dosage in millilitre (ml). The data of cured patients is summarized in **Table Q1**. The claim from the study suggested that the means for both genders who gets better would not be more than 50.

Table Q1

	Not Cured	2ml	5ml	10ml	Total
Female	60	54	61	46	221
Male	40	49	58	57	204
Total	100	103	119	103	425

- (a) Find the means and sample deviations for both female and male samples. (4 marks)
- (b) State the null and alternative hypothesis. (2 marks)
- (c) Identify the Type I and Type II errors that corresponds to the hypothesis above. (2 marks)
- (d) Test this claim using the critical value approach with 5 % significance for both males and females separately. (12 marks)

Q2 Our nation is found to be in the lead among other South Eastern Asian countries in obesity. The Ministry of Health has conducted a survey and discovered that Malaysians adore their food. Breakfast food especially is particular high in fat and sugar. In order to lead a healthier lifestyle, first, the ministry plans to learn, which of the two breakfast foods, i.e Nasi Lemak or Roti Canai, is more beloved, so that it can be modified to be healthier. **Table Q2** shows the results when the survey is done amongst Malaysians in the region of Batu Pahat.

Table Q2

	Nasi Lemak	Roti Canai
Sample size	11	13
Sample mean	2.16	4.82
Sample standard deviation	1.17	1.01

- (a) Identify the difference of mean and standard deviation of the sample mean if the mean of Roti Canai is greater than the mean of Nasi Lemak. (3 marks)

- (b) Find the Z-distribution probability of Roti Canai if the sample mean is between 2.45 and 2.85, exclusive. (7 marks)

- (c) Calculate the true variance for a 95% confidence interval of the Roti Canai using the χ^2 -distribution. (5 marks)

- (d) Calculate the ratio for both true variances of the breakfast foods with a given 90% confidence interval. (5 marks)

Q3 In order to increase tourism, the fastest internet service provider (ISP), Unifi™, has attempted to install an internet hubstation in the tourist area of Teluk Chempedak beach, so that locals and visitors can enjoy fast internet connection anywhere, anytime. Preliminary investigation of hub placement is measured at 5 different locations along the sandy coastline. The locations are taken at different distances to the main attraction point, i.e. the hotspot. These distances and the average speeds in megabits per second (Mbps) are detailed in **Table Q3**.

Table Q3

Distance to hotspot (km)	Average speed (Mbps)
10.0	9.3
8	9.675
6	10.25
4	10.225
2	11.5

- (a) Produce the equation of the least squares line that will predict the average speed in terms of the distance to the tourist hotspot. Interpret the results. (12 marks)
- (b) Identify the average speed when the relative distance is 20 km. (4 marks)
- (c) Find and interpret the Pearson correlation coefficient. (6 marks)

Q4 Let X is a random variable with probability density function (pdf) given as:

$$f(x) = \begin{cases} \frac{1}{8}\left(x + \frac{1}{4}\right), & 0 \leq x \leq 3 \\ 0, & \text{others} \end{cases}$$

- (a) Show that $\int_{-\infty}^{\infty} f(x)dx = 1$. (2 marks)
- (b) Find $P(X > 1)$. (3 marks)
- (c) Find $F(X)$. (5 marks)
- (d) Calculate the expected value of X and variance of X . (8 marks)
- (e) Calculate $Var\left(\frac{1}{3}X + 1\right)$. (2 marks)

- Q5** A Microcontroller subject class at the Faculty of Electrical and Electronics Engineering in UTHM has recorded that 75% of students taking test 1 failed. Assume that the distribution of the students who failed after taking test 1 is binomially distributed. As a preliminary study, 5 students are randomly selected,
- (a) Find the mean, variance and standard deviation of the distribution. (4 marks)
 - (b) Calculate the probability that only two of the students passed. (2 marks)
 - (c) Produce an approximate Poisson distribution probability to find at most two students failed. (5 marks)
 - (d) Now, in order to conduct the same study on a bigger sample, the number of randomly selected samples is increased to 6000 students. It is found that the probability of success of the students failing is now at 0.005. Produce an approximate normal probability distribution to find exactly 36 students failed after taking test 1. (9 marks)

– END OF QUESTIONS –

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2015/2016 PROGRAMME : BEV / BEJ
 COURSE : ENGINEERING COURSE CODE : BEE 31702/BWM 20502
 MATHEMATICS V

List of Formulas

Random Variables :

$$\sum_{i=-\infty}^{\infty} P(x_i) = 1, \quad E(X) = \sum_{\forall x} x \cdot P(x), \quad E(X^2) = \sum_{\forall x} x^2 \cdot P(x), \quad \int_{-\infty}^{\infty} f(x) dx = 1,$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot P(x) dx, \quad E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot P(x) dx, \quad Var(X) = E(X^2) - [E(X)]^2.$$

Special Probability Distributions :

$$P(x=r) = {}^n C_r \cdot p^r \cdot q^{n-r}, \quad r = 0, 1, \dots, n, \quad X \sim B(n, p), \quad P(X=r) = \frac{e^{-\mu} \cdot \mu^r}{r!}, \quad r = 0, 1, \dots, \infty,$$

$$X \sim P_0(\mu), \quad Z = \frac{X - \mu}{\sigma}, \quad Z \sim N(0, 1), \quad X \sim N(\mu, \sigma^2).$$

Sampling Distributions :

$$\bar{X} \sim N(\mu, \sigma^2/n), \quad Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1), \quad T = \frac{\bar{x} - \mu}{s/\sqrt{n}}, \quad \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right).$$

Estimations :

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2, \quad \left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - Z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + Z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}},$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2, v} \cdot S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2, v} \cdot S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where Pooled estimate of variance, $S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ with $v = n_1 + n_2 - 2$,

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2015/2016
 COURSE : ENGINEERING
 MATHEMATICS V

PROGRAMME : BEV / BEJ
 COURSE CODE : BEE 31702/BWM 20502

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2, v} \sqrt{\frac{1}{n} (s_1^2 + s_2^2)} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2, v} \sqrt{\frac{1}{n} (s_1^2 + s_2^2)} \text{ with } v = 2(n-1),$$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2, v} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2, v} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \text{ with } v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}},$$

$$\frac{(n-1) \cdot s^2}{\chi_{\alpha/2, v}^2} < \sigma^2 < \frac{(n-1) \cdot s^2}{\chi_{1-\alpha/2, v}^2} \text{ with } v = n-1,$$

$$\frac{s_1^2}{s_2^2} \cdot \frac{1}{f_{\alpha/2, v_1, v_2}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} \cdot f_{\alpha/2, v_2, v_1} \text{ with } v_1 = n_1 - 1 \text{ and } v_2 = n_2 - 1.$$

Hypothesis Testing :

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}, T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \text{ with } v = n_1 + n_2 - 2,$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}, T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{1}{n} (s_1^2 + s_2^2)}}, T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \text{ with}$$

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}; S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}; \chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Simple Linear Regressions :

$$S_{xy} = \sum x_i y_i - \frac{\sum x_i \cdot \sum y_i}{n}, S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n}, S_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n}, \bar{x} = \frac{\sum x}{n}, \bar{y} = \frac{\sum y}{n},$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}, \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}, \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x, r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}}, SSE = S_{yy} - \hat{\beta}_1 S_{xy}, MSE = \frac{SSE}{n-2},$$

$$T = \frac{\hat{\beta}_1 - \beta_1^*}{\sqrt{\frac{MSE}{S_{xx}}}} \sim t_{n-2}, T = \frac{\hat{\beta}_0 - \beta_0^*}{\sqrt{MSE \left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right)}} \sim t_{n-2}.$$

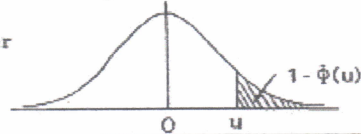
FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2015/2016 PROGRAMME : BEV / BEJ
 COURSE : ENGINEERING COURSE CODE : BEE 31702/BWM 20502
 MATHEMATICS V

Basic Distribution and Significance Table

AREAS IN TAIL OF THE NORMAL DISTRIBUTION

The function tabulated is $1 - \Phi(u)$ where $\Phi(u)$ is the cumulative distribution function of a standardised Normal variable u . Thus $1 - \Phi(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-u^2/2} du$ is the probability that a standardised Normal variable selected at random will be greater than a value of u ($= \frac{x - \mu}{\sigma}$)



$\frac{(x - \mu)}{\sigma}$.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									
3.1	.00097									
3.2	.00069									
3.3	.00048									
3.4	.00034									
3.5	.00023									
3.6	.00016									
3.7	.00011									
3.8	.00007									
3.9	.00005									
4.0	.00003									

REGAT CHOM KHAMMAS WONG SO
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FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2015/2016 PROGRAMME : BEV / BEJ
 COURSE : ENGINEERING COURSE CODE : BEE 31702/BWM 20502
 MATHEMATICS V

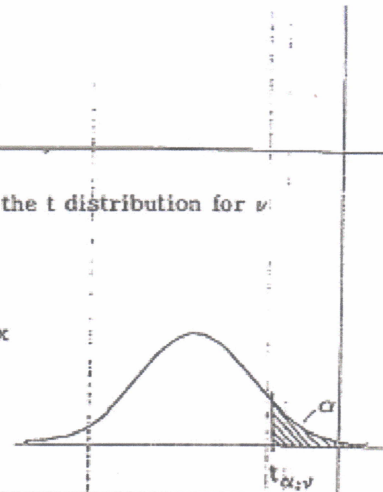
PERCENTAGE POINTS OF THE t DISTRIBUTION

The table gives the value of $t_{\alpha; \nu}$ - the 100α percentage point of the t distribution for ν degrees of freedom.

The values of t are obtained by solution of the equation:-

$$\alpha = \Gamma\left\{\frac{1}{2}(\nu+1)\right\} \left\{\Gamma\left(\frac{1}{2}\nu\right)\right\}^{-1} (\nu\pi)^{-1/2} \int_t^{\infty} (1+x^2/\nu)^{-(\nu+1)/2} dx$$

Note. The tabulation is for one tail only i.e. for positive values of t . For $|t|$ the column headings for α must be doubled.



$\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
$\nu = 1$	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2015/2016 PROGRAMME : BEV / BEJ
COURSE : ENGINEERING COURSE CODE : BEE 31702/BWM 20502
MATHEMATICS V

PERCENTAGE POINTS OF THE X^2 DISTRIBUTION

Table of X^2_{alpha; v} - the 100 alpha percentage point of the X^2 distribution for v degrees of freedom

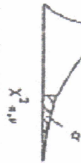


Table with columns for alpha (0.95 to 0.01) and degrees of freedom (v) from 1 to 100. The table contains numerical values representing the percentage points of the chi-square distribution.

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2015/2016 PROGRAMME : BEV / BEJ
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 MATHEMATICS V

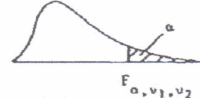
PERCENTAGE POINTS OF THE F DISTRIBUTION .

The table gives the values of $F_{\alpha; \nu_1, \nu_2}$ the 100 α percentage point of the F distribution having ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator.

For each pair of values of ν_1 and ν_2 , $F_{\alpha; \nu_1, \nu_2}$ is tabulated for $\alpha = 0.05, 0.025, 0.01, 0.001$, the 0.025 values being bracketed.

The lower percentage points of the distribution may be obtained from the relation:-

$$F_{1-\alpha; \nu_1, \nu_2} = 1/F_{\alpha; \nu_2, \nu_1}$$



e.g. $F_{.95; 12, 8} = 1/F_{.05; 8, 12} = 1/2.85 = 0.351$

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	10	12	24	∞	
1	161.4 (548) 4052 4053*	199.5 (800) 5000 5000*	215.7 (864) 5403 5404*	224.6 (900) 5625 5625*	230.2 (922) 5764 5764*	234.0 (937) 5859 5859*	236.8 (948) 5928 5929*	238.9 (957) 5981 5981*	241.9 (969) 6056 6056*	243.9 (977) 6106 6107*	249.0 (997) 6235 6235*	254.3 (1018) 6366 6366*	
2	18.5 (38.5) 98.5 998.5	19.0 (39.0) 99.2 999.0	19.2 (39.2) 99.2 999.2	19.2 (39.2) 99.3 999.2	19.3 (39.3) 99.3 999.3	19.3 (39.3) 99.3 999.3	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.4 (39.4) 99.4 999.4	19.5 (39.5) 99.5 999.5	19.5 (39.5) 99.5 999.5	
3	10.13 (17.4) 34.1 167.0	9.55 (16.0) 30.8 148.5	9.28 (15.4) 29.5 141.1	9.12 (15.1) 28.7 137.1	9.01 (14.9) 28.2 134.6	8.94 (14.7) 27.9 132.8	8.89 (14.6) 27.7 131.5	8.85 (14.5) 27.5 130.6	8.79 (14.4) 27.2 129.2	8.74 (14.3) 27.1 128.3	8.64 (14.1) 26.6 125.9	8.53 (13.9) 26.1 123.5	
4	7.71 (12.22) 21.2 74.14	6.94 (10.65) 18.0 61.25	6.59 (9.98) 16.7 56.18	6.39 (9.60) 16.0 53.44	6.26 (9.36) 15.5 51.71	6.16 (9.20) 15.2 50.53	6.09 (9.07) 15.0 49.66	6.04 (8.98) 14.8 49.00	5.96 (8.84) 14.5 48.05	5.91 (8.75) 14.4 47.41	5.77 (8.51) 13.9 45.77	5.63 (8.26) 13.5 44.05	
5	6.61 (10.01) 16.26 47.18	5.79 (8.43) 13.27 37.12	5.41 (7.76) 12.06 33.20	5.19 (7.39) 11.39 31.09	5.05 (7.15) 10.97 29.75	4.95 (6.98) 10.67 28.83	4.88 (6.85) 10.46 28.16	4.82 (6.76) 10.29 27.65	4.74 (6.62) 10.05 26.92	4.68 (6.52) 9.89 26.42	4.53 (6.28) 9.47 25.14	4.36 (6.02) 9.02 23.79	
6	5.99 (8.81) 13.74 35.51	5.14 (7.26) 10.92 27.00	4.76 (6.60) 9.78 23.70	4.53 (6.23) 9.15 21.92	4.39 (5.98) 8.75 20.80	4.28 (5.82) 8.47 20.03	4.21 (5.70) 8.26 19.46	4.15 (5.60) 8.10 19.03	4.06 (5.46) 7.87 18.41	4.00 (5.37) 7.72 17.99	3.84 (5.12) 7.31 16.90	3.67 (4.85) 6.88 15.75	
7	5.59 (8.07) 12.25 29.25	4.74 (6.54) 9.55 21.69	4.35 (5.89) 8.45 18.77	4.12 (5.52) 7.85 17.20	3.97 (5.29) 7.46 16.21	3.87 (5.12) 7.19 15.52	3.79 (4.99) 6.99 15.02	3.73 (4.90) 6.84 14.63	3.64 (4.76) 6.62 14.08	3.57 (4.67) 6.47 13.71	3.41 (4.42) 6.07 12.73	3.23 (4.14) 5.65 11.70	
8	5.32 (7.57) 11.26 25.42	4.46 (6.06) 8.65 18.49	4.07 (5.42) 7.59 15.83	3.84 (5.05) 7.01 14.39	3.69 (4.82) 6.63 13.48	3.58 (4.65) 6.37 12.86	3.50 (4.53) 6.18 12.40	3.44 (4.43) 6.03 12.05	3.35 (4.30) 5.81 11.54	3.28 (4.20) 5.67 11.19	3.12 (3.95) 5.28 10.30	2.93 (3.67) 4.86 9.34	
9	5.12 (7.21) 10.56 22.86	4.26 (5.71) 8.02 16.39	3.86 (5.08) 6.99 13.90	3.63 (4.72) 6.42 12.56	3.48 (4.48) 6.06 11.71	3.37 (4.32) 5.80 11.13	3.29 (4.20) 5.61 10.69	3.23 (4.10) 5.47 10.37	3.14 (3.96) 5.26 9.87	3.07 (3.87) 5.11 9.57	2.90 (3.61) 4.73 8.72	2.71 (3.33) 4.31 7.81	
10	4.96 (6.94) 10.04 21.04	4.10 (5.46) 7.56 14.91	3.71 (4.83) 6.55 12.55	3.48 (4.47) 5.99 11.28	3.33 (4.24) 5.64 10.48	3.22 (4.07) 5.39 9.93	3.14 (3.95) 5.20 9.52	3.07 (3.85) 5.06 9.20	2.98 (3.72) 4.85 8.74	2.91 (3.62) 4.71 8.44	2.74 (3.37) 4.33 7.64	2.54 (3.08) 3.91 6.76	
11	4.84 (6.72) 9.65 19.69	3.98 (5.26) 7.21 13.81	3.59 (4.63) 6.22 11.56	3.36 (4.28) 5.67 10.35	3.20 (4.04) 5.32 9.58	3.09 (3.88) 5.07 9.05	3.01 (3.76) 4.89 8.66	2.95 (3.66) 4.74 8.35	2.85 (3.53) 4.54 7.92	2.79 (3.43) 4.40 7.63	2.61 (3.17) 4.02 6.85	2.40 (2.88) 3.60 6.00	
12	4.75 (6.55) 9.33 18.64	3.89 (5.10) 6.93 12.97	3.49 (4.47) 5.95 10.80	3.26 (4.12) 5.41 9.63	3.11 (3.89) 5.06 8.89	3.00 (3.73) 4.82 8.38	2.91 (3.61) 4.64 8.00	2.85 (3.51) 4.50 7.71	2.75 (3.37) 4.30 7.29	2.69 (3.28) 4.16 7.00	2.51 (3.02) 3.78 6.25	2.30 (2.72) 3.36 5.42	