Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BEE 31602 / BWM 30602
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JUNE / JULY 2016
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	ANSWERS **ALL FIVE (5)** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 Distillation is a widely used method for separating mixtures based on differences in the conditions required to change the phase of components of the mixture. The data of a hypothetical distillation are tabulated in the following table where h is the number of samples and v is the volume of distillation.

h	0	1	2
$v=f(h)$	0	1	0

(a) Construct the divided-difference table for h_k, d_k, and b_k.

(6 marks)

(b) Formulate **TWO (2)** cubic spline functions of $S_o(x)$ and $S_1(x)$

(11 marks)

(c) Calculate the value of $f(1.5)$ in 4 decimal places.

(3 marks)

Q2 The current i of a circuit at time t is given in the table below:

t, sec	3	5	6	7	8
i (t), ampere	0.244	0.405	0.597	0.824	1.093

Given the voltage drop, $V_L(t) = L\frac{di}{dt}$ and the inductance, L is 3H.

(a) Give the **FOUR (4)** suitable methods that could be used to find $V_L(6)$.

(4 marks)

(b) Calculate $V_L(6)$ using the **FOUR (4)** suitable methods in 4 decimal places.

(16 marks)

Q3 (a) The π is the ratio of the circumference of a circle to the diameter. Jacob Bernoulli, one of many famous mathematicians of the infamous Bernoulli family, proved that $\pi = \int_0^1 \frac{4}{1+x^2} dx$. Compute the value of π using the $\pi = \int_0^1 \frac{4}{1+x^2} dx$ and trapezoidal rule with $n = 8$ in 4 decimal places.

(10 marks)

(b) The resistance force of a moving car is due to the constant tire resistance and the air resistance coefficient. An experiment is conducted in which the total resistance force can be measured for the velocity of different cars.

Calculate the resistance force by referring the following experimental data using suitable Simpson's rules in 4 decimal places.

Velocity (m/s)	2	5	8	11	14	17	20	23	26	29
Resistance Force (N)	137	133	223	198	345	264	338	450	439	482

(10 marks)

Q4 (a) A simple RL-electrical circuit consists of a constant resistance R (in ohms), a constant inductance L (in henrys) and an electromotive force $E\ (t)$ (in volts). According to Kirchoff's second law, the current i (in amperes) in the circuit satisfies the equation:

$$L\frac{di}{dt} + Ri = E(t)$$

Given $E\ (t) = 220$ volts, $L = 4$ henry, $R = 20$ ohms and $i = 0$ when $t = 0$,

(i) Analyze the current at $t = 0.04$ second using the fourth order Runge-Kutta method (RK4) with the step size $h = 0.02$ in 4 decimal places.

(8 marks)

(ii) Estimate the absolute error if the exact solution is $i = 11(1 - e^{-5t})$ in 4 decimal places.

(2 marks)

(b) Given the boundary value problem $y''(t) - 10y(t) = 0$, for $0 \le t \le 0.4$, with the boundary conditions $y(0) = 0$ and $y(0.4) = 1$, calculate the values of y (t) by using finite-difference method with $h = 0.1$ in 4 decimal places. [*Note*: $y''(t) = \frac{d^2y}{dt^2}$]

(10 marks)

Q5 (a) Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; 0 < x < 1, \; t > 0$$

with the boundary conditions $u(0,t) = u(1,t) = 0$ and the initial condition $u(x,0) = \sin 4\pi x$, $\frac{\partial u}{\partial t}(x,0) = 0$ for $0 \le x \le 1$, by using the finite-difference method with $\Delta x = h = 0.2$ and $\Delta t = k = 0.1$ until $t = 0.2$ in 4 decimal places.

(8 marks)

(b) Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}$, $0 < x < 1$, and $t > 0$ with the boundary conditions $u(0,t) = 20t^2$, $u(1,t) = 10t$, and the initial condition $u(x,0) = x(1-x)$ by using the explicit finite-difference method with $\Delta x = h = 0.5$ and $\Delta t = k = 0.1$ until $t = 0.3$ in 4 decimal places.

(12 marks)

-END OF QUESTIONS –

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FORMULAS

Interpolation by Cubic spline:

$$S_k(x)=\frac{m_k}{6h_k}(x_{k+1}-x)^3+\frac{m_{k+1}}{6h_k}(x-x_k)^3+\left(\frac{f_k}{h_k}-\frac{m_k}{6}h_k\right)(x_{k+1}-x)+\left(\frac{f_{k+1}}{h_k}-\frac{m_{k+1}}{6}h_k\right)(x-x_k)$$

where $k=0,1,2,\ldots,n-1$

$$\left.\begin{aligned} h_k &= x_{k+1}-x_k \\ d_k &= \frac{f_{k+1}-f_k}{h_k} \end{aligned}\right\}, \quad k=0,1,2,\ldots,n-1$$

$$b_k=6(d_{k+1}-d_k), \quad k=0,1,2,\ldots,n-2$$

$$m_0=0$$

$$m_n=0$$

$$h_k m_k+2(h_k+h_{k+1})m_{k+1}+h_{k+1}m_{k+2}=b_k, \quad k=0,1,2,\ldots,n-2$$

First Order Numerical differentiation:

$$f'(x)\approx\frac{f(x+h)-f(x-h)}{2h}$$

$$f'(x)\approx\frac{-3f(x)+4f(x+h)-f(x+2h)}{2h}$$

$$f'(x)\approx\frac{f(x-2h)-4f(x-h)+3f(x)}{2h}$$

$$f'(x)\approx\frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$$

Numerical Integration:

Trapezoidal rule: $\int_a^b f(x)dx\approx\frac{h}{2}\left[f_0+f_n+2\sum_{i=1}^{n-1}f_i\right]$

Simpson's $\frac{1}{3}$ rule: $\int_a^b f(x)dx\approx\frac{h}{3}\left[f_0+f_n+4\sum_{\substack{i=1\\ i\text{ odd}}}^{n-1}f_i+2\sum_{\substack{i=2\\ i\text{ even}}}^{n-2}f_i\right]$

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FORMULAS

Numerical Integration:

Simpson's $\frac{3}{8}$ rule: $\int_{a=x_0}^{b=x_n} f(x)dx \approx \frac{3h}{8}\left[f_0 + f_n + 3\sum_{i=1}^{n/3}(f_{3i-2} + f_{3i-1}) + 2\sum_{i=1}^{n/3-1} f_{3i}\right]$

Initial value problems:

Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Boundary value problems:

Finite difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h} \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Central time central space (CTCS) finite-difference method:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \Leftrightarrow \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$\frac{\partial u(x,0)}{\partial t} = \frac{u_{i,j+1} - u_{i,j-1}}{2k} = g(x_i)$$

$$u_{i,j+1} = \rho^2 u_{i-1,j} + (2 - 2\rho^2)u_{i,j} + \rho^2 u_{i+1,j} - u_{i,j-1} \qquad \rho^2 = \frac{k^2c^2}{h^2}$$

Forward time central space (FTCS) finite-difference method:

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x}\right)_{i,j} \Leftrightarrow \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

$$u_{i,j+1} = ru_{i-1,j} + (1 - 2r)u_{i,j} + ru_{i+1,j} \qquad r = \frac{kc^2}{h^2}$$