CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2015/2016

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BEE11403 / BWM10303
PROGRAMME	:	BACHELOR OF ELECTRONIC ENGINEERING WITH HONOURS
EXAMINATION DATE	:	DECEMBER 2015 / JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 A series RC circuit can be formulated as the following equation.

$$R\frac{dq}{dt}+\frac{q}{C}=E$$

Where q, R, C and E are the charge, the resistor, the capacitor, and the voltage supply of the circuit. When $t=0$, $q=0$.

(a) Apply the separation of variables method to find the solution of $q(t)$.
(6 marks)

(b) Apply the integrating factor of $e^{\frac{1}{RC}t}$ to find the solution of $q(t)$.
(6 marks)

(c) Calculate the $q(t)$ if $R = 5\ \Omega$, $C = 0.1\text{F}$, $E = 5$ Volts, $q(0) = 0$ and $q'(0) = 1$ using Laplace Transform.
(8 marks)

(d) Verify the answer of **Q1(c)** using the equation from **Q1(a)** or **Q1(b)**.
(2 marks)

Q2 For a series RLC circuit that contains an electromotive force E, a resistor R, an inductor L, and a capacitor C can be represented by a general equation as follows:

$$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{i}{C}=E$$

Given $E=2t\text{V}$, $L=1\text{H}$, $R=3\Omega, C=0.5\text{F}$, and initial condition of $i(0)=i'(0)=0$.

(a) Find the complementary solution of the equation, i_c.
(4 marks)

(b) Find the particular integral of the equation, i_p.
(7 marks)

(c) Find the current $i(t)$ using $i(t) = i_c + i_p$.
(7 marks)

(d) Verify the current $i(t)$ from **Q2(c)** using Laplace transform.
(14 marks)

Q3 For the following periodic signal.

$$f(t)=\begin{cases}0 & -\pi<t<0\\ t^2 & 0<t<\pi\end{cases} \text{ and } f(t)=f(t+2\pi)$$

(a) Sketch the given function when $0<t<5\pi$.

(4 marks)

(b) Formulate the trigonometric Fourier coefficients of a_0, a_n, and b_n.

(9 marks)

(c) Write out the first four terms of the Fourier Series, i.e. $n=0,1,2,3$.

(6 marks)

Q4 (a) Transform t^2e^{-4t} to s domain using Laplace transform.

(4 marks)

(b) Calculate $\mathcal{L}\{3t-(3t+1)u(t-3)\}$.

(5 marks)

(c) For $\dfrac{8s+13}{s^2+4s-5}$,

(i) Simplify this equation using partial fraction.

(4 marks)

(ii) From **Q4(c)(i)**, transform the equation into its time domain.

(2 marks)

Q5 Show $y(x)=c_0\sum_{n=0}^{\infty}\frac{x^n}{n!}$ for $\frac{dy}{dx}-y=0$ using power series method where $y(x)=\sum_{n=0}^{\infty}c_nx^n$.

(12 marks)

- END OF QUESTION -

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Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay''+by'+cy=0$.

Characteristic equation: $am^2+bm+c=0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y=Ae^{m_1x}+Be^{m_2x}$
2.	Real and equal roots: $m=m_1=m_2$	$y=(A+Bx)e^{mx}$
3.	Complex roots: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

Laplace Transform

$f(t)$	$F(s)$
a	$\dfrac{a}{s}$
e^{at}	$\dfrac{1}{s-a}$
$\sin at$	$\dfrac{a}{s^2+a^2}$
$\cos at$	$\dfrac{s}{s^2+a^2}$
$\sinh at$	$\dfrac{a}{s^2-a^2}$
$\cosh at$	$\dfrac{s}{s^2-a^2}$
t^n, $n=1,2,3,\ldots$	$\dfrac{n!}{s^{n+1}}$
$e^{at}f(t)$	$F(s-a)$
$t^nf(t)$, $n=1,2,3,\ldots$	$(-1)^n\dfrac{d^n}{ds^n}F(s)$
$H(t-a)$	$\dfrac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\delta(t-a)$	e^{-as}
$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
y	$Y(s)$
y'	$sY(s)-y(0)$
y''	$s^2Y(s)-sy(0)-y'(0)$

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Electrical's Formulae

1. Voltage drop across resistor, R (Ohm's Law): $v_R = iR$
2. Voltage drop across inductor, L (Faraday's Law): $v_L = L\frac{di}{dt}$
3. Voltage drop across capacitor, C (Coulomb's Law): $v_C = \frac{1}{C}\int i\,dt$
4. The relation between current, i and charge, q: $i = \frac{dq}{dt}$.

Fourier Series

Fourier series expansion of periodic function with period 2π	Half Range series
$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$	$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$
$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$	$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$	$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$
$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n\sin nx$	$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$

Table of Fourier Transform (Fourier Transform Pairs)

$f(t)$	$F(\omega)$	$f(t)$	$F(\omega)$
$\delta(t)$	1	$\text{sgn}(t)$	$\frac{2}{i\omega}$
$\delta(t-\omega_0)$	$e^{-i\omega_0\omega}$	$H(t)$	$\pi\delta(\omega)+\frac{1}{i\omega}$
1	$2\pi\delta(\omega)$	$e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{1}{\omega_0+i\omega}$
$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$t^n e^{-\omega_0 t}H(t)$ for $\omega_0>0$	$\frac{n!}{(\omega_0+i\omega)^{n+1}}$
$\sin(\omega_0 t)$	$i\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	$e^{-at}\sin(\omega_0 t)H(t)$ for $a>0$	$\frac{\omega_0}{(a+i\omega)^2+\omega_0^2}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$	$e^{-at}\cos(\omega_0 t)H(t)$ for $a>0$	$\frac{a+i\omega}{(a+i\omega)^2+\omega_0^2}$
$\sin(\omega_0 t)H(t)$	$\frac{\pi}{2}i[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$		
$\cos(\omega_0 t)H(t)$	$\frac{\pi}{2}[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]+\frac{i\omega}{\omega_0^2-\omega^2}$		