CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I

SESSION 2015/2016

COURSE NAME	:	ELECTROMAGNETIC FIELDS AND WAVES / ENGINEERING ELECTROMAGNETICS
COURSE CODE	:	BEB 20303 / BEF 22903
PROGRAMME	:	BACHELOR OF ELECTRONIC ENGINEERING WITH HONOURS / BACHELOR OF ELECTRICAL ENGINEERING WITH HONOURS
EXAMINATION DATE	:	DECEMBER 2015 / JANUARY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

CONFIDENTIAL

ANSWER **ALL** QUESTIONS

Q1 (a) State the procedures or steps for applying Gauss's law in order to calculate the Electric field intensity, $\vec{E}$ of symmetrical charge distributions.

(3 marks)

(b) A dielectric spherical shell is as shown in **FIGURE Q1 (a)** with volume charge density, ρ_v (c/m^3) for $a < R < b$ and 0 otherwise. (Inner radius = a and outer radius = b). Find the electric field intensity, $\vec{E}$ inside the spherical shell ($R < a$), in the spherical shell ($a < R < b$) and outside the spherical shell ($R > b$).

(10 marks)

(c) The centre of dielectric spherical shell similar to **FIGURE Q1 (a)** with volume charge density, $\rho_v = 2\ nC/m^3$ is located at (0, 2, -2) with inner radius, a = 1 m, outer radius b = 3m. In addition, a point charge is located at (3,-4, 3) with Q=12nC. Calculate the total electric field, $\vec{E}$ at (0, 2, 3) using Gauss's Law.

(12 marks)

Q2 (a) Two infinite lines are carrying identical current in same direction. Both of them are located at $x = -1,\ y = 1$ and $x = 1,\ y = -1$, respectively.

(i) Sketch the infinite lines and show the direction of current.

(3 marks)

(ii) Find the point where the magnetic field is equal to zero by using Ampere's law.

(6 marks)

(iii) By using the right hand rule, show that the magnetic field at (0, 0, 0) is equal to zero.

(2 marks)

(iv) Determine the magnetic field at (3,-3, 0) if both of the current are 3A.

(5 marks)

(v) Based on your answer in **Q2(a)(iv)**, justify the effect of the magnetic field observed at (3, -3, 0) due to the distance of the infinite lines.

(2 marks)

(b) The magnetic field of an infinitely long coaxial transmission line is represented in **FIGURE Q2 (b)**.

(i) Find the radius of the inner conductor, outer conductor, the thickness of the outer conductor, and the current flowing on the coaxial transmission line.

(5 marks)

(ii) If this infinite coaxial line is placed in between the parallel line at (0,0,0) as in **Q2 (a)**, predict what is the additional magnetic field that can be observed at (3,-3,0).

(2 marks)

Q3 (a) Faraday's law states that the induced electromotive force (emf), V_{emf} in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit. Differentiate between transformer electromotive force (emf) and motion electromotive force (emf). Propose an experiment to prove the Faraday's Law.

(8 marks)

(b) **FIGURE Q3 (b)** shows the rectangular loop with a conducting slide bar located at $x = 10t + 4t^3$. The separation between two rails is 40 cm. If the magnetic flux density, $\vec{B} = 0.8x^2\hat{z}$ Tesla, Calculate the voltmeter reading at $t = 0.8$s.

(9 marks)

(c) A conductor joining the two ends of a resistor as shown in **FIGURE Q3 (c)**. The time varying magnetic field is given by $\vec{B} = 0.4\cos(120\pi t)$ Tesla. Assume that the magnetic field produced by $I(t)$ is negligible. Calculate the induced electromotive force, $V_{ab}(t)$ in the circuit.

(8 marks)

Q4 (a) The propagation of a plane wave has differences characteristic depend the medium used. Elaborate the plane wave propagation characteristic for:

(i) Free space,
(3 marks)

(ii) Lossless dielectric,
(3 marks)

(iii) Good conductor.
(3 marks)

(b) The magnetic field of a wave propagating in free space is given by

$$\vec{H} = 0.1 \cos(2 \times 10^8 t - kx)\, \hat{y} \text{ A/m}$$

(i) Calculate the wave number, k, the wavelength, λ and the period of the wave, T.
(3 marks)

(ii) Determine the time t_1 it takes the wave to travel $\lambda/8$.
(1 marks)

(iii) Write the expression of the wave at time t_1 and sketch the wave.
(4 marks)

(c) A plane wave in nonmagnetic medium has $\vec{E} = 50 \sin(10^8 t + 2z)\, \hat{y}$ V/m

(i) Determine the direction of wave propagation.
(1 marks)

(ii) Calculate the wavelength, λ, frequency, f and relative permittivity ε_r.
(3 marks)

(iii) Formulate the corresponding magnetic field component, $\vec{H}$.
(4 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2015/2016
COURSE: ELECTROMAGNETIC FIELDS AND WAVES

PROGRAMME: BEJ / BEV
COURSE CODE: BEB 20303 / BEF 22903

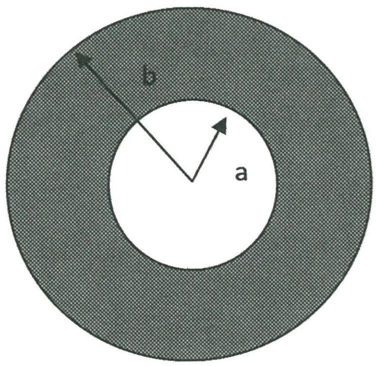

FIGURE Q1 (a)

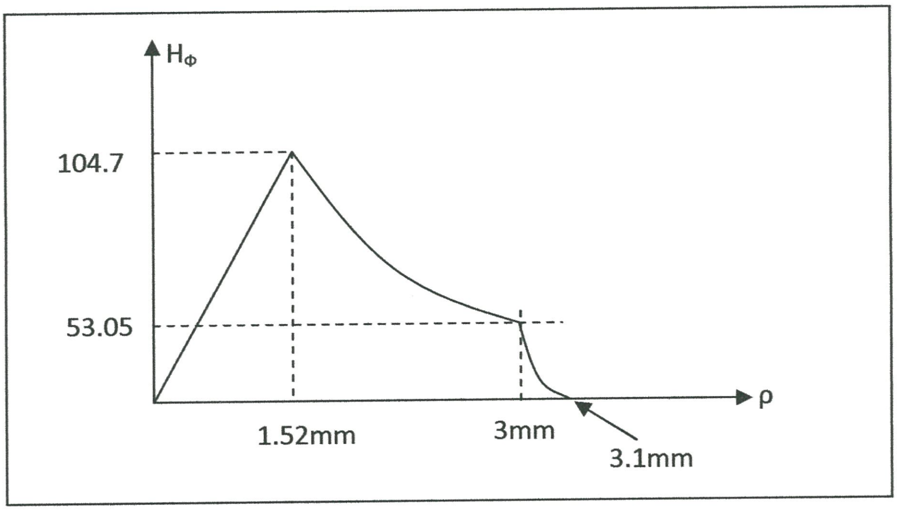

FIGURE Q2 (b)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2015/2016
COURSE: ELECTROMAGNETIC FIELDS AND WAVES

PROGRAMME: BEJ / BEV
COURSE CODE: BEB 20303 / BEF 22903

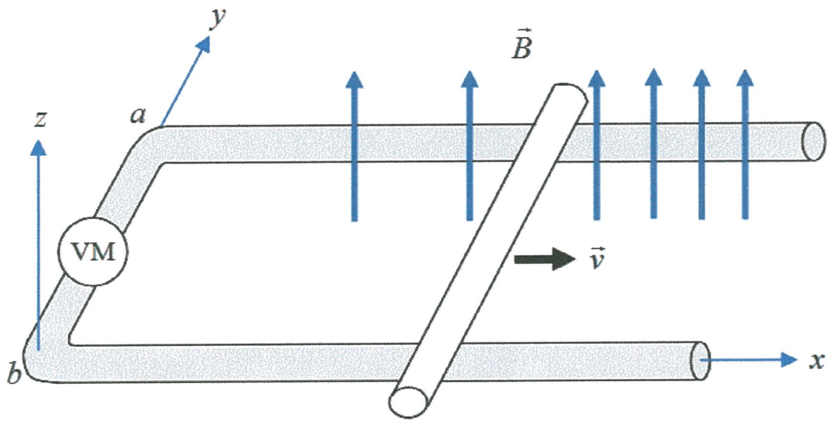

FIGURE Q3 (b)

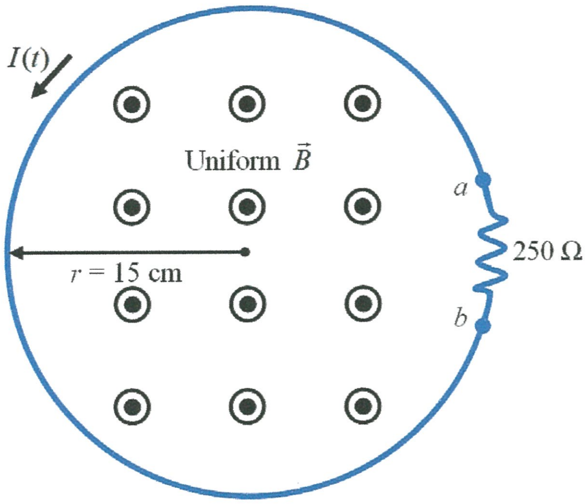

FIGURE Q3 (c)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2015/2016
COURSE: ELECTROMAGNETIC FIELDS AND WAVES

PROGRAMME: BEJ / BEV
COURSE CODE: BEB 20303 / BEF 22903

FORMULA

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial (rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial (R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial (A_\theta \sin\theta)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{r}\left(\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial (\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial (RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial (RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\varphi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2015/2016
COURSE: ELECTROMAGNETIC FIELDS AND WAVES
PROGRAMME: BEJ / BEV
COURSE CODE: BEB 20303 / BEF 22903

	Cartesian	**Cylindrical**	**Spherical**
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\varphi}}$
Magnitude $\vec{A}$	$\sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$	$\sqrt{A_r{}^2 + A_\phi{}^2 + A_z{}^2}$	$\sqrt{A_R{}^2 + A_\theta{}^2 + A_\phi{}^2}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1, y_1, z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1, \phi_1, z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1, \theta_1, \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\varphi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\varphi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\overrightarrow{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\varphi}}$
Differential surface, $\overrightarrow{ds}$	$\overrightarrow{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\overrightarrow{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\overrightarrow{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\overrightarrow{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\varphi}}$ $\overrightarrow{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\overrightarrow{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\overrightarrow{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume, $\overrightarrow{dv}$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2015/2016
PROGRAMME: BEJ / BEV
COURSE: ELECTROMAGNETIC FIELDS AND WAVES
COURSE CODE: BEB 20303 / BEF 22903

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to Spherical	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to Cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2015/2016 PROGRAMME: BEJ / BEV

COURSE: ELECTROMAGNETIC FIELDS AND WAVES COURSE CODE: BEB 20303 / BEF 22903

$$Q = \int \rho_\ell d\ell,$$

$$Q = \int \rho_s dS,$$

$$Q = \int \rho_v dv$$

$$\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$

$$\overline{E} = \frac{\overline{F}}{Q},$$

$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\overline{E} = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\overline{E} = \int \frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\overline{E} = \int \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\overline{D} = \varepsilon\overline{E}$$

$$\psi_e = \int \overline{D} \bullet d\overline{S}$$

$$Q_{enc} = \oint_S \overline{D} \bullet d\overline{S}$$

$$\rho_v = \nabla \bullet \overline{D}$$

$$V_{AB} = -\int_A^B \overline{E} \bullet d\overline{\ell} = \frac{W}{Q}$$

$$V = \frac{Q}{4\pi\varepsilon r}$$

$$V = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon r}$$

$$\oint \overline{E} \bullet d\overline{\ell} = 0$$

$$\nabla \times \overline{E} = 0$$

$$\overline{E} = -\nabla V$$

$$\nabla^2 V = 0$$

$$R = \frac{\ell}{\sigma S}$$

$$I = \int \overline{J} \bullet dS$$

$$d\overline{H} = \frac{Id\overline{\ell} \times \overline{R}}{4\pi R^3}$$

$$Id\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J} dv$$

$$\oint \overline{H} \bullet d\overline{\ell} = I_{enc} = \int \overline{J}_s dS$$

$$\nabla \times \overline{H} = \overline{J}$$

$$\psi_m = \int_s \overline{B} \bullet d\overline{S}$$

$$\psi_m = \oint \overline{B} \bullet d\overline{S} = 0$$

$$\psi_m = \oint \overline{A} \bullet d\overline{\ell}$$

$$\nabla \bullet \overline{B} = 0$$

$$\overline{B} = \mu\overline{H}$$

$$\overline{B} = \nabla \times \overline{A}$$

$$\overline{A} = \int \frac{\mu_0 Id\overline{\ell}}{4\pi R}$$

$$\nabla^2 \overline{A} = -\mu_0 \overline{J}$$

$$\overline{F} = Q(\overline{E} + \overline{u} \times \overline{B}) = m\frac{d\overline{u}}{dt}$$

$$d\overline{F} = Id\overline{\ell} \times \overline{B}$$

$$\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$$

$$\overline{m} = IS\hat{a}_n$$

$$V_{emf} = -\frac{\partial\psi}{\partial t}$$

$$V_{emf} = -\int \frac{\partial\overline{B}}{\partial t} \bullet d\overline{S}$$

$$V_{emf} = \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$$

$$I_d = \int J_d . d\overline{S}, J_d = \frac{\partial\overline{D}}{\partial t}$$

$$\gamma = \alpha + j\beta$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\overline{F}_1 = \frac{\mu I_1 I_2}{4\pi}\oint_{L1}\oint_{L2}\frac{d\overline{\ell}_1 \times (d\overline{\ell}_2 \times \hat{a}_{R_{21}})}{R_{21}^{\;2}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$

$$\tan\theta = \frac{\sigma}{\omega\varepsilon} = \frac{\overline{J}_s}{\overline{J}_{ds}}$$

$$\delta = \frac{1}{\alpha}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$$

$$\int \frac{dx}{(x^2 + c^2)^{3/2}} = \frac{x}{c^2(x^2 + c^2)^{1/2}}$$

$$\int \frac{xdx}{(x^2 + c^2)^{3/2}} = \frac{-1}{(x^2 + c^2)^{1/2}}$$

$$\int \frac{dx}{(x^2 \pm c^2)^{1/2}} = ln\left(x + \sqrt{x^2 \pm c^2}\right)$$

$$\int \frac{dx}{(x^2 + c^2)} = \frac{1}{c}\tan^{-1}\left(\frac{x}{c}\right)$$

$$\int \frac{xdx}{(x^2 + c^2)} = \frac{1}{2}ln(x^2 + c^2)$$

$$\int \frac{xdx}{(x^2 + c^2)^{1/2}} = \sqrt{x^2 + c^2}$$