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# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER I
## SESSION 2015/2016

| | | |
|---|---|---|
| COURSE NAME | : | DIGITAL SIGNAL PROCESSING |
| COURSE CODE | : | BEB 30503 |
| PROGRAMME | : | BACHELOR OF ELECTRONIC ENGINEERING WITH HONOURS |
| EXAMINATION DATE | : | DECEMBER 2015 / JANUARY 2016 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWER **ALL** QUESTIONS IN **SECTION A,** AND **ONE (1)** QUESTION IN **SECTION B** |

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

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## SECTION A

**Q1** The output signal of a proposed system in **Figure Q1** is defined as:

$$y(n) = -r(n+3) + 2r(n+2) - u(n+2) - r(n) - 3u(n-2) + 3u(n-3)$$

(a) If $x_1[n] = (-1)^n$, determine the fundamental period of this input signal.

(2 marks)

(b) Rewrite $y[n]$ as a sequence number.

(7 marks)

(c) Calculate the signal of $x_1[n]$, if the signal of $x_2[n]$ is defined as:

$$x_2(n) = tri\left(\frac{n-2}{2}\right) + 2\delta(n-3)$$

(11 marks)

**Q2** An important concept in communication applications is the correlation between two signals. The function $r_{xx}[n]$ is usually referred to the autocorrelation function of the signal $x[n]$, while $r_{xh}[n]$ is often called as a cross-correlation function. If the input signal, $x[n] = \{1, \overset{\Downarrow}{2}, 0, 1\}$ and impulse response, $h[n] = \{-1, 0, 1, \overset{\Downarrow}{1}\}$:

(a) Demonstrate the relationship between $r_{xh}[n]$ and $r_{hx}[n]$.

(12 marks)

(b) Calculate the odd part of $r_{xx}[n]$.

(8 marks)

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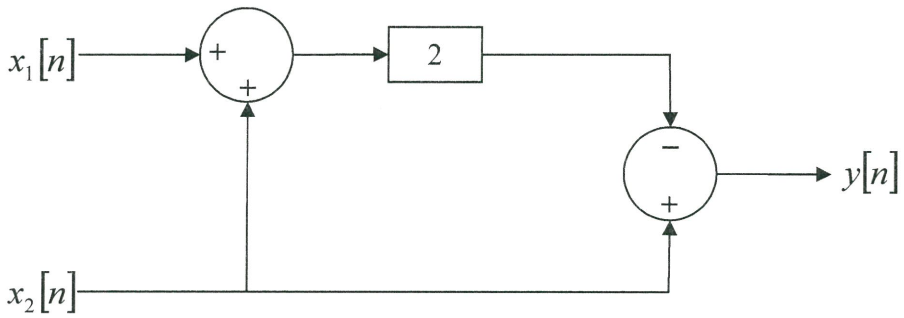


**FIGURE Q1**

**Q3** The analog to digital converter system with the input signal, $x(t)=\cos(28\pi t)+\sin(2\pi t)+\sin(22\pi t)$ for $0\le t\le 0.5$, sampled at sampling frequency of 10 Hz.

(a) Produce the reconstructed signal, $x_a(t)$.

(5 marks)

(b) Calculate the digital signal, $x_c[n]$. Show all the working step, using rounding techniques, with 4 levels of quantization and the dynamic range of 2.5V.

(15 marks)

**Q4** (a) List **FOUR (4)** properties of Discrete Fourier Transform (DFT).

(2 marks)

(b) Calculate IDFT of $Y_{DFT}[k]=\{\underset{\Downarrow}{1}0,-2+2j,-2,-2-2j\}$ using decimation in time Fast Fourier Transform (FFT) algorithm.

(14 marks)

(c) Using Parseval's relation, prove an energy of both signal is equal to 6J.

$$m[n]=\{1,\ \mathbf{A},\ \overset{\Downarrow}{1},\ 2\}\quad\Leftrightarrow\quad M_{DFT}[k]=\{0,\ \mathbf{B},\ \overset{\Downarrow}{4},\ -2j\}$$

(4 marks)

## SECTION B

**Q5** (a) Assume that $x[n]$ represent a right-sight signal. Compute the $x[n]$ of the following z-transform using partial fractions.

$$X(z)=\frac{3z}{(z^2-1.5z+0.5)}$$

(5 marks)

(b) The differential equation of a digital communication system is described as:

$$y(n)=2y(n-1)+x(n-2)+3x(n)-5x(n+1)$$

Determine the above system whether it is stable or not.

(5 marks)

(c) Design FIR filter with stop-band frequency of 5 kHz, pass-band frequency of 6 kHz and sampling frequency of 10 kHz. Choose the suitable window for your FIR filter with $N = 5$.

(10 marks)

**Q6** (a) Calculate the inverse $z$-transform of $X(z) = \dfrac{z^2}{(z^2 - 0.25)}$.

(5 marks)

(b) The $z$-transform of $x[n]$ is given as $X(z) = \dfrac{4z}{(z+0.5)^2}, |z| > 0.5$. Find the $z$-transform of the following using properties and specify the Region of Convergence (ROC).

(i) $y[n] = (2)^n x[n]$

(3 marks)

(ii) $z[n] = [n-2]x[n]$

(2 marks)

(c) Design FIR filter with pass-band frequency of 5 kHz, stop-band frequency of 6 kHz and sampling frequency of 10 kHz. Choose the suitable window for your FIR filter with $N = 5$. Calculate the transfer function of causal sequence, $H_c[n]$.

(10 marks)

**- END OF QUESTIONS -**

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**TABLE 1**: Properties of the $N$-Sample DFT

| **Property** | **Signal** | **DFT** |
|---|---|---|
| Shift | $x[n-n_o]$ | $X_{DFT}[k]e^{-j2\pi k n_o/N}$ |
| Shift | $x[n-0.5N]$ | $(-1)^k X_{DFT}[k]$ |
| Modulation | $x[n]e^{j2\pi n k_o/N}$ | $X_{DFT}[k-k_o]$ |
| Modulation | $(-1)^n x[n]$ | $X_{DFT}[k-0.5N]$ |
| Folding | $x[-n]$ | $X_{DFT}[-k]$ |
| Product | $x[n]y[n]$ | $\frac{1}{N}X_{DFT}[k]\otimes Y_{DFT}[k]$ |
| Convolution | $x[n]\otimes y[n]$ | $X_{DFT}[k]Y_{DFT}[k]$ |
| Correlation | $x[n]\otimes\otimes y[n]$ | $X_{DFT}[k]Y^{*}_{DFT}[k]$ |
| Central Ordinates | $x[0]=\frac{1}{N}\sum_{k=0}^{N-1}X_{DFT}[k],\qquad X_{DFT}[0]=\sum_{n=0}^{N-1}x[n]$ | |
| Central Ordinates | $x\left[\frac{N}{2}\right]=\frac{1}{N}\sum_{k=0}^{N-1}(-1)^{k}X_{DFT}[k]$ ($N$ even), $X_{DFT}\left[\frac{N}{2}\right]=\sum_{n=0}^{N-1}(-1)^{n}x[n]$ ($N$ even) | |

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**TABLE 2**: Properties of z-transform.

| **Property** | **Signal** | **z-transform** |
|---|---|---|
| Linearity | $a_1x_1(n) + a_2x_2(n)$ | $a_1X_1(z) + a_2X_2(z)$ |
| Time reversal | $x(-n)$ | $X(z^{-1})$ |
| Time shifting | i) $x(n-k)$<br>ii) $x(n+k)$ | i) $z^{-k}X(z)$<br>ii) $z^{k}X(z)$ |
| Convolution | $x_1(n) * x_2(n)$ | $X_1(z)X_2(z)$ |
| Correlation | $r_{x_1x_2}(l) = \sum_{n=-\infty}^{\infty} x_1(n)x_2(n-l)$ | $R_{x_1x_2}(z) = X_1(z)X_2(z^{-l})$ |
| Scaling | $a^n x(n)$ | $X(a^{-1}z)$ |
| Differentiation | $nx(n)$ | $z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$ |
| Time differentiation | $x(n) - x(n-1)$ | $X(z)(1 - z^{-1})$ |
| Time integration | $\sum_{k=0}^{\infty} X(k)$ | $X(z) = \left(\frac{z}{z-1}\right)$ |
| Initial value theorem | $\lim_{n \to 0} x(n)$ | $\lim_{\lvert z \rvert \to \infty} X(z)$ |
| Final value theorem | $\lim_{n \to \infty} x(n)$ | $\lim_{\lvert z \rvert \to 1} \left(\frac{z-1}{z}\right) X(z)$ |

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**TABLE 3**: z-Transform Pairs

| **Signal $x(t)$** | **Sequence $x(n)$** | **z-Transform $X(z)$** |
|---|---|---|
| $\delta(t)$ | $\delta(n)$ | 1 |
| $\delta(t - k)$ | $\delta(n - k)$ | $z^{-k}$ |
| $u(t)$ | $u(n)$ | $\frac{1}{1-z^{-1}}=\frac{z}{z-1}$ |
| | $-u(-n - 1)$ | $\frac{1}{1-z^{-1}}=\frac{z}{z-1}$ |
| $r(t) = tu(t)$ | $nu(n)$ | $\frac{z^{-1}}{\left(1-z^{-1}\right)^2}=\frac{z}{(z-1)^2}$ |
| | $a^n u(n)$ | $\frac{1}{1-az^{-1}}=\frac{z}{z-a}$ |
| | $-a^n u(-n - 1)$ | $\frac{1}{1-az^{-1}}=\frac{z}{z-a}$ |
| | $na^n u(n)$ | $\frac{az}{(z-a)^2}$ |
| | $-na^n u(-n - 1)$ | $\frac{az}{(z-a)^2}$ |

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**TABLE 4**: Windows for FIR filter design.

| **Window** | **Expression** $w_N[n]$, $-0.5(N-1) \le n \le 0.5(N-1)$ |
|---|---|
| Boxcar | $1$ |
| Cosine | $\cos\left(\frac{n\pi}{N-1}\right)$ |
| Riemann | $\operatorname{sinc}^L\left(\frac{2n}{N-1}\right)$, $L>0$ |
| Bartlett | $1-\frac{2\lvert n\rvert}{N-1}$ |
| Von Hann (Hanning) | $0.5+0.5\cos\left(\frac{2n\pi}{N-1}\right)$ |
| Hamming | $0.54+0.46\cos\left(\frac{2n\pi}{N-1}\right)$ |

**Infinite Summation Formula**

$$\sum_{k=0}^{\infty}\alpha^k=\frac{1}{1-\alpha}, \qquad |\alpha|<1$$

$$\sum_{k=1}^{\infty}\alpha^k=\frac{\alpha}{1-\alpha}, \qquad |\alpha|<1$$

$$\sum_{k=1}^{\infty}k\alpha^k=\frac{\alpha}{(1-\alpha)^2}, \qquad |\alpha|<1$$

$$\sum_{k=1}^{\infty}k^2\alpha^k=\frac{\alpha^2+\alpha}{(1-\alpha)^3}, \qquad |\alpha|<1$$

$$\sum_{k=-\infty}^{\infty}e^{-\alpha|k|}=\frac{1+e^{-\alpha}}{1-e^{-\alpha}}, \qquad \alpha>0$$