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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BEE 21503
PROGRAMME	:	BACHELOR OF ELECTRICAL ENGINEERING WITH HONOURS AND BACHELOR OF ELECTRONIC ENGINEERING WITH HONOURS
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 (a) Find the local extrema and saddle points (if exists) for the function

$$f(x,y)=2x^4+y^2-2xy+20.$$

(10 marks)

(b) Using cylindrical coordinates, evaluate $\int_0^4 \int_0^{\sqrt{16-y^2}} \int_0^{\sqrt{16-x^2-y^2}} z\,dz\,dx\,dy$.

(10 marks)

Q2 (a) Suppose that a particle moves along a circular helix such that its position at time t is given by a vector valued function $\mathbf{r}(t)=\sin t\,\mathbf{i}+\cos t\,\mathbf{j}+2t\,\mathbf{k}$.

(i) Find the unit tangent vector $\mathbf{T}(t)$.

(ii) Find the unit normal vector $\mathbf{N}(t)$.

(iii) Sketch the graph of $\mathbf{r}(t)$, for $0 \le t \le 2\pi$.

(12 marks)

(b) Assume that the velocity of a moving particle is

$$\mathbf{v}(t)=(t+2)\mathbf{i}+t^2\mathbf{j}+e^{-t/3}\mathbf{k}$$

and the position at $t=0$ is $\mathbf{r}(0)=4\mathbf{i}-3\mathbf{k}$. Find the particle position at $t=1$.

(8 marks)

Q3 (a) Use Green's theorem to evaluate the integral

$$\oint_C x^2y\,dx+\left(y+xy^2\right)dy,$$

where C is the path in the counterclockwise sense along the boundary of the region defined by $y=x^2$ and $x=y^2$.

(10 marks)

(b) Given the vector field $\mathbf{F}(x,y,z)=2xy\mathbf{i}+\left(x^2+z^2\right)\mathbf{j}+2yz\mathbf{k}$.

(i) Show that $\mathbf{F}$ is a conservative vector field.

(3 marks)

(ii) Find its potential function $\phi(x,y,z)$.

(5 marks)

(iii) Hence, find the work done in this field to move an object from a point $(1,0,0)$ to $(0,1,2\pi)$.

(2 marks)

Q4 (a) Evaluate the integral

$$\int_C (x+y)\,dx + xy\,dy$$

where C consists of two line segments, from $(0,\ 0)$ to $(1,\ 0)$ along the line $y=0$, and from $(1,\ 0)$ to $(1,\ 1)$ along the line $x=1$.

(8 marks)

(b) Use Stokes' theorem to evaluate $\oint_C \mathbf{F}\cdot d\mathbf{r}$ where

$$\mathbf{F}(x,y,z) = xz\,\mathbf{i} + xy^2\,\mathbf{j} + 3xz\,\mathbf{k}$$

along the curve C where C is the intersection between the cylinder $x^2+y^2=4$ and the plane $x+z=3$ oriented counter clockwise.

(12 marks)

Q5 (a) Evaluate $\iint_S 3\left(x^2+y^2\right)dS$, where S is the surface of the solid bounded by $z=\sqrt{9-x^2-y^2}$ that lies above the $xy-$plane.

(10 marks)

(b) Let S be the surface of sphere $x^2+y^2+z^2=1$ and $\mathbf{F}(x,y,z) = 7x\,\mathbf{i} - z\,\mathbf{k}$.

(i) Find the divergence of $\mathbf{F}(x,y,z)$.

(ii) Use Gauss's Theorem to evaluate

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS.$$

(10 marks)

- END OF QUESTIONS -

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Formulae

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(y/x)$, and
$\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$,
$\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, $x^2 + y^2 + z^2 = \rho^2$,

$0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$, and
$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

Directional derivative: $D_{\mathbf{u}} f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

the **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

the curvature: $\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

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Gauss Theorem: $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_G \nabla\cdot\mathbf{F}\,dV$

Stokes' Theorem: $\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS$

Arc length : If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t\in[a,b]$, then the **arc length**

$$s = \int_a^b \|\mathbf{r}'(t)\|\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t\in[a,b]$, then the **arc length**

$$s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$$

Tangent Plane :

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

Extreme of two variable functions

$$D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - (f_{xy}(x,y))^2$$

Case1: If $D(a,b)>0$ and $f_{xx}(x,y)<0$ then f has local maximum at (a,b)

Case2: If $D(a,b)>0$ and $f_{xx}(x,y)>0$ then f has local minimum at (a,b)

Case3: If $D(a,b)<0$ then f has a saddle point at (a,b)

Case4: If $D(a,b)=0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x,y)\,dA$, where $\delta(x,y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\,\delta(x,y)\,dA$,

(ii) about x-axis, $M_x = \iint_R y\,\delta(x,y)\,dA$

Centre of mass, $(\bar{x},\bar{y}) = \left(\dfrac{M_y}{m}, \dfrac{M_x}{m}\right)$

Moment inertia: (i) $I_y = \iint_R x^2\delta(x,y)\,dA$,

(ii) $I_x = \iint_R y^2\delta(x,y)\,dA$,

(iii) $I_o = \iint_R (x^2+y^2)\delta(x,y)\,dA$

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In 3-D: Solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$. If $\delta(x,y,z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x,y,z)\,dV$

Centre of gravity, $(\bar{x},\bar{y},\bar{z}) = \left(\dfrac{M_{yz}}{m}, \dfrac{M_{xz}}{m}, \dfrac{M_{xy}}{m}\right)$

Moment inertia

(i) about x-axis: $I_x = \iiint_G (y^2+z^2)\delta(x,y,z)\,dV$

(ii) about y-axis: $I_y = \iiint_G (x^2+z^2)\delta(x,y,z)\,dV$

(iii) about z-axis: $I_z = \iiint_G (x^2+y^2)\delta(x,y,z)\,dV$