UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	SIGNALS & SYSTEMS
COURSE CODE	:	BEB 20203
PROGRAMME	:	BEJ
EXAMINATION DATE	:	DECEMBER 2014/ JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A: ANSWER **ALL** QUESTIONS SECTION B: ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **FIFTEEN (15)** PAGES

SECTION A: ANSWER ALL QUESTIONS

Q1 (a) Given $g(t)$ in **Figure Q1 (a)**. Express $g(t)$ as sum of triangle (*tri*) and unit step function. Sketch both triangle function and unit step function.

(4 marks)

(b) Signal $x(t)$ in **Figure Q1 (b)** is applied to an amplifier that has a gain of 3 and introduces a bias of -1 as shown in **Figure Q1 (c)**. Sketch the amplifier output signal, $y(t)$.

(6 marks)

Q2 (a) Test whether the signal $x(t) = 2\cos\left(10\pi t + \frac{\pi}{6}\right)$ is periodic or not. If the signal is periodic determine its fundamental period.

(3 marks)

(b) The non-zero Fourier series coefficients in exponential form of a continuous-time periodic signal *f(t)* with fundamental time period $T = 8$ are

$$F_1 = F_{-1}^* = 2,\ F_3 = F_{-3}^* = 4j\,.$$

(i) Find the fundamental frequency.

(ii) Write the exponential signal equation.

(iii) Express the signal in sinusoidal form.

(4 marks)

(b) Calculate the average power supplied to a network if the applied voltage and resulting current are given by

$v(t) = 100\sin 30t + 80\sin 60t + 40\sin 90t$ (Volts)

$i(t) = 12\sin(30t + 65^0) + 20\sin(60t + 45^0) + 15\sin(90t + 25^0)$ (Amperes)

(3 marks)

Q3 (a) A non-periodic input signal of a high pass filter is written as

$$w(t) = u(t+2) - 3u(t) + 2u(t-2).$$

(i) Sketch $f(t) = w(-t)$

(2 marks)

(ii) Derive $F(\omega)$ using the definition of Fourier Transform.

(6 marks)

(b) Explain the duality property of Fourier transform with the aid of a simple example.

(2 marks)

Q4 (a) **Figure Q4 (a)** shows a LTI system. Determine the Laplace transform of the system using the properties of Laplace transform.

(4 marks)

(b) Investigate the causality and the stability of a LTI system with the system function

$$H(s) = \frac{s-1}{(s+1)(s-2)}.$$

(6 marks)

SECTION B: ANSWER THREE (3) QUESTIONS ONLY

Q5 (a) The output of the system given in **Figure Q5(a)** is $y(t) = x(t)u(t)$. Determine whether the system is

(i) Linear or non linear
(ii) Time variant or time invariant

Show all the required steps in your assessment of the system.

(4 marks)

(b) Impulse response for LTI system is $h(t) = 1$, for $-1 \le t \le 2$. This system is used in modeling of digital to analog convertor. The input signal of the system is specified as

$$x(t) = 2e^{-0.5t}u(t)$$

Determine the output of the system $y(t)$ using graphical method of convolution signal.

(16 marks)

Q6 (a) The Fourier series of a periodic function *f(t)* is a representation that resolves *f(t)* into the dc component and ac components comprising an infinite series of harmonic sinusoids.

(i) explain the definition of a periodic function.

(i) Define **TWO (2)** applications of Fourier series in electrical field.

(4 marks)

(b) Consider a single sinusoidal signal which is $v(t) = 10\cos 2\pi 1000t$ V.

(i) Convert the signal into exponential Fourier series form; and

(ii) Draw the spectrum.

(4 marks)

(c) Given a periodic signal shown in **Figure Q6(c)**. Determine the trigonometric Fourier series for the given signal.

(12 marks)

Q7 (a) The impulse responses of two cascaded Linear Time Invariant (LTI) system as shown in **Figure Q7(a)** are $h_{1(t)} = 3\delta(3t-6)$ and $h_2(t) = e^{-5t}u(t)$ respectively. The input signal to the system is $x(t) = e^{-2t}u(t)$. By using appropriate properties of Fourier Transform,

(i) determine the total Frequency Response, $H_T(\omega)$of the system above

(5 marks)

(ii) compute the output response, $y(t)$.

(10 marks)

(b) Determine the total energy dissipated by a resistor in a RC circuit using **Parseval's Relation** if the voltage across the 2-Ω resistor is given by $v_R(t) = 10\,e^{-2t}u(t)\ V$. [Hint: $\int_{-\infty}^{+\infty}|x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2\,d\omega$]

(5 marks)

Q8 (a) Consider the following LTI system as shown in **Figure Q8(a)**. The system has a response of $H_1(s) = \frac{s}{(s+1)(s+a)}$ and $H_2(s) = \frac{b}{s}$, respectively.

(i) Determine a and b such that the overall transfer function is

$$H(s) = \frac{s}{(s+4)(s+5)}$$

(5 marks)

(ii) Determine the output y(t) of the system with the above transfer function to the unit-step input x(t)=u(t).

(5 marks)

(b) A series RLC circuit is illustrated in **Figure Q8(b).** The relationship between the input and the output can be written in the form of differential equation which is

$$RCy' + LCy'' + y(t) = x(t).$$

Given that the values of of R, L and C are 2 Ω, 1 H, 2 F respectively, determine using Laplace Transform,

(i) Impulse response of the system circuit, *h*(*t*).

(5 marks)

(ii) Output response, $y(t)$ for $x(t) = e^{-2t}u(t)$.

(5 marks)

END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2014/2015 PROGRAMME: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

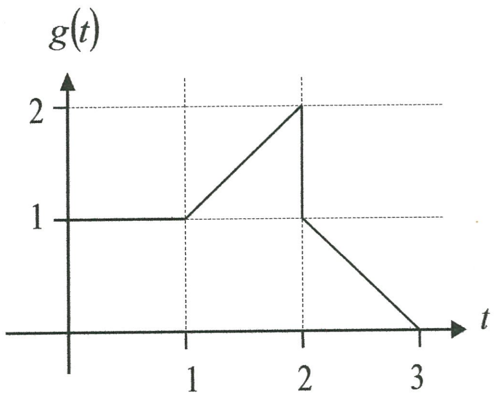

Figure Q1(a)

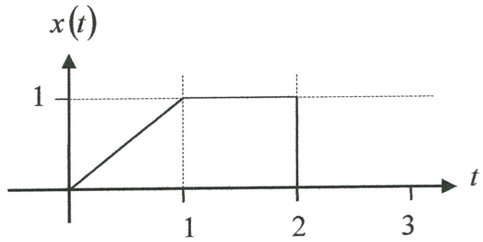

Figure Q1(b)

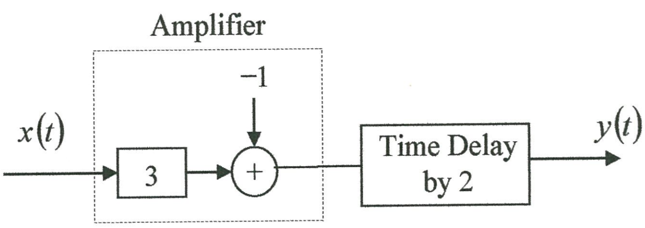

Figure Q1(c)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2014/2015
COURSE NAME: SIGNALS AND SYSTEMS

PROGRAMME: BEJ
COURSE CODE: BEB 20203

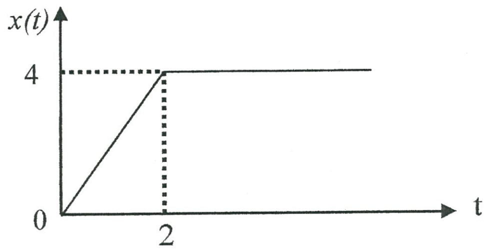

Figure Q4(a)

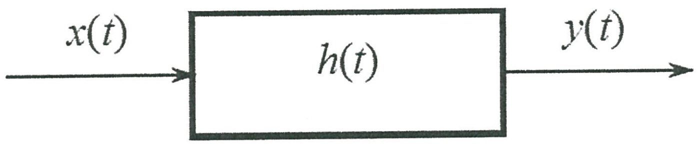

Figure Q5(a)

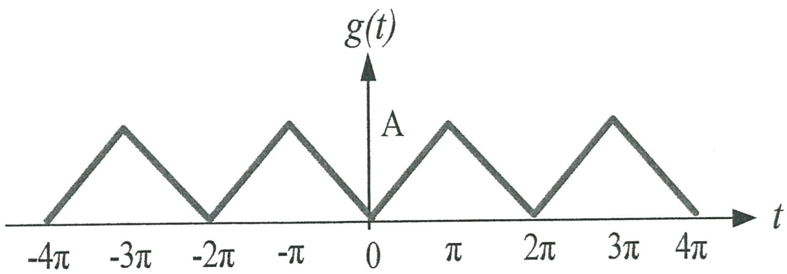

Figure Q6(c)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2014/2015
COURSE NAME: SIGNALS AND SYSTEMS

PROGRAMME: BEJ
COURSE CODE: BEB 20203

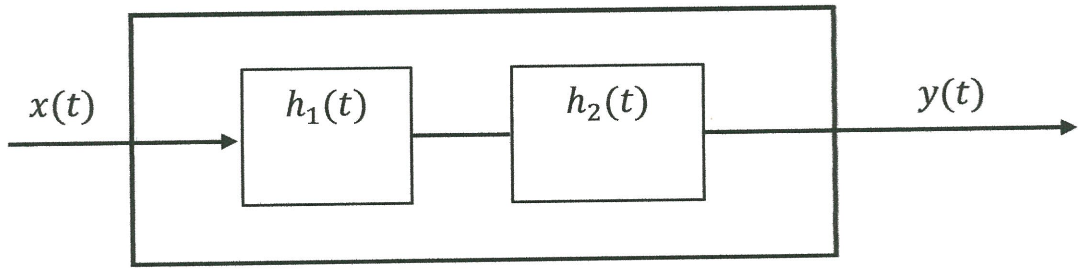

Figure Q7(a)

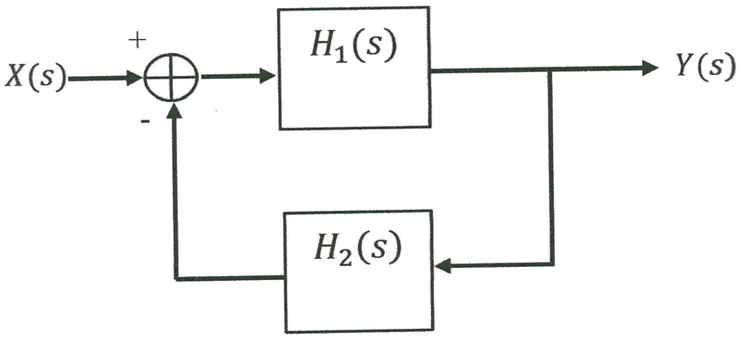

Figure Q8(a)

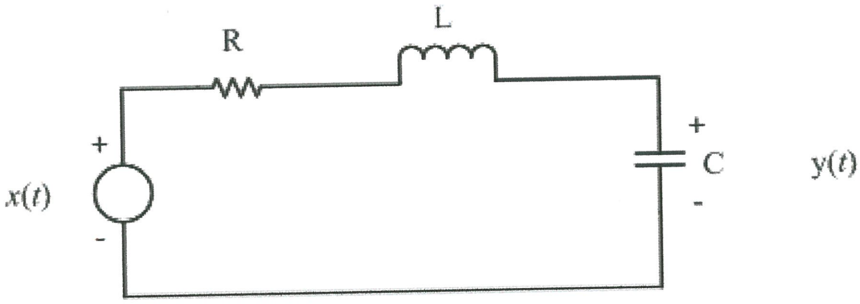

Figure Q8(b)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2014/2015 PROGRAMME: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

INDEFINITE INTEGRALS

$$\int \cos at\, dt = \frac{1}{a}\sin at$$

$$\int \sin at\, dt = -\frac{1}{a}\cos at$$

$$\int t\cos at\, dt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at$$

$$\int t\sin at\, dt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

EULER'S IDENTITY

$$e^{\pm j\pi/2} = \pm j \quad ; \qquad A\angle \pm\theta = Ae^{\pm j\theta}$$

$$e^{\pm jk\pi} = \cos(k\pi) \quad ; \qquad e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \quad ; \qquad \sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2014/2015 PROGRAMME: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

FOURIER SERIES

Exponential	$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$ $x_n = \frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t}$
Trigonometric	$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\frac{2\pi}{T}t + b_n \sin n\frac{2\pi}{T}t$ $a_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t) \cos n\frac{2\pi}{T}t$ $b = \frac{2}{T}\int_{\alpha}^{\alpha+T} x(t) \sin n\frac{2\pi}{T}t$

FOURIER TRANSFORM

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

INVERSE FOURIER TRANSFORM

$$x(t) = F^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$$

LAPLACE TRANSFORM

$$\mathcal{L}(x(t)) = X(s) = \int_{0}^{\infty} x(t)e^{-st} dt$$

INVERSE LAPLACE TRANSFORM

$$x(t) = \mathcal{L}^{-1}(X(s)) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st} ds$$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2014/2015
COURSE NAME: SIGNALS AND SYSTEMS

PROGRAMME: BEJ
COURSE CODE: BEB 20203

TABLE 1: Trigonometric Identities

Trigonometric identities	
$\sin \alpha = \cos\left(\alpha - \frac{\pi}{2}\right)$	$\cos\alpha = \sin(\alpha + \frac{\pi}{2})$
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 2: Values of cosine, sine and exponential functions for integral multiple of π

Function	**Value**
$\cos 2n\pi$	1
$\sin 2n\pi$	0
$\cos n\pi$	$(-1)^n$
$\sin n\pi$	0
$\cos\frac{n\pi}{2}$	$\begin{cases}(-1)^{n/2}, & n = even \\ 0, & n = odd\end{cases}$
$\sin\frac{n\pi}{2}$	$\begin{cases}(-1)^{(n-1)/2}, & n = odd \\ 0, & n = even\end{cases}$
$e^{j2n\pi}$	1
$e^{jn\pi}$	$(-1)^n$
$e^{jn\pi/2}$	$\begin{cases}(-1)^{n/2}, & n = even \\ j(-1)^{(n-1)/2}, & n = odd\end{cases}$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2014/2015 PROGRAMME: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 3: Fourier Transform Pairs

Time domain, $f(t)$	Frequency domain, $F(\omega)$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$u(t+\tau)-u(t-\tau)$	$2\frac{\sin\omega\tau}{\omega}$
$\|t\|$	$\frac{-2}{\omega^2}$
$\text{sgn}(t)$	$\frac{2}{j\omega}$
$e^{-\alpha t}u(t)$	$\frac{1}{\alpha+j\omega}$
$e^{\alpha t}u(-t)$	$\frac{1}{\alpha-j\omega}$
$t^n e^{-\alpha t}u(t)$	$\frac{n!}{(\alpha+j\omega)^{n+1}}$
$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\alpha^2+\omega^2}$
$e^{j\omega_o t}$	$2\pi\delta(\omega-\omega_o)$
$\sin\omega_o t$	$j\pi[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]$
$\cos\omega_o t$	$\pi[\delta(\omega+\omega_o)+\delta(\omega-\omega_o)]$
$e^{-\alpha t}\sin\omega_o t\, u(t)$	$\frac{\omega_o}{(\alpha+j\omega)^2+\omega_o^2}$
$e^{-\alpha t}\cos\omega_o t\, u(t)$	$\frac{\alpha+j\omega}{(\alpha+j\omega)^2+\omega_o^2}$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2014/2015 PROGRAMME: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 4: Fourier Transform Properties

Property	Time domain, *f(t)*	Frequency domain, $F(\omega)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a\rvert} F(\frac{\omega}{a})$
Time Shift	$f(t-a)$	$e^{-j\omega a} F(\omega)$
Frequency Shift	$e^{j\omega_o t} f(t)$	$F(\omega - \omega_o)$
Modulation	$\cos(\omega_0 t)\, f(t)$	$\frac{1}{2}[F(\omega+\omega_o) + F(\omega-\omega_o)]$
Time Differentiation	$\frac{df}{dt}$ $\frac{d^n f}{dt^n}$	$j\omega F(\omega)$ $(j\omega)^n F(\omega)$
Time Integration	$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$
Frequency Differentiation	$t^n f(t)$	$j^n \frac{d^n}{d\omega^n} F(\omega)$
Reversal	$f(-t)$	$F(-\omega)$ or $F^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Convolution in *t*	$f_1(t) * f_2(t)$	$F_1(\omega) F_2(\omega)$
Convolution in ω	$f_1(t) f_2(t)$	$F_1(\omega) * F_2(\omega)$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2014/2015
COURSE NAME: SIGNALS AND SYSTEMS

PROGRAMME: BEJ
COURSE CODE: BEB 20203

TABLE 5: Laplace Transform

$x(t),\, t > 0$	$X(s)$	ROC
$\delta(t)$	1	*All s*
$u(t)$	$\frac{1}{s}$	$Re(s) > 0$
t	$\frac{1}{s^2}$	$Re(s) > 0$
t^n	$\frac{n!}{s^{n+1}}$	$Re(s) > 0$
e^{-at}	$\frac{1}{s+a}$	$Re(s) > -a$
te^{-at}	$\frac{1}{(s+a)^2}$	$Re(s) > -a$
$cos\, bt$	$\frac{s}{s^2+b^2}$	$Re(s) > 0$
$sin\, bt$	$\frac{b}{s^2+b^2}$	$Re(s) > 0$
$e^{-at}\, cos\, bt$	$\frac{s+a}{(s+a)^2+b^2}$	$Re(s) > -a$
$e^{-at}\, sin\, bt$	$\frac{b}{(s+a)^2+b^2}$	$Re(s) > -a$
$tcos\, bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$Re(s) > 0$
$tsin\, bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s) > 0$
$tsin\, bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s) > 0$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2014/2015 PROGRAMME: BEJ
COURSE NAME: SIGNALS AND SYSTEMS COURSE CODE: BEB 20203

TABLE 6: Laplace Transform Properties

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathcal{R}e\{s\} > 0\}$

Initial- and Final-Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s \to \infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \longrightarrow \infty$, then

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$$