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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BEE21503/BWM20403
PROGRAMME	:	BDD/BEJ/BEV/BFF
EXAMINATION DATE	:	DECEMBER 2014 / JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) Find the rate of change of volume of a cylinder with radius 6 cm and height 14 cm if the increasing rate of radius is 0.3 cms^{-1} and the decreasing rate of height is 0.4 cms^{-1}.

(6 marks)

(b) Show that the function $z = e^x \sin y + e^y \cos x$ satisfies Laplace equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

(6 marks)

(c) Determine the local extremum of the function $f(x, y) = xy^2 - 6x^2 - 3y^2$.

(8 marks)

Q2 (a) Compute the multiple integrals:

(i) $\displaystyle\int_0^1 \int_0^{\sqrt{4-z^2}} \int_0^y xy \, dxdydz,$

(3 marks)

(ii) $\displaystyle\iint_R \left(8 - x^2 - y^2\right) dA$, where $R = \{(x, y) = -1 \le x \le 1 \text{ and } 0 \le y \le 2\}$.

(4 marks)

(b) Evaluate the following triple integral using spherical coordinates.

$$I = \int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} 3x^2 + 3y^2 + 3z^2 \, dzdydx$$

(5 marks)

(c) Given a lamina that occupies the region bounded by $y = \sqrt{x}$, $x = 9$ and $y = 0$, and has density function $\delta(x, y) = x + y$. Find

(i) its mass.

(3 marks)

(ii) its coordinate $\bar{y}$ of its center of mass.

(5 marks)

Q3 (a) A scalar function is given as $f(x,y) = (1+xy)^{\frac{3}{2}}$.

(i) Find the gradient of $f(x,y)$ at (3,1).

(2 marks)

(ii) Find the directional derivative of $f(x,y)$ at (3,1) in the direction of vector $\mathbf{u} = 2\mathbf{i} + \mathbf{j}$.

(3 marks)

(iii) Find a unit vector in the direction in which $f(x,y)$ increases most rapidly at (3,1) and find the rate of change of $f(x,y)$ at (3,1) in that direction.

(4 marks)

(iv) In what direction is $f(x,y)$ decreasing most rapidly at point (3,1) and what is the maximum rate of decrease?

(3 marks)

(b) The electric field distribution in a room is given by $\boldsymbol{E}(x,y,z) = xy^2z\,\mathbf{i} + x^2yz\,\mathbf{j} + xyz\,\mathbf{k}$.

(i) Find the divergence and curl of the electric field.

(5 marks)

(ii) Show that $\nabla \bullet (\nabla \times \boldsymbol{E}) = 0$.

(3 marks)

Q4 (a) Given that the velocity vector $\mathbf{v}(t) = -a\sin t\,\mathbf{i} + a\cos t\,\mathbf{j} + \sqrt{1-a^2}\,\mathbf{k}$. Find its position vector $\mathbf{r}(t)$, if $\mathbf{r}(0) = a\mathbf{i}$. Then find its arc length from $(a,\ 0,\ 0)$ to $\left(-a,\ 0,\ \pi\sqrt{1-a^2}\right)$.

(8 marks)

(b) The displacement of a particle at time t is given by $\mathbf{r}(t) = t\,\mathbf{i} + 2\sin\pi t\,\mathbf{j} + 2\cos\pi t\,\mathbf{k}$, $0 \le t \le 2\pi$. Find the velocity $\mathbf{v}(t)$, the unit tangent vector $\mathbf{T}(t)$, the unit normal vector $\mathbf{N}(t)$, binormal vector $\mathbf{B}(t)$ and curvature κ at any time t.

(12 marks)

Q5 (a) Evaluate $\int_c x^2\,dx + (x+y)dy$, where C is a path as shown in **FIGURE Q6(a)**. Can Green's theorem be used to evaluate this integral?

(5 marks)

(b) Apply the Green's theorem to evaluate

$$\oint_c (2x^2 + xy)\,dx + (x^3 + 3xy^2)\,dy,$$

where C is the boundary of the region between $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

(5 marks)

(c) Use Gauss Theorem (or Divergence Theorem) to find the flux of the vector field $\mathbf{F}\,(x,y,z) = 2x^3\mathbf{i} + y^2\mathbf{j} + 2\,z^3\mathbf{k}$ across surfaces σ of the solid G enclosed by circular cylinder $x^2 + y^2 = 16$, planes $z = 0$ and $z = 3$, oriented outward.

(5 marks)

(d) Use Stoke's Theorem to show that $\oint_c \mathbf{F} \bullet d\boldsymbol{r} = -1/3$, if $\mathbf{F}\,(x,y,z) = 3xz\,\mathbf{i} + xy\,\mathbf{j} + 3xz\,\mathbf{k}$ and C is the boundary of the portion of plane $2x + y + z = 2$ in first octant, oriented counterclockwise, and upward.

(5 marks)

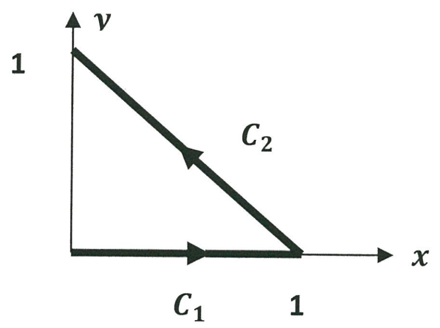

FIGURE Q6(a)

- END OF QUESTIONS -

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Formulae

Polar coordinate

$x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)\,dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate

$x = r\cos\theta$, $y = r\sin\theta$, $z = z$ and $\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate

$x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$, then $x^2 + y^2 + z^2 = \rho^2$, for $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$, and $\iiint_G f(x,y,z)\,dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

Directional derivative in $D_{\mathbf{u}}$ (in $\mathbf{u}$ direction)

$$D_{\mathbf{u}} f(x,y) = \left(f_x\mathbf{i} + f_y\mathbf{j}\right)\cdot\mathbf{u}$$

Divergence and Curl of $\mathbf{F}$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

$$\nabla\cdot\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} \text{ and}$$

$$\nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Vector Valued Functions $\mathbf{r}(t)$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

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the curvature: $\kappa = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}\right) dA$

Gauss Theorem: $\iint_\sigma \mathbf{F}\bullet\mathbf{n}\,dS = \iiint_G \nabla\bullet\mathbf{F}\,dV$

Stokes' Theorem: $\oint_C \mathbf{F}\bullet d\mathbf{r} = \iint_\sigma (\nabla\times\mathbf{F})\bullet\mathbf{n}\,dS$,

where $\mathbf{n} = \dfrac{-\dfrac{\partial z}{\partial x}\mathbf{i} - \dfrac{\partial z}{\partial y}\mathbf{j} + \mathbf{k}}{\sqrt{\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 + 1}}$ and $dS = \sqrt{\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 + 1}\,dA$.

Arc length s

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t\in[a,b]$, then $s = \int_a^b \|\mathbf{r}'(t)\|\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t\in[a,b]$, then $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$

Tangent Plane

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Second Derivative Test for Extreme of two variable functions

$G(x,y) = f_{xx}(x,y)f_{yy}(x,y) - (f_{xy}(x,y))^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

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Lamina

Mass: $m = \iint_R \delta(x, y)\,dA$, where $\delta(x, y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x, y)\,dA$,

(ii) about x-axis, $M_x = \iint_R y\delta(x, y)\,dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\dfrac{M_y}{m}, \dfrac{M_x}{m}\right)$

Moment inertia: (i) $I_y = \iint_R x^2\delta(x, y)\,dA$, (ii) $I_x = \iint_R y^2\delta(x, y)\,dA$, and

(iii) $I_o = \iint_R \left(x^2 + y^2\right)\delta(x, y)\,dA$

In 3-D: Solid

Mass, $m = \iiint_G \delta(x, y, z)\,dV$. If $\delta(x, y, z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x, y, z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x, y, z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x, y, z)\,dV$

Centre of gravity, $(\bar{x}, \bar{y}, \bar{z}) = \left(\dfrac{M_{yz}}{m}, \dfrac{M_{xz}}{m}, \dfrac{M_{xy}}{m}\right)$

Moment inertia

(i) about x-axis: $I_x = \iiint_G \left(y^2 + z^2\right)\delta(x, y, z)\,dV$

(ii) about y-axis: $I_y = \iiint_G \left(x^2 + z^2\right)\delta(x, y, z)\,dV$

(iii) about z-axis: $I_z = \iiint_G \left(x^2 + y^2\right)\delta(x, y, z)\,dV$