UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2013/2014

COURSE NAME	:	ELECTROMAGNETIC FIELDS AND WAVES
COURSE CODE	:	BEB 20303
PROGRAMME	:	BEJ
EXAMINATION DATE	:	JUNE 2014
DURATION	:	3 HOURS
INSTRUCTION	:	**SECTION A** – ANSWER **ALL** QUESTIONS **SECTION B** – ANSWER **TWO (2)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **THIRTEEN (13)** PAGES

SECTION A – ANSWER **ALL** QUESTIONS

Q1 (a) List **ONE (1)** surrounding phenomenon on magnetostatic concept and discuss the working principles of the phenomenon based on the definition of Biot Savart's Law.

(5 marks)

(b) State **THREE (3)** similarities and difference between Electric Field and Magnetic Field

(6 marks)

(c) A rectangular conducting loop shown in **Figure Q1 (c)** lies in the z = 0 plane and carries a current I in the counterclockwise direction. Prove that at the center of the loop, the magnetic field intensity is given by $H = \frac{8.1\,I}{4\pi a}\hat{z}$.

(14 marks)

Q2 (a) Indicate whether electromotive force (EMF) induced in the loop of each of the following situations is clockwise, counterclockwise, or zero (EMF not induced). Justify your answer.

(i) Magnetic field through a wire loop is pointed upwards and increasing with time, as shown in **Figure Q2 (a)(i).**

(ii) A rectangular wire loop is pulled away from a long wire carrying current I in the direction shown in **Figure Q2 (a)(ii).**

(iii) A rectangular wire loop is pulled upward though a uniform magnetic field that point out of the screen penetrating the bottom half of the loop, as shown in **Figure Q2 (a)(iii).**

(iv) A rectangular wire loop is pulled sideways though a uniform magnetic field that point out of the screen penetrating the bottom half of the loop, as shown in **Figure Q2 (a)(iv).** Justify your answers.

(10 marks)

(b) A plane of wingspan of 30 m flies through a vertical magnetic field of strength, $B = 5 \text{ X } 10^{-4}$ T. Calculate the electromotive force (EMF) induced across the wing tips if the plane velocity is 150 m/s. Briefly explain why horizontal component of the Earth's magnetic field does not contribute to the EMF between the wing tips of the plane.

(7 marks)

(c) A square loop with length l = 5 cm on each side is placed in a uniform magnetic field, $B = 5 \text{ X } 10^{-6}$ T pointing into the page. During a time interval, Δt = 2s, the loop is pulled from its two edges and turned into a rhombus with angle, $\theta = 60°$, as shown in the **Figure Q2 (c)**. Assuming that the total resistance of the loop is R = 10 Ω, calculate the average induced current in the loop I and its direction.

(8 marks)

SECTION B – ANSWER **TWO (2)** QUESTIONS ONLY

Q3 (a) **'An isolated magnetic charge does not exist'**. Justify this statement.

(5 marks)

(b) Consider an infinitely long coaxial transmission line consisting of two concentric cylinders as shown in **Figure Q3(a)**.

(i) By assuming the current distribution is uniformly distributed in both conductors, determine the magnetic field intensity, H along the Amperian path for all possible **SIX (6)** regions.

(16 marks)

(ii) Draw the magnetic flux lines which indicates the intensity of the magnetic field intensity, H.

(4 marks)

Q4 A spherical capacitor is formed by two metallic spheres of radius a (inner sphere) and b (outer sphere) with the separation between spheres d is as shown in **Figure Q4**. The charge on the inner sphere is +Q and that on the outer sphere is –Q. The region in between the spheres is filled with dielectric material with dielectric constant of ε_r.

(a) State the law that can be used to find the total electric flux in the region between the spheres. Explain the law in the form of a sentence.

(3 marks)

(b) Using the definition of law in **Q4 (a)**, deduce the general formulation for calculating the electric field intensity in the region between the spheres.

(6 marks)

(c) Calculate the capacitance of the system.

(5 marks)

(d) What is the capacitance of the isolated inner sphere? State clearly your assumption for the solution.

(4 marks)

(e) If the separation between the two spheres d is very small such that $d << a$, what is the capacitance of this system? State clearly your approximation for the solution.

(4 marks)

(f) From Q4 (b) – Q4 (e), how a capacitance value of a two conductors system can be controlled?

(3 marks)

Q5 (a) Consider a magnetic field intensity, $\vec{H} = 25\sin(2 \times 10^8 t + 6x)\hat{a}_y \ mA/m$. If the wave propagating through a nonmagnetic medium ($\mu = \mu_o$), determine:

(i) The direction of wave propagation, k,

(ii) The permittivity of the medium, ε,

(iii) The electric field intensity, E_o.

(10 marks)

(b) Given a 60-MHz plane wave travelling in the (-) x- direction in dry soil with relative permittivity, $\varepsilon_r = 4$ has an electric field polarized along the (+) z- direction. By assuming the dry soil to be approximately lossless, and given that the magnetic field has a peak value of 10 (mA/m) and that its value was measured to be 7 (mA/m) at t = 0 and x = -0.75, develop a complete expressions for the wave's electric and magnetic fields.

(15 marks)

- **END OF QUESTIONS** -

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER II/2013/2014 PROGRAMME: BEJ

COURSE: ELECTROMAGNETIC FIELDS AND WAVES COURSE CODE: BEB 20303

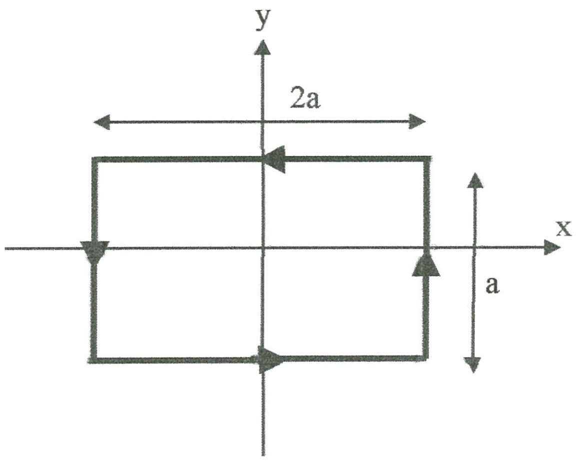

FIGURE Q1 (c)

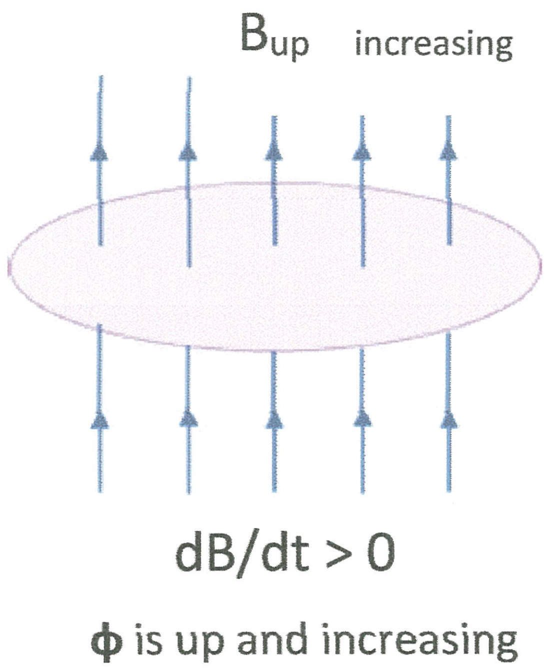

FIGURE Q2 (a) (i)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER II/2013/2014 PROGRAMME: BEJ

COURSE: ELECTROMAGNETIC FIELDS AND WAVES COURSE CODE: BEB 20303

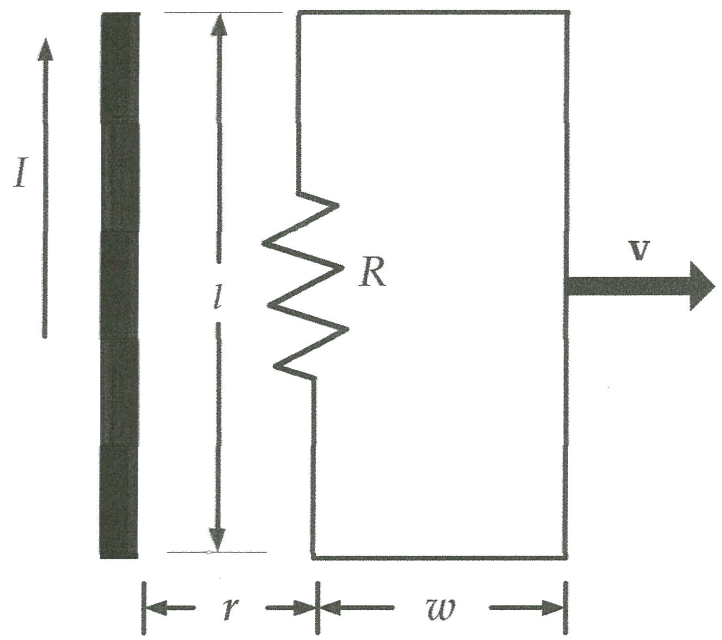

FIGURE Q2 (a) (ii)

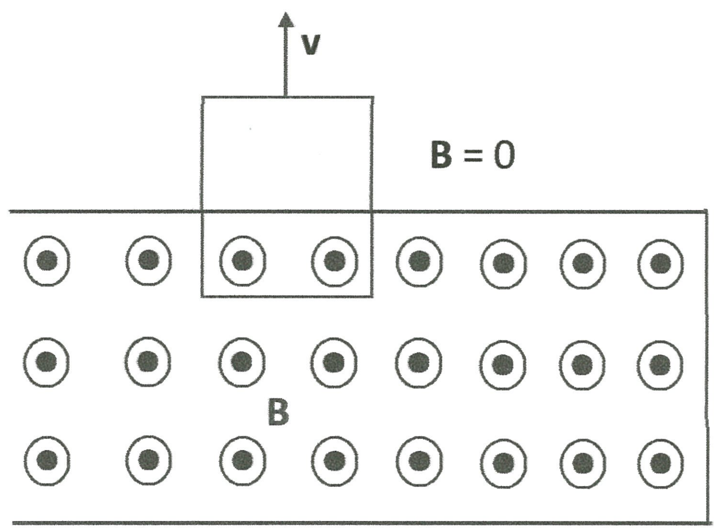

FIGURE Q2 (a) (iii)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER II/2013/2014 PROGRAMME: BEJ

COURSE: ELECTROMAGNETIC FIELDS AND WAVES COURSE CODE: BEB 20303

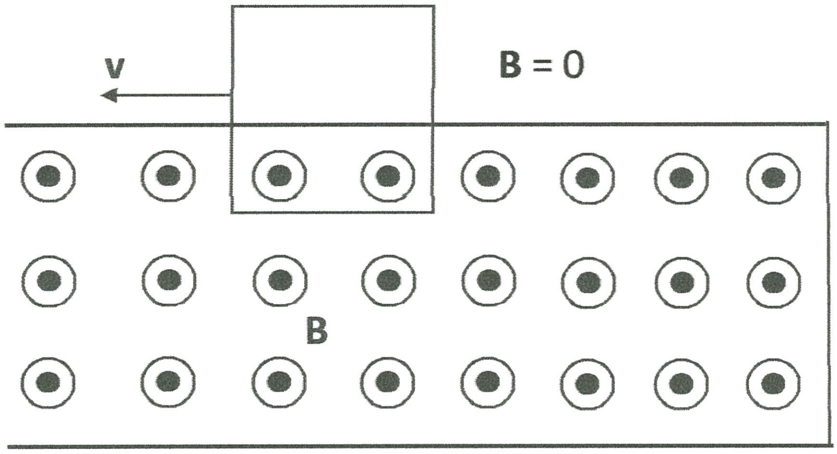

FIGURE Q2 (a) (iv)

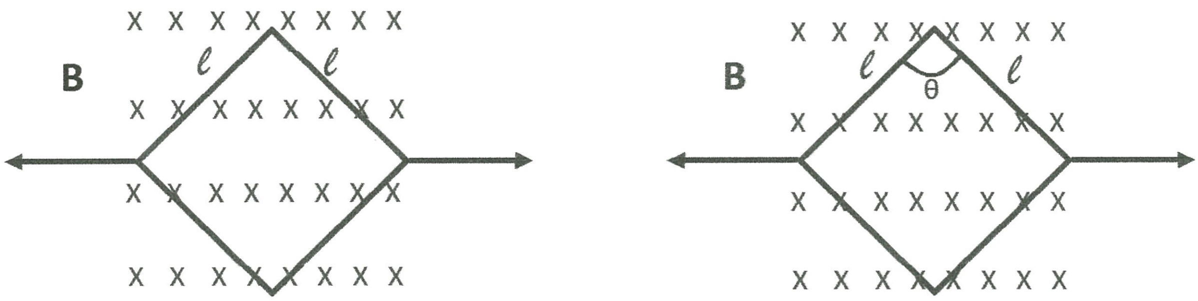

FIGURE Q2 (c)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER II/2013/2014 PROGRAMME: BEJ

COURSE: ELECTROMAGNETIC FIELDS AND WAVES COURSE CODE: BEB 20303

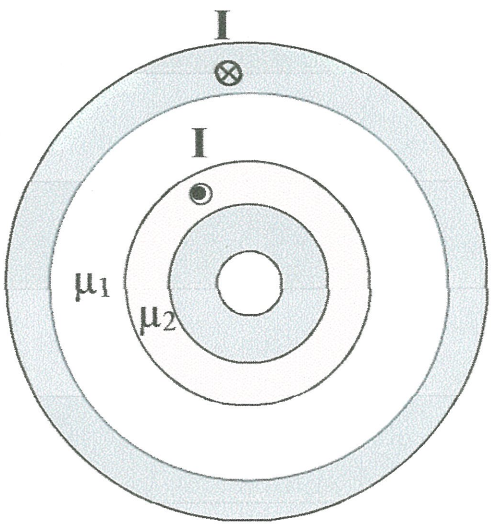

FIGURE Q3 (a)

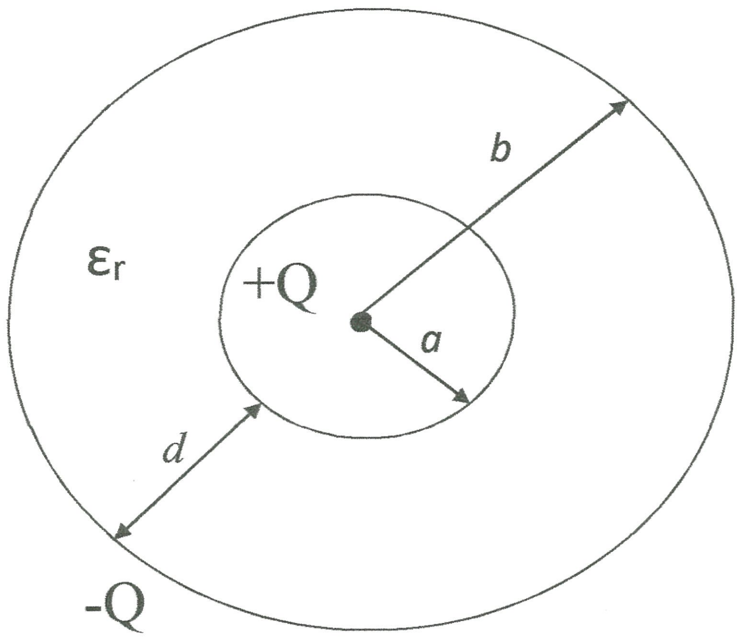

FIGURE Q4

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER II/2013/2014 PROGRAMME: BEJ
COURSE: ELECTROMAGNETIC FIELDS AND WAVES COURSE CODE: BEB 20303

Formula

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial (rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial (R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial (A_\theta \sin\theta)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\phi}} + \frac{1}{r}\left(\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial (\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial (RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial (RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER II/2013/2014 PROGRAMME: BEJ
COURSE: ELECTROMAGNETIC FIELDS AND WAVES COURSE CODE: BEB 20303

	Cartesian	**Cylindrical**	**Spherical**
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\varphi}}$
Magnitude $\vec{A}$	$\sqrt{A_x^{\;2} + A_y^{\;2} + A_z^{\;2}}$	$\sqrt{A_r^{\;2} + A_\phi^{\;2} + A_z^{\;2}}$	$\sqrt{A_R^{\;2} + A_\theta^{\;2} + A_\phi^{\;2}}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1, y_1, z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1, \phi_1, z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1, \theta_1, \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\varphi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\varphi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\overrightarrow{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\varphi}}$
Differential surface, $\overrightarrow{ds}$	$\overrightarrow{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\overrightarrow{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\overrightarrow{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\overrightarrow{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\varphi}}$ $\overrightarrow{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\overrightarrow{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\overrightarrow{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume, $\overrightarrow{dv}$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER II/2013/2014 PROGRAMME: BEJ
COURSE: ELECTROMAGNETIC FIELDS AND WAVES COURSE CODE: BEB 20303

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\varphi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to Spherical	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}}=\hat{\boldsymbol{\varphi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to Cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\varphi}}=\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER II/2013/2014 PROGRAMME: BEJ
COURSE: ELECTROMAGNETIC FIELDS AND WAVES COURSE CODE: BEB 20303

$$Q = \int \rho_\ell d\ell,$$
$$Q = \int \rho_s dS,$$
$$Q = \int \rho_v dv$$
$$\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$
$$\overline{E} = \frac{\overline{F}}{Q},$$
$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{D} = \varepsilon\overline{E}$$
$$\psi_e = \int \overline{D} \bullet d\overline{S}$$
$$Q_{enc} = \oint_S \overline{D} \bullet d\overline{S}$$
$$\rho_v = \nabla \bullet \overline{D}$$
$$V_{AB} = -\int_A^B \overline{E} \bullet d\overline{\ell} = \frac{W}{Q}$$
$$V = \frac{Q}{4\pi\varepsilon r}$$
$$V = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon r}$$
$$\oint \overline{E} \bullet d\overline{\ell} = 0$$
$$\nabla \times \overline{E} = 0$$
$$\overline{E} = -\nabla V$$
$$\nabla^2 V = 0$$
$$R = \frac{\ell}{\sigma S}$$
$$I = \int \overline{J} \bullet dS$$

$$d\overline{H} = \frac{Id\overline{\ell} \times \overline{R}}{4\pi R^3}$$
$$Id\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J} dv$$
$$\oint \overline{H} \bullet d\overline{\ell} = I_{enc} = \int \overline{J}_s dS$$
$$\nabla \times \overline{H} = \overline{J}$$
$$\psi_m = \int_s \overline{B} \bullet d\overline{S}$$
$$\psi_m = \oint \overline{B} \bullet d\overline{S} = 0$$
$$\psi_m = \oint \overline{A} \bullet d\overline{\ell}$$
$$\nabla \bullet \overline{B} = 0$$
$$\overline{B} = \mu\overline{H}$$
$$\overline{B} = \nabla \times \overline{A}$$
$$\overline{A} = \int \frac{\mu_0 Id\overline{\ell}}{4\pi R}$$
$$\nabla^2 \overline{A} = -\mu_0 \overline{J}$$
$$\overline{F} = Q(\overline{E} + \overline{u} \times \overline{B}) = m\frac{d\overline{u}}{dt}$$
$$d\overline{F} = Id\overline{\ell} \times \overline{B}$$
$$\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$$
$$\overline{m} = IS\hat{a}_n$$
$$V_{emf} = -\frac{\partial\psi}{\partial t}$$
$$V_{emf} = -\int \frac{\partial\overline{B}}{\partial t} \bullet d\overline{S}$$
$$V_{emf} = \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$$
$$I_d = \int J_d . d\overline{S}, J_d = \frac{\partial D}{\partial t}$$
$$\gamma = \alpha + j\beta$$
$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$
$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\overline{F}_1 = \frac{\mu I_1 I_2}{4\pi}\oint_{L1}\oint_{L2}\frac{d\overline{\ell}_1 \times (d\overline{\ell}_2 \times \hat{a}_{R_{21}})}{R_{21}^{\ 2}}$$
$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$
$$tan\,2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$
$$tan\,\theta = \frac{\sigma}{\omega\varepsilon} = \frac{\overline{J}_s}{\overline{J}_{ds}}$$
$$\delta = \frac{1}{\alpha}$$
$$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{Fm}^{-1}$$
$$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$$

$$\int \frac{dx}{(x^2 + c^2)^{3/2}} = \frac{x}{c^2 (x^2 + c^2)^{1/2}}$$
$$\int \frac{xdx}{(x^2 + c^2)^{3/2}} = \frac{-1}{(x^2 + c^2)^{1/2}}$$
$$\int \frac{dx}{(x^2 \pm c^2)^{1/2}} = ln\left(x + \sqrt{x^2 \pm c^2}\right)$$
$$\int \frac{dx}{(x^2 + c^2)} = \frac{1}{c}tan^{-1}\left(\frac{x}{c}\right)$$
$$\int \frac{xdx}{(x^2 + c^2)} = \frac{1}{2}ln(x^2 + c^2)$$
$$\int \frac{xdx}{(x^2 + c^2)^{1/2}} = \sqrt{x^2 + c^2}$$