UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	SIGNALS & SYSTEMS
COURSE CODE	:	BEB 20203
PROGRAMME	:	BEJ / BEV
EXAMINATION DATE	:	JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **SECTION A** AND **THREE (3)** QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **FOURTEEN (14)** PAGES

SECTION A: ANSWER ALL QUESTIONS

Q1 (a) Any time-varying physical phenomenon that is intended to convey information is a signal.

(i) Based on your understanding, define the meaning of Continuous Time Signal (CTS).

(2 marks)

(ii) Suggest a method to convert a Continuous Time Signal into a Discrete Time Signal.

(2 marks)

(b) A continuous-time signal is given as $x(t) = e^{-2t}\, u(t)$, where $u(t)$ is a unit step function.

(i) Sketch the signal $x(t)$.

(2 marks)

(ii) Determine and sketch the even and odd parts of $x(t)$.

(6 marks)

(iii) Calculate the energy and power of $x(t)$.

(6 marks)

(iv) Identify whether the signal $x(t)$ is a power or energy signal.

(2 marks)

Q2 (a) The block diagram representation of an electrical system is shown in **Figure Q2(a)**, where $x(t)$ is the input signal, $h(t)$ is the impulse response of the system and $y(t)$ is the output signal. The relationship between $y(t)$ and $x(t)$ is given as

$$y(t) = \log[2x(t-1)]$$

Classify the system whether it is memoryless, causal, linear, time-invariant and stable. You must show and explain the steps used in making your decision.

(5 marks)

(b) The impulse response, $h(t)$ of a linear time-invariant (LTI) system is given as,

$$h(t) = u(t) - u(t-2),$$

where $u(t)$ is a unit-step function. The input $x(t)$ is given as,

$$x(t) = \begin{cases} t+1 & : -1 \le t < 0 \\ -t+1 & : 0 \le t \le 1 \\ 0 & : \text{elsewhere} \end{cases}$$

Determine and sketch the output responses $y(t)$ using the graphical solution of the convolution integral.

(15 marks)

SECTION B: ANSWER THREE (3) QUESTIONS ONLY

Q3 A rectangular pulse train $x(t)$ as shown in **Figure Q3** is given by

$$x(t) = \begin{cases} 1, & -2\times10^{-3} \le t \le 0 \\ 0, & 0 < t \le 2\times10^{-3} \end{cases}$$

with a period $T = 4\times10^{-3}$ s

(a) Determine the exponential Fourier Series representation of $x(t)$.

(8 marks)

(b) Sketch the amplitude spectrum for $0 \le n \le 6$, where n is the coefficient index.

(4 marks)

(c) The signal $x(t)$ in **Q3(a)** is passed through an ideal low pass filter that has a cut off frequency of 1 kHz. Let $y(t)$ be the output of the low pass filter and the gain of the filter is unity.

(i) Calculate the total power of the input signal $x(t)$.

(3 marks)

(ii) Analyse the differences between the output signal $y(t)$ and input signal $x(t)$ by comparing the frequency components and total power of both signals.

(5 marks)

Q4 (a) An exponential signal is given by $x(t) = \begin{cases} e^{-0.5t} & \text{for } t \geq 0 \\ e^{0.5t} & \text{for } t < 0 \end{cases}$

(i) Sketch the exponential signal *x(t)*.

(1 mark)

(ii) Derive the Fourier transform of *x(t)*.

(5 marks)

(iii) Plot the magnitude spectrum and phase spectrum of *x(t)*.

(6 marks)

(b) Consider a continuous-time linear time invariant (LTI) system $\frac{dy(t)}{dt} + 2y(t) = x(t)$. Use Fourier transform to determine,

(i) the impulse response $h(t)$ of the system.

(2 marks)

(ii) the output $y(t)$ for $x(t) = e^{-t}u(t)$.

(6 marks)

Q5 (a) A time-shifted signal $x(t)$ is given in **Figure Q5(a).**

(i) Write the expression for the signal *x(t)*.

(1 mark)

(ii) Find the bilateral Laplace transform of the signal and specify the region of convergence.

(6 marks)

(b) Using the linearity properties of Laplace transform, prove that the unilateral Laplace transform of $x(t) = \cos(bt)$ is $X(s) = \frac{s}{s^2 + b^2}$. (Hint: Use Euler's relation.)

(3 marks)

(c) A system H_1 is made of a simple RL circuit as shown in **Figure Q5(c)** where R = 4 Ω and L = 0.5 H. The voltage input to the system is $v(t)$ and the output current is $i(t)$.

(i) Using Kirchoff's voltage law, write down the first order differential equation for the circuit.

(2 marks)

(ii) Show that the output voltage to input voltage transfer function, $\frac{V_O(s)}{V_i(s)}$ for the RL circuit is

$$H_1 = \frac{s}{s+8}.$$

(5 marks)

(iii) The system is cascaded with another system H_2 where its transfer function is $H_2 = \frac{1}{2s+1}$, thus forming an interconnected system. Draw the block diagram of the interconnected system and determine the overall transfer function of the system.

(3 marks)

Q6 (a) Unit impulse $\delta(t)$ is an imaginary signals often used in the study of signals and systems.

(i) Describe the two characteristics of unit impulse.

(2 marks)

(ii) Using an appropriate figure, describe how a continuous time signal can be sampled using the unit impulse's sampling property.

(3 marks)

(b) Linear time invariant (LTI) system is an important class of system.

(i) Describe the two conditions needed for a system to be linear.

(2 marks)

(ii) Explain the meaning of 'impulse response'. How does an impulse response relates the input and the output of a LTI system?

(3marks)

(c) The exponential Fourier series expansion of signal $x(t)$ is given by

$$x(t) = \frac{1}{2} + \sum_{n=-\infty}^{+\infty} \frac{2}{\pi n} sin\left(\frac{n\pi}{2}\right) e^{jn\pi t} \quad , \; n \neq 0$$

(i) Determine the period T of the signal $x(t)$.

(2 marks)

(ii) Show that the magnitude of the Fourier series coefficient is an even function.

(4 marks)

(iii) Discuss what would happen to signal $x(t)$ in time and frequency domain if the period T increases to infinity.

(4 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2013/2014
COURSE NAME: SIGNALS & SYSTEMS

PROGRAMME: BEJ/BEV
COURSE CODE: BEB 20203

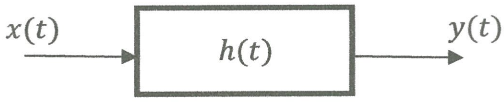

FIGURE Q2(a)

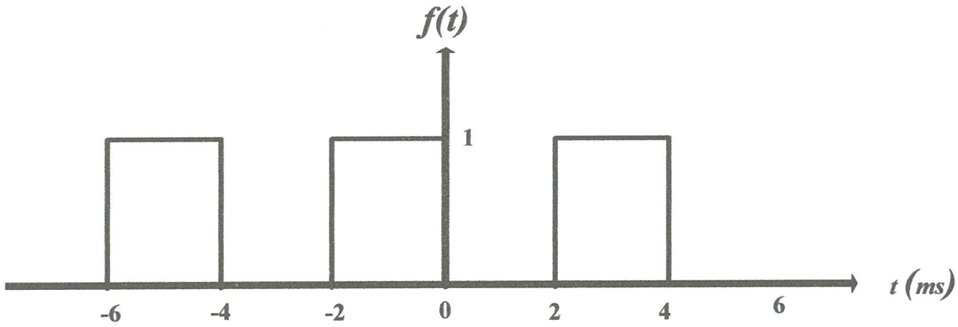

FIGURE Q3

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2013/2014
COURSE NAME: SIGNALS & SYSTEMS

PROGRAMME: BEJ/BEV
COURSE CODE: BEB 20203

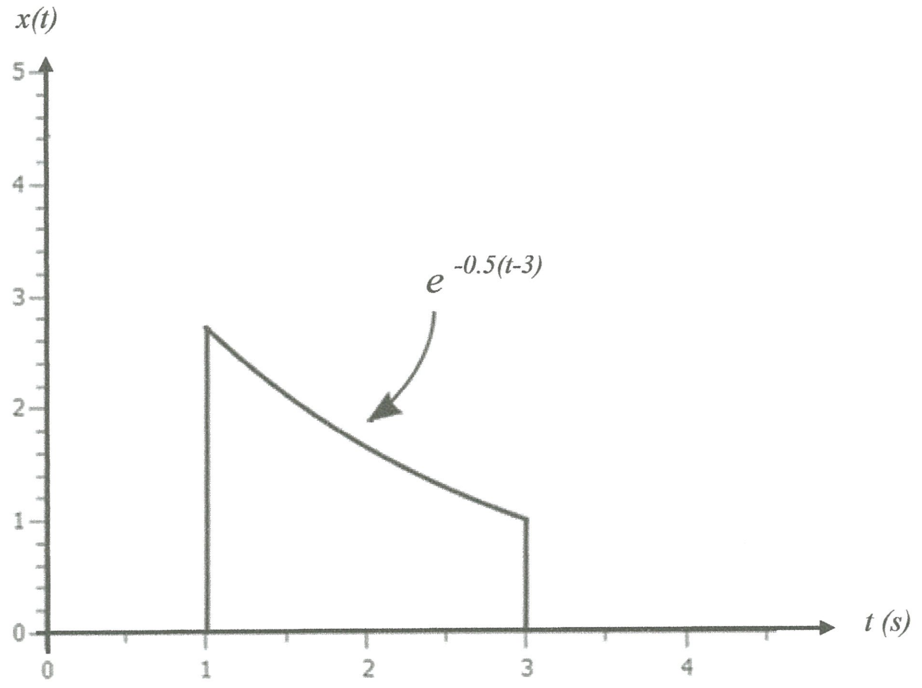

FIGURE Q5(a)

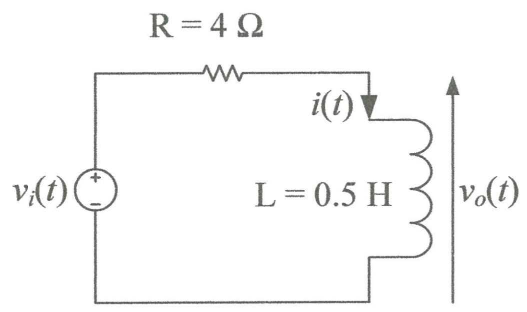

FIGURE Q5(c)

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2013/2014
COURSE NAME: SIGNALS & SYSTEMS

PROGRAMME: BEJ/BEV
COURSE CODE: BEB 20203

INDEFINITE INTEGRALS

$$\int \cos at\, dt = \frac{1}{a} \sin at$$

$$\int \sin at\, dt = -\frac{1}{a} \cos at$$

$$\int t \cos at\, dt = \frac{1}{a^2} \cos at + \frac{1}{a} t \sin at$$

$$\int t \sin at\, dt = \frac{1}{a^2} \sin at - \frac{1}{a} t \cos at$$

EULER'S IDENTITY

$$e^{\pm j\pi/2} = \pm j \quad ; \qquad A\angle \pm\theta = Ae^{\pm j\theta}$$

$$e^{\pm jk\pi} = \cos(k\pi) \quad ; \qquad e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \quad ; \qquad \sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2013/2014
COURSE NAME: SIGNALS & SYSTEMS

PROGRAMME: BEJ/BEV
COURSE CODE: BEB 20203

TABLE 1: Trigonometric Identities

Trigonometric identities	
$\sin \propto \; = \; \cos \; (\propto - \frac{\pi}{2})$	$\cos\alpha = \sin(\alpha + \frac{\pi}{2})$
$\sin(\propto \pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin 2\alpha \; = 2\sin\alpha\cos\alpha$	$\cos\; 2\alpha = 2\, cos^2\; \alpha - 1$
$\cos 2\alpha \; = 1 - 2\, sin^2\; \alpha$	$\cos 2\alpha = \; cos^2\,\alpha - \; sin^2\alpha$

TABLE 2: Values of cosine, sine and exponential functions for integral multiple of π

Function	**Value**
$\cos 2n\pi$	1
$\sin\; 2n\pi$	0
$\cos\; n\pi$	$(-1)^n$
$\sin\; n\pi$	0
$\cos\dfrac{n\pi}{2}$	$\begin{cases}(-1)^{n/2}, & n = even \\ 0, & n = odd\end{cases}$
$\sin\dfrac{n\pi}{2}$	$\begin{cases}(-1)^{(n-1)/2}, & n = odd \\ 0, & n = even\end{cases}$
$e^{j2n\pi}$	1
$e^{jn\pi}$	$(-1)^n$
$e^{jn\pi/2}$	$\begin{cases}(-1)^{n/2}, & n = even \\ j(-1)^{(n-1)/2}, & n = odd\end{cases}$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2013/2014
COURSE NAME: SIGNALS & SYSTEMS

PROGRAMME: BEJ/BEV
COURSE CODE: BEB 20203

TABLE 3: Fourier Transform Pairs

Time domain, *f(t)*	Frequency domain, $F(\omega)$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$u(t+\tau)-u(t-\tau)$	$2\frac{\sin\omega\tau}{\omega}$
$\lvert t\rvert$	$\frac{-2}{\omega^2}$
$\operatorname{sgn}(t)$	$\frac{2}{j\omega}$
$e^{-\alpha t}u(t)$	$\frac{1}{\alpha+j\omega}$
$e^{\alpha t}u(-t)$	$\frac{1}{\alpha-j\omega}$
$t^n e^{-\alpha t}u(t)$	$\frac{n!}{(\alpha+j\omega)^{n+1}}$
$e^{-\alpha\lvert t\rvert}$	$\frac{2\alpha}{\alpha^2+\omega^2}$
$e^{j\omega_o t}$	$2\pi\delta(\omega-\omega_o)$
$\sin\omega_o t$	$j\pi[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]$
$\cos\omega_o t$	$\pi[\delta(\omega+\omega_o)+\delta(\omega-\omega_o)]$
$e^{-\alpha t}\sin\omega_o t\, u(t)$	$\frac{\omega_o}{(\alpha+j\omega)^2+\omega_o^2}$
$e^{-\alpha t}\cos\omega_o t\, u(t)$	$\frac{\alpha+j\omega}{(\alpha+j\omega)^2+\omega_o^2}$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2013/2014 PROGRAMME: BEJ/BEV
COURSE NAME: SIGNALS & SYSTEMS COURSE CODE: BEB 20203

TABLE 4: Fourier Transform Properties

Property	Time domain, *f(t)*	Frequency domain, $F(\omega)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a \rvert} F(\frac{\omega}{a})$
Time Shift	$f(t-a)$	$e^{-j\omega a} F(\omega)$
Frequency Shift	$e^{-j\omega_o t} f(t)$	$F(\omega - \omega_o)$
Modulation	$\cos(\omega_0 t)\, f(t)$	$\frac{1}{2}[F(\omega + \omega_o) + F(\omega - \omega_o)]$
Time Differentiation	$\frac{df}{dt}$ $\frac{d^n f}{dt^n}$	$j\omega F(\omega)$ $(j\omega)^n F(\omega)$
Time Integration	$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(\omega)}{j\omega} + \pi\, F(0)\delta(\omega)$
Frequency Differentiation	$t^n f(t)$	$j^n \frac{d^n}{d\omega^n} F(\omega)$
Reversal	$f(-t)$	$F(-\omega)$ or $F^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Convolution in *t*	$f_1(t) * f_2(t)$	$F_1(\omega) F_2(\omega)$
Convolution in ω	$f_1(t) f_2(t)$	$F_1(\omega) * F_2(\omega)$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2013/2014 PROGRAMME: BEJ/BEV
COURSE NAME: SIGNALS & SYSTEMS COURSE CODE: BEB 20203

TABLE 5: Laplace Transform

Signal	**Waveform** $f(t)$	**Transform** $F(s)$
Impulse	$\delta(t)$	1
Step	$u(t)$	$\frac{1}{s}$
Ramp	$t\,u(t)$	$\frac{1}{s^2}$
Power of t	$\frac{t^{n-1}}{(n-1)!}u(t) \quad n=1,2,3\ldots$	$\frac{1}{s^n}$
Exponential	$e^{-\alpha t}\,u(t)$	$\frac{1}{s+\alpha}$
Damped ramp	$t\,e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$
Damped Power of t	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t) \quad n=1,2,3\ldots$	$\frac{1}{(s+\alpha)^n}$
Sine	$\sin\beta t\ \ u(t)$	$\frac{\beta}{s^2+\beta^2}$
Cosine	$\cos\beta t\ \ u(t)$	$\frac{s}{s^2+\beta^2}$
Sinusoid	$\sqrt{a^2+b^2}\cos\left(\beta t-\tan^{-1}\frac{b}{a}\right)u(t)$	$\frac{a\,s+b\,\beta}{s^2+\beta^2}$
Damped Sine	$e^{-\alpha t}\sin\beta t\ u(t)$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$
Damped Cosine	$e^{-\alpha t}\cos\beta t\ u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$

FINAL EXAMINATION

SEMESTER/SESSION: SEMESTER I/2013/2014
PROGRAMME: BEJ/BEV
COURSE NAME: SIGNALS & SYSTEMS
COURSE CODE: BEB 20203

TABLE 6: Laplace Transform Properties

Property	Signal	Laplace Transform	ROC
	$x(t)$ $x_1(t)$ $x_2(t)$	$X(s)$ $X_1(s)$ $X_2(s)$	R R_1 R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathcal{R}e\{s\} > 0\}$

Initial- and Final-Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s \to \infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \longrightarrow \infty$, then

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$$