UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME : FINITE ELEMENT METHOD
COURSE CODE : BDA 40303 / BDA 4033
PROGRAMME : BDD
EXAMINATION DATE : DECEMBER 2013/JANUARY 2014
DURATION : 3 HOURS
INSTRUCTION : ANSWER **FOUR (4)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **FOURTEEN (14)** PAGES

Q1 A 10 m beam is fixed at node 1 and has an elastic support at node 3. The beam carries a 50 kN/m uniformly distributed load from node 1 to node 2 and a 25 kN concentrated load at node 3. Given the Young's Modulus, E = 200 GPa, moment of inertia, $I = 2 \times 10^{-4}\ m^4$ and the spring stiffness, k = 200 kN/m.

(a) Illustrate the finite element model of this beam. Please label the nodes, elements, constraints and loads clearly.

(4 marks)

(b) Assemble the global matrices for the stiffness matrix and load vector. Use Direct Elimination method to accommodate the constraints into the static equation.

(10 marks)

(c) Determine the nodal displacements and rotations.

(7 marks)

(d) Determine the reaction forces for the beam shown in **FIGURE Q1.**

(4 marks)

Q2 A long bar of trapezoid cross section, having thermal conductivity of 1.5 W/m°C, is subjected to the boundary conditions shown in **FIGURE Q2**. Top side of the bar is maintained at a uniform temperature of 200°C; left side is insulated, and the inclined and bottom sides are subjected to a convection process with $T_\infty = 50\,°C$ and $h = 40\,W/m^2°C$. By referring to the connectivity table given in **FIGURE Q2**, and letting node 1 as origin (0,0);

(a) Define the element types used in the analysis.

(3 marks)

(b) Calculate the thermal conductivity, convection and thermal loading matrices for each element.

(12 marks)

(c) Determine the temperature distribution in the bar.

(10 Marks)

Q3 Liquid with dynamic viscosity of $\mu = 0.5\ \text{N} \bullet \text{s/m}^2$ and density of $\rho = 1000\ \text{kg/m}^3$ flows through the piping network shown in the accompanying **FIGURE Q3**. Determine the pressure distribution in the system if the flow rate at node 1 is $25 \times 10^{-4} \text{m}^3/\text{s}$. For the given conditions, the flow is laminar throughout the system. We assumed that the pressure at node 1 is 40kPa and at node 4 is -4kPa.

(a) Construct a table to discretize the given piping network of **FIGURE Q3** into 4 elements and 4 nodes, as numbered.

(4 marks)

(b) Determine the elemental flow resistance $[R]^{(e)}$ for each element.

(8 marks)

(c) Assemble the global matrices for resistance matrix $[R]^{(G)}$, pressure force matrix $\{F_P\}^{(G)}$ and the unknown nodal pressure matrix $\{P\}^{(G)}$.

(5 marks)

(d) Estimate the nodal pressure distribution, P at each node of the network according to the global finite element equations:

$$[R]^{(G)}\{P\}^{(G)} = \{F_P\}^{(G)}$$

(3 marks)

(e) By roughly estimated pressure of part (d) above, determine the flow rate Q in each node.

(5 marks)

Q4 A fluid flows in a narrow channel measured 5 m long, 10 cm high and 50 cm wide. The pressure drop along the channel is measured to be 100 Pa/m. The density of the fluid is constant at 1000 kg/m^3 while the fluid viscosity varies with the depth due to the temperature difference between the fluid and the ambient temperature as shown in **TABLE Q4.**

TABLE Q4: Viscosity of the fluid as a function of channel depth.

Level (cm)	**Viscosity (kg/ms)**
0	0.01
2.5	0.005
5	0.0025
7.5	0.005
10	0.01

(a) Draw the finite element model to represent the fluid flows problem.
(4 marks)

(b) Prepare the table to calculate the elemental flow resistance and forcing matrix.
(4 marks)

(c) Determine the velocity of the fluid at level 2.5 cm, 5 cm and 7.5 cm.
(8 marks)

(d) Sketch the velocity profile of the flow.
(4 marks)

(e) Calculate the mass flow rate of the fluid through the channel.
(5 marks)

Q5 A steel plate with a square hole in the middle of the plate is subjected to a tensile stress as shown in **FIGURE Q5 (a)**. The modulus elasticity of the plate is 210 GPa with Poisson's ratio 0.25. The thickness of the plate is 5 mm.

(a) For this particular problem, are you going to analyse using plane stress or plane strain problem? Explain briefly your selection and the conditions for the selected problem.

(4 marks)

(b) For a faster solution, you are requested to analyse the displacement by using a quarter model only as in **FIGURE Q5 (b)**. You need to draw the quarter model complete with the meshing. Your meshing lines must be clear and visible. The meshing must contain triangular 3 nodes and quadrilateral 4 nodes.

(5 marks)

(c) Explain the boundary conditions that you have to apply to nodes in line AB and CD.

(2 marks)

(a) Based on the data, specify precisely the point forces at nodes 1, 2, 3, 4 and 5 for two dimensional elements, if points 1,2,3,4 and 5 are equally spaced.

(6 marks)

(b) Considering your meshing, calculate the stiffness matrix of one of any triangular CST elements in your meshing. You have to clearly mention the coordinates of the nodes.

(8 marks)

- END OF QUESTION -

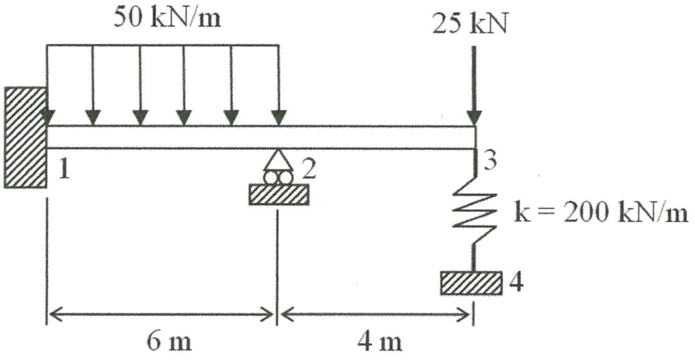

FIGURE Q1

Element	Node-i	Node-j	Node-m	Node-n
1	1	2	3	4
2	2	5	3	

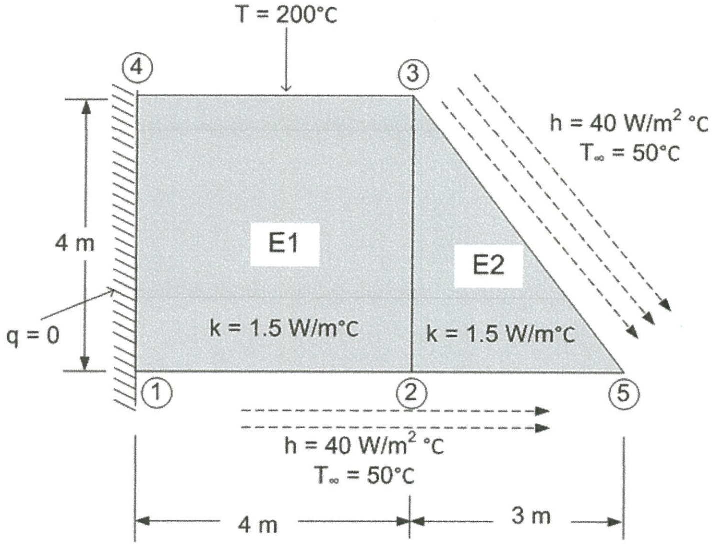

FIGURE Q2

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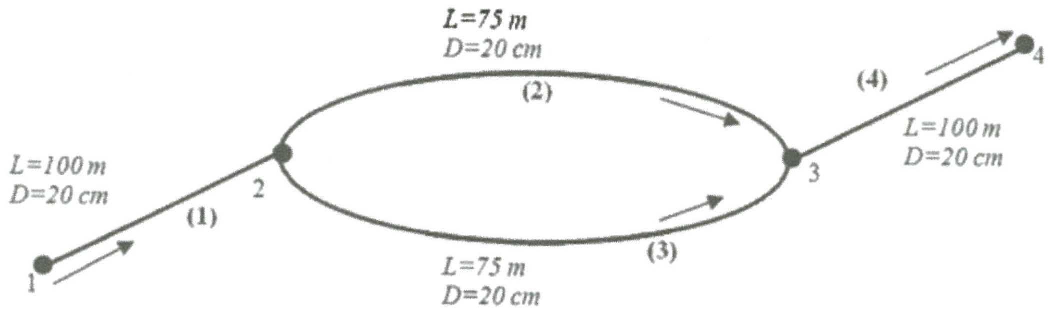

FIGURE Q3

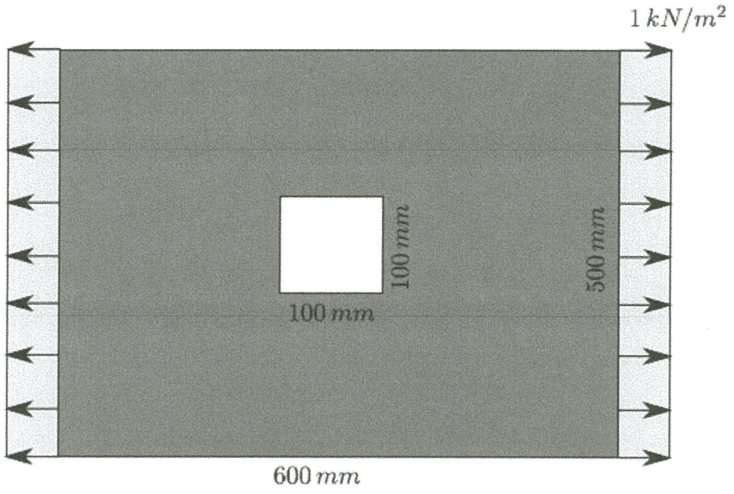

FIGURE Q5 (a)

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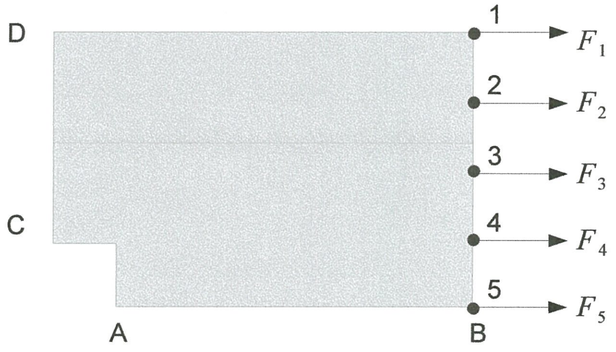

FIGURE Q5 (b)

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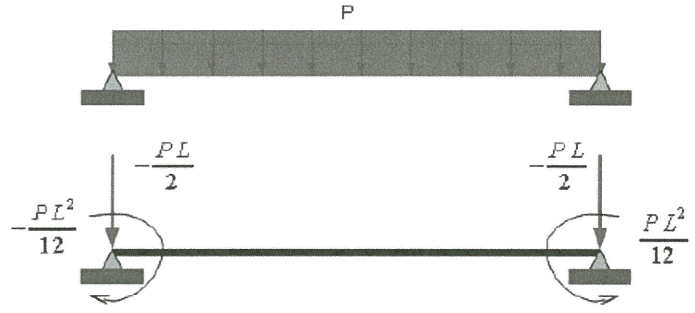
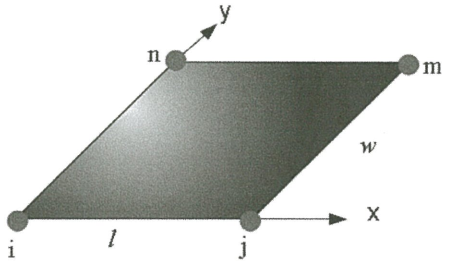
USEFUL FORMULA

BEAM ELEMENT:

$$[K^e] = \frac{E^e I^e}{(L^e)^3} \begin{array}{c} \\ \left[\begin{array}{cccc} v_i & \theta_i & v_j & \theta_j \\ 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{array} \right] \end{array} \begin{array}{c} \\ v_i \\ \theta_i \\ v_j \\ \theta_j \end{array}$$



BILINEAR RECTANGULAR HEAT TRANSFER:

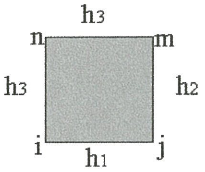


$$[K^e] = \frac{k_x w}{6l} \begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{k_y l}{6w} \begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

Additional conductance matrix due to convection

$$[K^e]=\frac{h_3 L_{mn}}{6}\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&2&1\\0&0&1&2\end{bmatrix}$$

$$[K^e]=\frac{h_4 L_{ni}}{6}\begin{bmatrix}2&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&2\end{bmatrix}$$



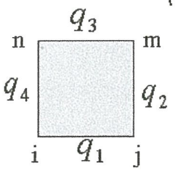
$$[K^e]=\frac{h_2 L_{jm}}{6}\begin{bmatrix}0&0&0&0\\0&2&1&0\\0&1&2&0\\0&0&0&0\end{bmatrix}$$

$$[K^e]=\frac{h_1 L_{ij}}{6}\begin{bmatrix}2&1&0&0\\1&2&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

Thermal load heat flux

$$\{F^e\}=\frac{q_3 l_{mn}}{2}\begin{Bmatrix}0\\0\\1\\1\end{Bmatrix}$$

$$\{F^e\}=\frac{q_4 l_{ni}}{2}\begin{Bmatrix}1\\0\\0\\1\end{Bmatrix}$$



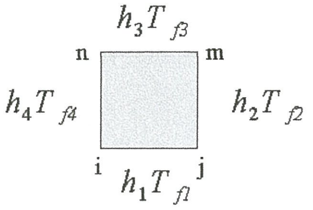
$$\{F^e\}=\frac{q_2 l_{jm}}{2}\begin{Bmatrix}0\\1\\1\\0\end{Bmatrix}$$

$$\{F^e\}=\frac{q_1 l_{ij}}{2}\begin{Bmatrix}1\\1\\0\\0\end{Bmatrix}$$

Thermal load due to heat loss

$$\{F^e\} = \frac{h_3 T_{f3} L_{mn}}{2} \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{F^e\} = \frac{h_4 T_{f4} L_{ni}}{2} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$$



$$\{F^e\} = \frac{h_2 T_{f2} L_{jm}}{2} \begin{Bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{Bmatrix}$$

$$\{F^e\} = \frac{h_1 T_{f1} L_{ij}}{2} \begin{Bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$

TRIANGULAR HEAT TRANSFER:

$$[K^e] = \frac{k_X}{4A} \begin{bmatrix} y_{23}^2 & y_{31} y_{23} & y_{12} y_{23} \\ y_{23} y_{31} & y_{31}^2 & y_{12} y_{31} \\ y_{23} y_{12} & y_{31} y_{12} & y_{12}^2 \end{bmatrix} + \frac{k_Y}{4A} \begin{bmatrix} x_{32}^2 & x_{13} x_{32} & x_{21} x_{32} \\ x_{32} x_{13} & x_{13}^2 & x_{21} x_{13} \\ x_{32} x_{21} & x_{13} x_{21} & x_{21}^2 \end{bmatrix}$$

$$y_{ij} = y_i - y_j \qquad x_{ij} = x_i - x_j$$

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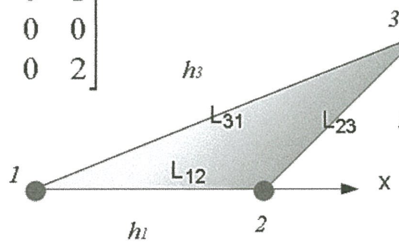
Additional conductance matrix due to convection

$$[K^e]=\frac{h_3 L_{31}}{6}\begin{bmatrix}2 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 2\end{bmatrix}$$

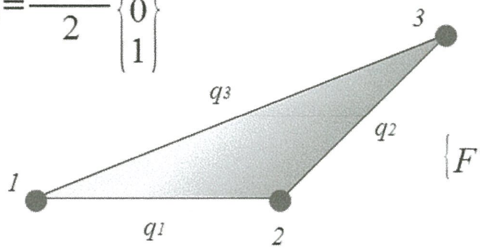


$$[K^e]=\frac{h_2 L_{23}}{6}\begin{bmatrix}0 & 0 & 0\\ 0 & 2 & 1\\ 0 & 1 & 2\end{bmatrix}$$

$$[K^e]=\frac{h_1 L_{12}}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 2 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Thermal load heat flux

$$\{F^e\}=\frac{q_3 L_{31}}{2}\begin{Bmatrix}1\\ 0\\ 1\end{Bmatrix}$$

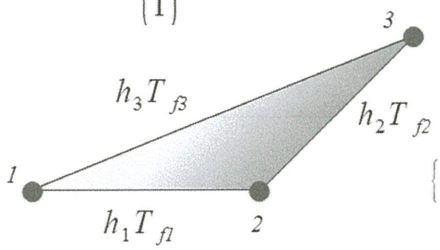


$$\{F^e\}=\frac{q_2 L_{23}}{2}\begin{Bmatrix}0\\ 1\\ 1\end{Bmatrix}$$

$$\{F^e\}=\frac{q_1 L_{12}}{2}\begin{Bmatrix}1\\ 1\\ 0\end{Bmatrix}$$

Thermal load due to heat loss

$$\{F^e\} = \frac{h_3 T_{f3} L_{31}}{2} \begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}$$



$$\{F^e\} = \frac{h_2 T_{f2} L_{23}}{2} \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{F^e\} = \frac{h_1 T_{f1} L_{12}}{2} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

PIPE FLOW NETWORK: FLOW RESISTANCE MATRIX

$$[R] = \begin{bmatrix} C & -C \\ -C & C \end{bmatrix}$$

$$C = \frac{\pi D^4}{128\, L\, \mu}$$

FLUID MECHANICS PROBLEM

$$[k]\{u\} = \{f\}$$

$$[k^e] = \frac{\mu}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\{f^e\} = \frac{\frac{-dp}{dx} L}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

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CST ELEMENT
Plane Stress:

$$[E]=\frac{E}{(1-\nu^2)}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}$$

Plane Strain:

$$[E]=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}$$

$$[B]=\frac{1}{2A}\begin{bmatrix}y_{23} & 0 & y_{31} & 0 & y_{12} & 0\\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21}\\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12}\end{bmatrix} \qquad A=\frac{1}{2}\begin{vmatrix}1 & x_1 & y_1\\ 1 & x_2 & y_2\\ 1 & x_3 & y_3\end{vmatrix}$$

$$x_{ij}=x_i-x_j \qquad y_{ij}=y_i-y_j$$