UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2011/2012

COURSE NAME	: ENGINEERING ELECTROMAGNETICS
COURSE CODE	: BEF 22903
PROGRAMME	: BEF
EXAMINATION DATE	: JUNE 2012
DURATION	: 3 HOURS
INSTRUCTION	: ANSWER **FIVE (5)** QUESTIONS ONLY

THIS PAPER CONSISTS OF **TEN** (10) PAGES

Q1 (a) State TWO (2) fundamental laws governing the electrostatic field. Then, give TWO (2) examples of electrostatic application.

(4 marks)

(b) A simple experiment has been setup using rulers and glass rods. Explain how the experiment can prove the characteristic of electric charges. Use diagrams to support your answers.

(8 marks)

(c) A spherical shell centered at the origin extend up to 2 m radius. If the volume charge density is $\rho_v = 2r^2 \sin\theta \cos\phi^2$ C/m^3, find the total charge contain in the shell.

(8 marks)

Q2 (a) Using your own words, explain how the behavior of electric field can be visualized using field lines between two point charges of an equal magnitude but opposite signs. Use diagrams to support your answers.

(6 marks)

(b) Two infinite uniform line charges of 5 nC/m lie along the positive and negative x and y axes in free space. Another finite line charge 10 nC/m lies along z axes as shown in Figure Q2 (b). Determine the total force, $\vec{F}$ exerted on 2 nC charge at P(2, - 4, - 1).

(14 marks)

Q3 (a) Describe the meaning of Gauss's Law. Then, state the formula that relates Gauss's Law to Divergence Theorem.

(5 marks)

(b) Cylindrical surfaces at $r \leq 2m$, $2m \leq r \leq 4m$ and $r \geq 4m$ carry uniform charge densities of 20nC/m^2, -4nC/m^2 and 0 C/m respectively. By applying Gauss's Law, find $\vec{D}$ everywhere.

(15 marks)

Q4 (a) Define the work done in moving a point charge from point A to point B in an electric field.

(4 marks)

(b) Find the work done in carrying a 6C charge from M(1, 8, 5) to N(2, 18, 6) along path $y = 3x^2 + z$ and $z = x + 4$. Given that the field vector is $\vec{E} = -8xy\,\hat{x} - 4x^2\,\hat{y} + \hat{z}$ V/m.

(16 marks)

Q5 (a) Distinguish between linear, isotropic and homogeneous dielectric materials. Then, explain the importance of boundary conditions in evaluating the electric field.

(8 marks)

(b) Given that two homogeneous isotropic dielectric media with dielectric constants $\varepsilon_{r1} = 3$ and $\varepsilon_{r2} = 2$ are separated by x-y plane. At a common point, $\vec{E}_1 = \hat{x} - 5\hat{y} - 4\hat{z}$ V/m. Find $\vec{E}_2$, $\vec{D}_2$ and the angle make with the normal.

(12 marks)

Q6 (a) Magnetic field is characterized by $\vec{B}$ and $\vec{H}$ vectors. Briefly explain the meaning of these two vectors and how are they related.

(6 marks)

(b) Consider a triangular loop carrying an 8A current is placed on y = 0 plane as shown in Figure Q6(b). Calculate the magnetic field intensity $\vec{H}$, at point P(2,3,0). Then, find the magnetic flux density $\vec{B}$.

(14 marks)

Q7 (a) A square loop of wire in the $x = 0$ plane carrying current 2 mA in the field of an infinite filament on the z-axis as shown in Figure Q7 (b). Calculate the total force on the loop.

(12 marks)

(b) All moving charged particles produce magnetic field and therefore forces are generated as well. Based on your understanding, explain the main characteristics of magnetic force.

(8 marks)

Q8 (a) Faraday's Law is a basic law of electromagnetism relating to the operating principles of transformers. Draw structure of a transformer and explain its operation.

(7 marks)

(b) Consider an inductor which is formed by winding, N = 20 turns of a thin conducting wire into a square loop centered at the origin and having 10 cm sides oriented parallel to the x-y plane. It is connected to a resistor, R as shown in Figure Q2 (b). In the presence of a magnetic field, $\vec{B} = B_0\, x^2 \cos 10^3 t\, \hat{z}$ and $B_0 = 100\ T$. Calculate;

(i) The magnetic flux, ϕ linking a single turn of the inductor,
(ii) The transformer emf, $V^{tr}{}_{emf}$,
(iii) The polarity of $V^{tr}{}_{emf}$ at $t = 0$,
(iv) The induced current in the circuit for R = 1 kΩ (assume the wire resistance to be negligibly small).

(13 marks)

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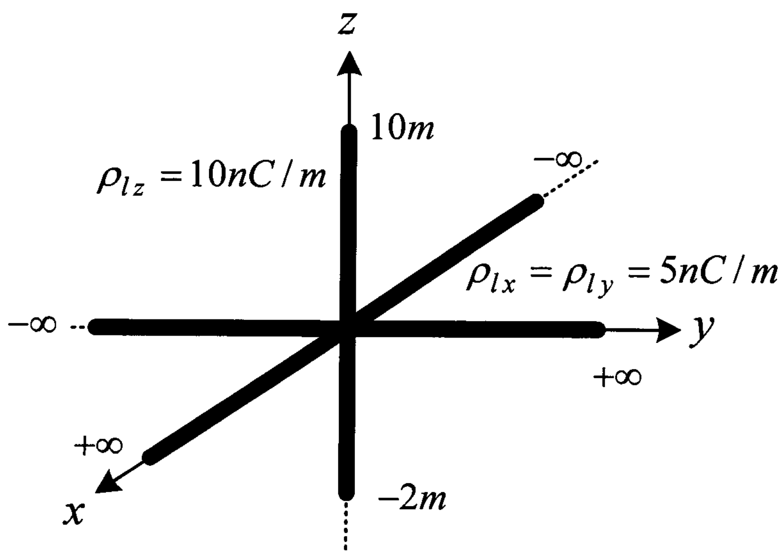

FIGURE Q2(b)

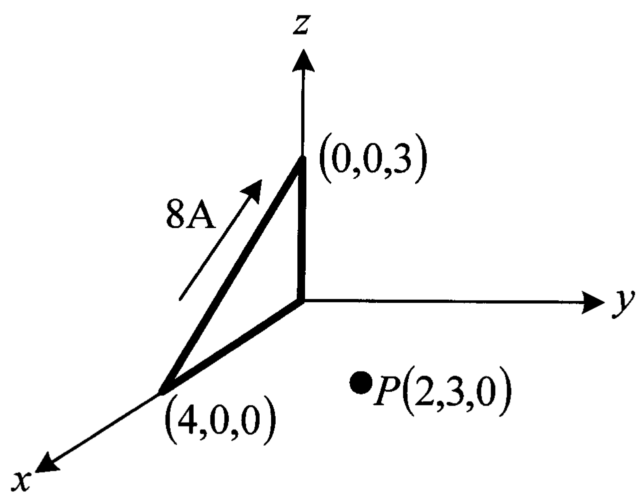

FIGURE Q6(b)

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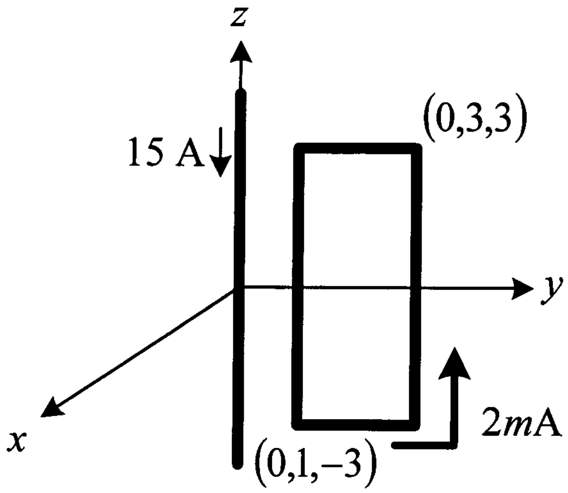

FIGURE Q7(b)

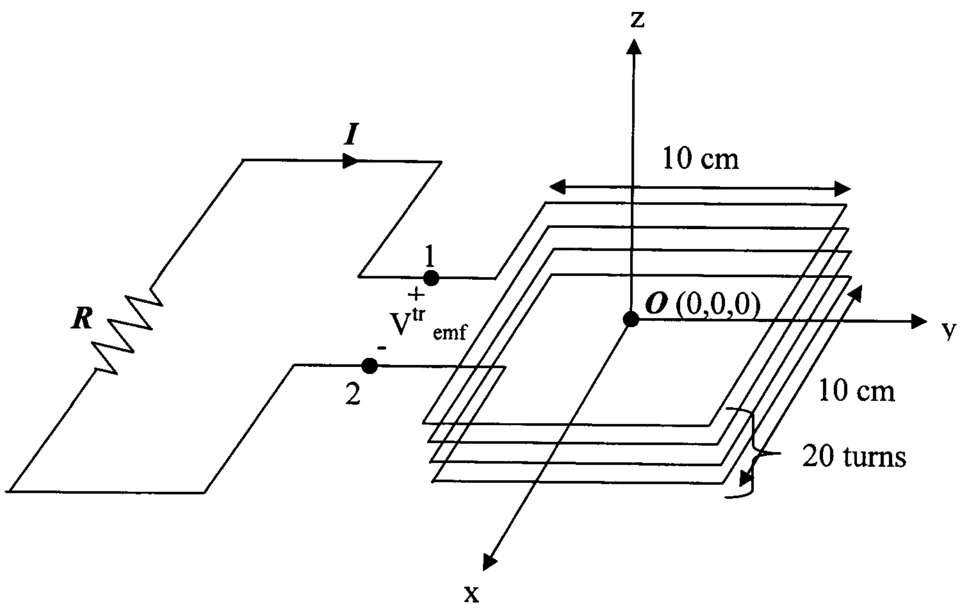

FIGURE Q8(b)

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Formula

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial (rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial (R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial (A_\theta \sin\theta)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\phi}} + \frac{1}{r}\left(\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial (\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial (RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial (RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

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	Cartesian	Cylindrical	Spherical
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\varphi}}$
Magnitude $\vec{A}$	$\sqrt{A_x^{\ 2} + A_y^{\ 2} + A_z^{\ 2}}$	$\sqrt{A_r^{\ 2} + A_\phi^{\ 2} + A_z^{\ 2}}$	$\sqrt{A_R^{\ 2} + A_\theta^{\ 2} + A_\phi^{\ 2}}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1, y_1, z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1, \phi_1, z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1, \theta_1, \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\varphi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\varphi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\vec{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\, d$
Differential surface, $\vec{ds}$	$\vec{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\vec{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\vec{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\vec{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\vec{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\varphi}}$ $\vec{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\vec{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\vec{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\vec{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume, $\vec{dv}$	$dx\,dy\,\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to Spherical	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to Cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

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$Q = \int \rho_\ell d\ell,$

$Q = \int \rho_s dS,$

$Q = \int \rho_v dv$

$\bar{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$

$\bar{E} = \frac{\bar{F}}{Q},$

$\bar{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\bar{E} = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\bar{E} = \int \frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\bar{E} = \int \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\bar{D} = \varepsilon\bar{E}$

$\psi_e = \int \bar{D} \bullet d\bar{S}$

$Q_{enc} = \oint_S \bar{D} \bullet d\bar{S}$

$\rho_v = \nabla \bullet \bar{D}$

$V_{AB} = -\int_A^B \bar{E} \bullet d\bar{\ell} = \frac{W}{Q}$

$V = \frac{Q}{4\pi\varepsilon r}$

$V = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon r}$

$\oint \bar{E} \bullet d\bar{\ell} = 0$

$\nabla \times \bar{E} = 0$

$\bar{E} = -\nabla V$

$\nabla^2 V = 0$

$R = \frac{\ell}{\sigma S}$

$I = \int \bar{J} \bullet dS$

$d\bar{H} = \frac{Id\bar{\ell} \times \bar{R}}{4\pi R^3}$

$Id\bar{\ell} \equiv \bar{J}_s dS \equiv \bar{J}dv$

$\oint \bar{H} \bullet d\bar{\ell} = I_{enc} = \int \bar{J}_s dS$

$\nabla \times \bar{H} = \bar{J}$

$\psi_m = \int_s \bar{B} \bullet d\bar{S}$

$\psi_m = \oint \bar{B} \bullet d\bar{S} = 0$

$\psi_m = \oint \bar{A} \bullet d\bar{\ell}$

$\nabla \bullet \bar{B} = 0$

$\bar{B} = \mu\bar{H}$

$\bar{B} = \nabla \times \bar{A}$

$\bar{A} = \int \frac{\mu_0 Id\bar{\ell}}{4\pi R}$

$\nabla^2 \bar{A} = -\mu_0 \bar{J}$

$\bar{F} = Q(\bar{E} + \bar{u} \times \bar{B}) = m\frac{d\bar{u}}{dt}$

$d\bar{F} = Id\bar{\ell} \times \bar{B}$

$\bar{T} = \bar{r} \times \bar{F} = \bar{m} \times \bar{B}$

$\bar{m} = IS\hat{a}_n$

$V_{emf} = -\frac{\partial\psi}{\partial t}$

$V_{emf} = -\int \frac{\partial\bar{B}}{\partial t} \bullet d\bar{S}$

$V_{emf} = \int (\bar{u} \times \bar{B}) \bullet d\bar{\ell}$

$I_d = \int J_d . d\bar{S}, J_d = \frac{\partial\bar{D}}{\partial t}$

$\gamma = \alpha + j\beta$

$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$

$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$

$\bar{F}_1 = \frac{\mu I_1 I_2}{4\pi}\oint_{L1}\oint_{L2}\frac{d\bar{\ell}_1 \times (d\bar{\ell}_2 \times \hat{a}_{R_{21}})}{R_{21}^2}$

$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$

$tan\,2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$

$tan\,\theta = \frac{\sigma}{\omega\varepsilon} = \frac{\bar{J}_s}{\bar{J}_{ds}}$

$\delta = \frac{1}{\alpha}$

$\varepsilon_0 = 8.854 \times 10^{-12}$ Fm^{-1}

$\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}

$\int \frac{dx}{(x^2 + c^2)^{3/2}} = \frac{x}{c^2(x^2 + c^2)^{1/2}}$

$\int \frac{xdx}{(x^2 + c^2)^{3/2}} = \frac{-1}{(x^2 + c^2)^{1/2}}$

$\int \frac{dx}{(x^2 \pm c^2)^{1/2}} = ln\left(x + \sqrt{x^2 \pm c^2}\right)$

$\int \frac{dx}{(x^2 + c^2)} = \frac{1}{c}tan^{-1}\left(\frac{x}{c}\right)$

$\int \frac{xdx}{(x^2 + c^2)} = \frac{1}{2}ln(x^2 + c^2)$

$\int \frac{xdx}{(x^2 + c^2)^{1/2}} = \sqrt{x^2 + c^2}$