UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2009/10

SUBJECT NAME	:	DIGITAL CONTROL
SUBJECT CODE	:	BER 4113/BEM 4713
COURSE	:	4BEE
EXAMINATION DATE	:	NOVEMBER 2009
DURATION	:	2 ½ HOURS
INSTRUCTION	:	ANSWER 4 (FOUR) QUESTIONS ONLY

THIS PAPER CONSISTS OF 10 PAGES

Q1 (a) A unity feedback closed loop system plant with the digital controller is shown in Figure Q1. Design a digital controller with an inter-sampling interval of 1 s to deadbeat control a plant having the transfer function

$$G_p(s) = \frac{1}{s(s+1)}.$$

(13 marks)

(b) Assuming that the input signal is a unit step, obtain and draw the output response of the system in Q1 (a).

(12 marks)

Q2 The open loop pulse transfer function of a digital closed-loop system with a unity feedback is given by

$$G(z) = \frac{K(z + 0.9355)}{(z - 0.2308)(z - 1)}$$

The sampling period is T = 0.1 s.

(a) Plot the root locus as the gain K is varied from 0 to ∞. (Please use the graph scale 5 cm : 1 unit)

(12 marks)

(b) Determine the critical value of gain K for stability.

(3 marks)

(c) What is value of the gain K will yield a damping ratio ζ of the closed-loop poles equal to 0.5.

(6 marks)

(d) With the gain K set to yield ζ = 0.5, state whether the sampling period T=0.1 second is suitable for this system. Justify your answer.

(4 marks)

Note that any point on the locus of constant ζ on the z plane is given by

$$Log\, e^{r} = \frac{-\zeta\theta}{\sqrt{(1-\zeta^2)}}$$

where r and θ are the magnitude and phase of the point respectively with reference to the origin.

Q3 (a) A system with the characteristic equation $P(z) = 0$ is given by

$$P(z) = a_0 z^n + a_1 z^{n-1} + ... + a_{n-1} z + a_n$$

where $a_0 > 0$.

A general Form of the Jury Stability Table is given in the Table Q3.

According to the Jury Stability Test, what are the criterions to be fulfilled in order to make the system stable?

(7 marks)

(b) A digital control system having a following characteristic equation:

$$P(z) = z^4 - 1.2z^3 + 0.07z^2 + 0.3z - 0.08 = 0$$

By using the Jury Stability Test, examine the stability of the system.

(18 marks)

Q4 (a) Consider the discrete-time control system shown in Figure Q4. Show that the Pulse Transfer Function of the system is given by

$$\frac{C(z)}{R(z)} = \frac{G_1G_2(z)}{1 + G_1G_2(z)}$$

(12.5 marks)

(b) If $G_1(s) = \dfrac{1 - e^{-Ts}}{s}$ and $G_2(s) = \dfrac{K}{s}$, obtain the output sequence c(kT) of the system when it is subjected to a unit-step input. Assume that the sampling period T is 1 sec.

(12.5 marks)

Q5 The state and output equations of a discrete-time control system are respectively given by

$$\underline{x}(k+1) = G\underline{x}(k) + H\underline{u}(k)$$
$$y(k) = C\underline{x}(k)$$

Show that the z-transform of the state vector $\underline{x}(k)$ is given by

$$\underline{X}(z) = [zI - G]^{-1} z\, \underline{x}(0) + [zI - G]^{-1} H\underline{U}(z)$$

where $\underline{x}(0)$ are the initial conditions for the state $\underline{x}(k)$.

(4 marks)

The G, H and C matrices of a discrete-time control system are respectively given by

$$G = \begin{bmatrix} 0 & 1 \\ -0.24 & -1 \end{bmatrix}, H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Obtain the state $\underline{x}(k)$ and the output $y(k)$ in a closed form when the input $u(k) = 1$ for $k = 0, 1, 2, \ldots.$ Assume that the initial state is given by

$$\underline{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(21 marks)

Q6 The state equation of a discrete-time control system is given by:

$$\underline{x}(k+1) = Gx(k) + Hu(k)$$

(a) State the condition for state variable feedback can be applied to this system and give a method to test the condition.

(5 marks)

(b) Given that

$$G = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.16 & 0.84 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Design the state feedback matrix K such that when the control signal is given by

$$u(k) = -Kx(k)$$

the states $\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ for k = 3, 4, 5, ... for any non-zero initial condition $\underline{x}(0)$.

(20 marks)

FINAL EXAMINATION

SEMESTER/SESSION : SEMESTER 1/2009/10 COURSE : 4 BEE

SUBJECT : KAWALAN DIGIT SUBJECT CODE : BER4113/BEM4713

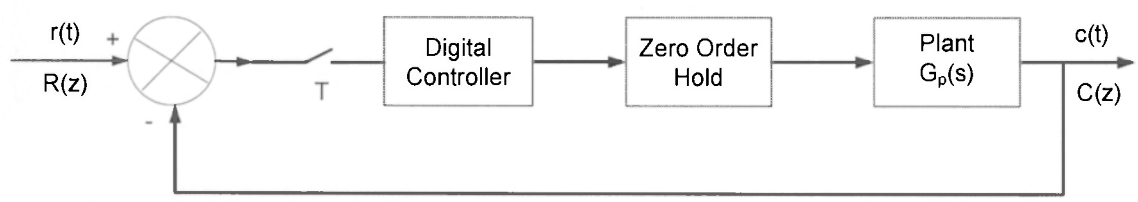

Figure Q1

Table Q3

Row	z^0	z^1	z^2	z^3	. . .	z^{n-2}	z^{n-1}	z^n
1	a_n	a_{n-1}	a_{n-2}	a_{n-3}	. . .	a_2	a_1	a_0
2	a_0	a_1	a_2	a_3	. . .	a_{n-2}	a_{n-1}	a_n
3	b_{n-1}	b_{n-2}	b_{n-3}	b_{n-4}	. . .	b_1	b_0	
4	b_0	b_1	b_2	b_3	. . .	b_{n-2}	b_{n-1}	
5	c_{n-2}	c_{n-3}	c_{n-4}	c_{n-5}	. . .	c_0		
6	c_0	c_1	c_2	c_3	. . .	c_{n-2}		
⋮	⋮							
$2n-5$	p_3	p_2	p_1	p_0				
$2n-4$	p_0	p_1	p_2	p_3				
$2n-3$	q_2	q_1	q_0					

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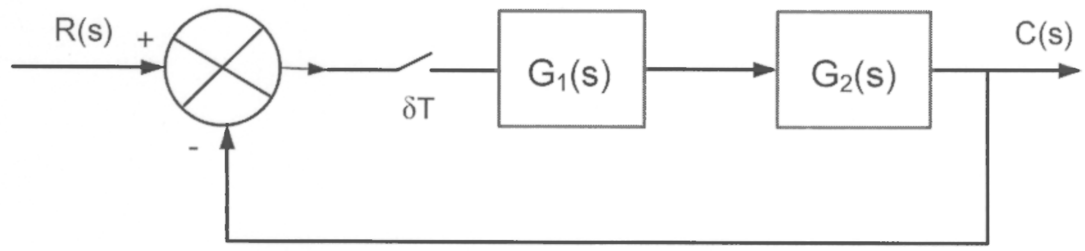

Figure Q4

Table 1: Table of z Transform

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ 1, $k = 0$ 0, $k \neq 0$	1
2.	—	—	$\delta_0(n-k)$ 1, $n = k$ 0, $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT}(1 + e^{-aT} z^{-1}) z^{-1}}{(1 - e^{-aT} z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{[(aT - 1 + e^{-aT}) + (1 - e^{-aT} - aTe^{-aT}) z^{-1}] z^{-1}}{(1 - z^{-1})^2 (1 - e^{-aT} z^{-1})}$
14.	$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1} \sin \omega T}{1 - 2z^{-1} \cos \omega T + z^{-2}}$
15.	$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1 - z^{-1} \cos \omega T}{1 - 2z^{-1} \cos \omega T + z^{-2}}$
16.	$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$	$e^{-akT} \sin \omega kT$	$\frac{e^{-aT} z^{-1} \sin \omega T}{1 - 2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
17.	$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$	$e^{-akT} \cos \omega kT$	$\frac{1 - e^{-aT} z^{-1} \cos \omega T}{1 - 2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
18.			a^k	$\frac{1}{1 - az^{-1}}$
19.			a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1 - az^{-1}}$
20.			ka^{k-1}	$\frac{z^{-1}}{(1 - az^{-1})^2}$
21.			$k^2 a^{k-1}$	$\frac{z^{-1}(1 + az^{-1})}{(1 - az^{-1})^3}$
22.			$k^3 a^{k-1}$	$\frac{z^{-1}(1 + 4az^{-1} + a^2 z^{-2})}{(1 - az^{-1})^4}$
23.			$k^4 a^{k-1}$	$\frac{z^{-1}(1 + 11az^{-1} + 11a^2 z^{-2} + a^3 z^{-3})}{(1 - az^{-1})^5}$
24.			$a^k \cos k\pi$	$\frac{1}{1 + az^{-1}}$
25.			$\frac{k(k-1)}{2!}$	$\frac{z^{-2}}{(1 - z^{-1})^3}$
26.		$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}$		$\frac{z^{-m+1}}{(1 - z^{-1})^m}$
27.			$\frac{k(k-1)}{2!} a^{k-2}$	$\frac{z^{-2}}{(1 - az^{-1})^3}$
28.	$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!} a^{k-m+1}$			$\frac{z^{-m+1}}{(1 - az^{-1})^m}$

Table 2: z Transform of $x(k+m)$ and $x(k-m)$

Discrete function	z Transform
$x(k+4)$	$z^4X(z) - z^4x(0) - z^3x(1) - z^2x(2) - zx(3)$
$x(k+3)$	$z^3X(z) - z^3x(0) - z^2x(1) - zx(2)$
$x(k+2)$	$z^2X(z) - z^2x(0) - zx(1)$
$x(k+1)$	$zX(z) - zx(0)$
$x(k)$	$X(z)$
$x(k-1)$	$z^{-1}X(z)$
$x(k-2)$	$z^{-2}X(z)$
$x(k-3)$	$z^{-3}X(z)$
$x(k-4)$	$z^{-4}X(z)$