UNIVERSITI
TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2009/2010

SUBJECT NAME	: ELECTROMAGNETIC FIELDS AND WAVES
SUBJECT CODE	: BEE 2263
COURSE	: 2 BEE
DATE OF EXAMINATION	: NOVEMBER 2009
DURATION	: 3 HOURS
INSTRUCTION	: ANSWER FIVE (5) QUESTIONS ONLY

THIS PAPER CONSISTS OF 10 PAGES

Q1 (a) Compare the differences between electrostatic fields and magnetostatic fields. Your answer should include all important laws and expressions or equations that relate to each of them.

(9 marks)

(b) A current distribution in free space gives rise to the magnetic potential $\overline{A}$ and is given by;

$$\overline{A}=(2x^2y+yz)\hat{x}+(xy^2-xz^3)\hat{y}-(6xyz-2x^2y^2)\hat{z} \text{ Wb/m}$$

(i) Calculate magnetic flux density $\overline{B}$

(3 marks)

(ii) Find the magnetic flux through a loop given by $x = 1\,\text{m}$, $0 < y < 2\,\text{m}$, and $0 < z < 2\,\text{m}$

(4 marks)

(iii) Show that $\nabla \bullet \overline{A} = 0$ and $\nabla \bullet \overline{B} = 0$

(4 marks)

Q2 (a) Two extensive homogeneous isotropic dielectrics meet on plane z = 0. For $z > 0$, $\varepsilon_{r1} = 2.5$ and $z < 0$, $\varepsilon_{r2} = 3.2$. A uniform electric field $E_1 = 2\hat{x}+4\hat{y}+3\hat{z}\ kV/m$ exists for $z > 0$. Determine,

(i) E_2 for $z < 0$,
(ii) The angles E_1 and E_2 make with the interface.

(10 marks)

(b) If the electric flux density $D_1 = 10\hat{x}-12\hat{y}+6\hat{z}\ nC/m^2$ is applied, find

(i) The flux density D_2 in dielectric 2.
(ii) The angle 0_2 between E_2 and normal.

(10 marks)

Q3 (a) Consider an infinitely long line charge with charge density 4 nC/m exists at $y = -3$ m, $x=0$.

(i) Determine the direction of electric field intensity at the origin due to the line charge. Sketch your diagram.

(1 mark)

(ii) Calculate electric field intensity due to the line charge measured at the origin.

(3 marks)

(iii) A charge $Q = 10$ nC exist somewhere along the y-axis. Where must you locate Q so that the total electric field is zero at the origin?

(3 marks)

(iv) Suppose instead of the 10 nC charge of part (iii), a charge Q is located at (0, 5 m, 0), what value of Q will result in a total electric field intensity of zero at the origin?

(3 marks)

(b) A vector field $\overline{D} = r^3\hat{r}$ C/m^2 exist in the region of a cylindrical surface defined by $r = 2$ m with height extending between $z = 0$ and $z = 5$ m. Verify divergence theorem by evaluating the following:

(i) $\oint_S \overline{D} \bullet d\overline{s}$

(5 marks)

(ii) $\int (\nabla \bullet \overline{D}) dv$

(5 marks)

Q4 An inductor is formed by winding N turns of a thin conducting wire into a circular loop of radius a. The inductor loop is in the x-y plane with its center at the origin, and it is connected to a resistor R as shown in Figure Q4. In the presence of a magnetic field given by $\overline{B} = B_0(3\hat{y} + 4\hat{z})\sin\omega t$ Wb/m^2 where ω is the angular frequency, find;

(a) the magnetic flux linking a single turn of the inductor

(4 marks)

(b) the transformer V_{emf}, given that $N = 15, B_0 = 0.15$ T, $a = 12$ cm and $\omega = 10^3$ rad/s

(7 marks)

(c) the polarity of V_{emf} at $t = 0$

(5 marks)

(d) the induced current in the circuit for $R = 1.2$ kΩ.

(4 marks)

Q5 (a) Show that the magnetic field intensity for semi-infinite line carrying current I along the z-axis is given by $\overline{H} = \frac{I}{4\pi r}\hat{\phi}\ (\text{Am}^{-1})$.

(12 marks)

(b) A thin ring of radius 6 cm is placed on plane z = 2 cm so that its center is (0, 0, 2 cm). If the ring carries 55 mA along $a_ø$, find the magnetic field intensity at,

(i) (0, 0, -2 cm)
(ii) (0, 0, 15 cm)

(8 marks)

Q6 (a) Polarization occurs when electric field is applied on a dielectric. Illustrate the polarization process and show quantitatively the impact of polarization on the electric flux density inside the dielectric.

(5 marks)

(b) A parallel plate capacitor is made of two similar rectangular thin conductive sheets of area 6 mm^2. The plates are separated by a distance of 2 mm and filled with a dielectric of relative permittivity 2.5.
A 5 volt dry cell battery is connected between the terminals of the capacitor.

(i) Calculate the capacitance of the capacitor
(ii) What is the electric field intensity in the capacitor?
(iii) Determine the amount of charge on each plate.
(iv) Calculate the electric energy density stored in the capacitor.
(v) Calculate the electric energy stored in the capacitor.

(10 marks)

(c) A student requires an inductor which has a self inductance of 5 mH for his final year project. He uses a solenoid made of insulated copper which has 100 turns. What is the length required for the solenoid if the wire is wound on a cylindrical dowel of radius 2 cm? Propose a method to reduce the number of turns if all other parameters remain unchanged.

(5 marks)

Q7 (a) List and explain the Maxwell's equations for time varying electric and magnetic fields. Illustrate an experiment that can describe the significance of ONE (1) of the Maxwell's equations.

(8 marks)

(b) The electric field phasor of a uniform plane wave is given by $\vec{E} = \hat{z}10e^{j0.2y}$ (V/m). If the phase velocity of the wave is 1.5 x 10^8 m/s and the relative permeability of the medium is $\mu_r = 2.4$, find:

(i) the wavelength,
(ii) the wavenumber,
(iii) the frequency f of the wave,

(iv) the relative permittivity of the medium,

(v) the magnetic field $\vec{H}(y,t)$,

(vi) the average power density carried by the wave.

Plot the $\vec{E}(y,t)$ and $\vec{H}(y,t)$, as a function of y at t=0.

(12 marks)

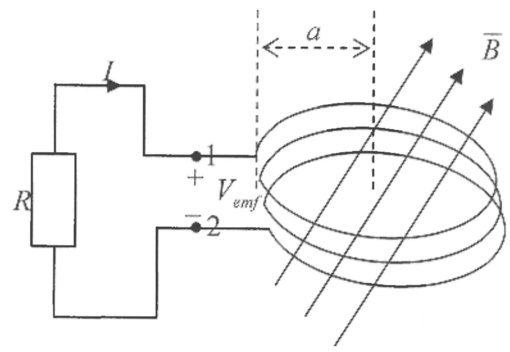

Figure Q4

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\varphi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial(rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial\left(R^2 A_R\right)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial\left(A_\theta \sin\theta\right)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{r}\left(\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial(\sin\theta\, A_\varphi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial(RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial(RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\varphi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

	Cartesian	Cylindrical	Spherical
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\varphi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\varphi\hat{\boldsymbol{\varphi}}$
Magnitude $\vec{A}$	$\sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1, y_1, z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1, \phi_1, z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1, \theta_1, \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\varphi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\varphi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\vec{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\varphi}}$
Differential surface, $\vec{ds}$	$\vec{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\vec{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\vec{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\vec{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\vec{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\varphi}}$ $\vec{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\vec{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\vec{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\vec{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume, $\vec{dv}$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\varphi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\varphi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to Spherical	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\varphi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\varphi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\varphi}}=\hat{\boldsymbol{\varphi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to Cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\varphi}}=\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

$Q = \int \rho_\ell d\ell,$

$Q = \int \rho_s dS,$

$Q = \int \rho_v dv$

$\overline{F}_{12} = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$

$\overline{E} = \dfrac{\overline{F}}{Q},$

$\overline{E} = \dfrac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\overline{E} = \int \dfrac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\overline{E} = \int \dfrac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\overline{E} = \int \dfrac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\overline{D} = \varepsilon\overline{E}$

$\psi_e = \int \overline{D} \bullet d\overline{S}$

$Q_{enc} = \oint_S \overline{D} \bullet d\overline{S}$

$\rho_v = \nabla \bullet \overline{D}$

$V_{AB} = -\int_A^B \overline{E} \bullet d\overline{\ell} = \dfrac{W}{Q}$

$V = \dfrac{Q}{4\pi\varepsilon r}$

$V = \int \dfrac{\rho_\ell d\ell}{4\pi\varepsilon r}$

$\oint \overline{E} \bullet d\overline{\ell} = 0$

$\nabla \times \overline{E} = 0$

$\overline{E} = -\nabla V$

$\nabla^2 V = 0$

$R = \dfrac{\ell}{\sigma S}$

$I = \int \overline{J} \bullet dS$

$d\overline{H} = \dfrac{Id\overline{\ell} \times \overline{R}}{4\pi R^3}$

$Id\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J} dv$

$\oint \overline{H} \bullet d\overline{\ell} = I_{enc} = \int \overline{J}_s dS$

$\nabla \times \overline{H} = \overline{J}$

$\psi_m = \int_s \overline{B} \bullet d\overline{S}$

$\psi_m = \oint \overline{B} \bullet d\overline{S} = 0$

$\psi_m = \oint \overline{A} \bullet d\overline{\ell}$

$\nabla \bullet \overline{B} = 0$

$\overline{B} = \mu\overline{H}$

$\overline{B} = \nabla \times \overline{A}$

$\overline{A} = \int \dfrac{\mu_0 Id\overline{\ell}}{4\pi R}$

$\nabla^2 \overline{A} = -\mu_0 \overline{J}$

$\overline{F} = Q(\overline{E} + \overline{u} \times \overline{B}) = m\dfrac{d\overline{u}}{dt}$

$d\overline{F} = Id\overline{\ell} \times \overline{B}$

$\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$

$\overline{m} = IS\hat{a}_n$

$V_{emf} = -\dfrac{\partial \psi}{\partial t}$

$V_{emf} = -\int \dfrac{\partial \overline{B}}{\partial t} \bullet d\overline{S}$

$V_{emf} = \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$

$I_d = \int J_d . d\overline{S}, J_d = \dfrac{\partial \overline{D}}{\partial t}$

$\gamma = \alpha + j\beta$

$\alpha = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\dfrac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$

$\beta = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\dfrac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$

$\overline{F}_1 = \dfrac{\mu I_1 I_2}{4\pi}\oint_{L1}\oint_{L2} \dfrac{d\overline{\ell}_1 \times (d\overline{\ell}_2 \times \hat{a}_{R_{21}})}{R_{21}^{\;2}}$

$|\eta| = \dfrac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\dfrac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$

$\tan 2\theta_\eta = \dfrac{\sigma}{\omega\varepsilon}$

$\tan\theta = \dfrac{\sigma}{\omega\varepsilon} = \dfrac{\overline{J}_s}{\overline{J}_{ds}}$

$\delta = \dfrac{1}{\alpha}$

$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{Fm}^{-1}$

$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$

$\int \dfrac{dx}{(x^2 + c^2)^{3/2}} = \dfrac{x}{c^2(x^2 + c^2)^{1/2}}$

$\int \dfrac{xdx}{(x^2 + c^2)^{3/2}} = \dfrac{-1}{(x^2 + c^2)^{1/2}}$

$\int \dfrac{dx}{(x^2 \pm c^2)^{1/2}} = \ln\left(x + \sqrt{x^2 \pm c^2}\right)$

$\int \dfrac{dx}{(x^2 + c^2)} = \dfrac{1}{c}\tan^{-1}\left(\dfrac{x}{c}\right)$

$\int \dfrac{xdx}{(x^2 + c^2)} = \dfrac{1}{2}\ln(x^2 + c^2)$

$\int \dfrac{xdx}{(x^2 + c^2)^{1/2}} = \sqrt{x^2 + c^2}$