UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2008/2009

SUBJECT NAME	:	DIGITAL SIGNAL PROCESSING
SUBJECT CODE	:	BEE 3213
COURSE	:	3 BEE / 3 BEI
EXAMINATION DATE	:	APRIL 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** OUT OF SEVEN (7) QUESTIONS.

THIS QUESTION PAPER CONTENT ELEVEN (11) PAGES

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Q1 (a) Given $x[n] = 2r[n+2] - 7\delta[n+1] + 4tri(\frac{n+1}{2}) - 4rect(\frac{n-1}{4}) +$

$$(-2)^n(u[n] - u[n-4]) - 2r[n-3] - 10u[n-4]$$

(i) Separate x[n] as its odd and even parts and sketch both of them.

(7 marks)

(ii) Find y[n] for $y[n] = x[2n - \frac{7}{4}]$ using step interpolation

(3 marks)

(b) Classify the following as left-sided, right-sided or two-sided signals. Represent those signals in a numeric sequence to support your answer.

(i) $z[n] = x[n+1]y[n]$ where $x[n] = 0.5(\delta[n] + \delta[n+1] - 2\delta[n+3])$ and $y[n] = 3 + u[n-3]$

(ii) $h[n] = (n+5)u[-n-1]$

(iii) $z[n] = cos[185n\pi k - \frac{2\pi}{3}]$ where $k = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & elsewhere \end{cases}$

(6 marks)

(c) Given a discrete signal $x[n] = \{1, 6, a, 7, -4, b\}$. If $x_{av} = 1.83$ and $P_{av} = 19.17$, find the value of a and b if the value b > 0.

(4 marks)

Q2 Convolution is a method of finding the zero-state response of relaxed linear time-invariant (LTI) systems, while correlation is a measure of similarity between two signals.

(a) Given $x[n] = \{-3, 12, 6, 0, -9\}$ for $-2 \le n \le 2$ and $h[n] = \begin{cases} (3^n - 2) & -1 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$

(i) Determine the convolution of x[n] and h[n] using sliding strip method

(6 marks)

(ii) Assume that x[n] and h[n] is periodic signal. Find the convolution using cyclic method

(6 marks)

(b) A system have an input signal of $x[n] = 6\,rect\left(\frac{2-n}{2N}\right) + \{\overset{\Downarrow}{0}, -8, -5, 2, 4\}$ and impulse response $h[n] = (2n+1)(u[n] - u[n-2]) - 3\delta[n-3]$.

(i) Find the correlation $r_{xh}[n]$ for the above signal for N=3

(6 marks)

(ii) Based on answer from Q2(b)(i), find $r_{hx}[n]$

(2 marks)

Q3 (a) As an electrical engineer, you are required to make an analysis of digital signal. If you are given an analog signal, with aid of diagram, explain the first step before you can proceed this task.

(4 marks)

(b) Consider a sinusoidal signal $g(t) = \frac{5}{2}\sin(800\pi t + 60°)$ V is sampled at about 25% above the Nyquist Rate. Given a dynamic range of this signal is ±3V and it will be encoded with 4 bits per sample. Determine for $-1 < n < 6$:

(i) The discrete signal g[n]

(ii) The quantized signal using both truncate and rounding techniques gQ1[n] and gQ2[n]

(iii) The encoded signal for both quantization techniques gc1[n] and gc2[n]

(iv) The quantization error for both quantization techniques ε1[n] and ε2[n]

(v) Find the Signal-to-Noise Ratio (SNR) for both quantized signals

(vi) Based on Q3(b) (v), comment the performance of both quantized signals and which quantization technique give a minimum error

(vii) Suggest another way to improve the Signal-to-Noise Ratio (SNR)

(16 marks)

Q4 (a) Let $X_{DFT}[k] = \{2,\ 0.5+0.866j,\ 0.5-0.866j\}$. Compute its Inverse Discrete Fourier Transform (IDFT)

(2 marks)

(b) Given the Discrete Fourier Transform (DFT) pair of a discrete signal,

$z[n] = \{\ 1, -2,\ 3,\ 0,\ -1,\ 1,\ 2\} \Longleftrightarrow$

$Z_{DFT}[k] = \{\ 4, \boxed{A}\ , -3.23+5.55j, 3.72+2.32j, \boxed{B}, \boxed{C}, 1.01-0.74j\}$.

By using the properties of DFT;

(i) Find A, B and C

(ii) Determine the sequence $w[n] = z[-n]$ and its DFT

(3 marks)

(c) The Discrete Fourier Transform (DFT) of a discrete signal, c[n] is given by $C_{DFT}[k] = \{6,\ 3-1j, -8,\ 3+1j\}$. Apply Decimation in Time (DIT) Fast Fourier Transform (FFT) algorithm to get its discrete signal, c[n].

(15 marks)

Q5 (a) Differentiate the region of convergence (ROC) of the following signals:

(i) Causal and anti-causal finite signal.

(ii) Causal and anti-causal infinite signal.

(4 marks)

(b) (i) Given $u[n] \xleftrightarrow{z} \frac{z}{z-1}$ with its ROC : $|z| > 1$. Modify the signal u[n] so that its ROC: $|z| < 1$ and support your answer using defining relation.

(4 marks)

(ii) By using the properties of z-transform, determine the z-transform and the ROC of the signal $x[n] = \left[2\left(\frac{2}{3}\right)^n + \frac{5}{6}\right]u[n]$ (4 marks)

(c) Let $X(z) = \frac{1+3z^{-1}}{1-3z^{-1}+z^{-2}}$. Calculate the inverse z-transform of X(z) by using long division method for the following cases:

(i) Right-sided signal

(ii) Left-sided signal

(Notes : Show the first 5 terms only)

(8 marks)

Q6 (a) List down four applications area of the DSP processor in modern life.

(2 marks)

(b) A lowpass filter $\frac{3(z+1)^2}{31z^2-26z+7}$ operates at S = 10 kHz, and its cutoff frequency is fc = 2 kHz. Use this filter to design a lowpass filter with a cutoff frequency of 1 kHz.

(10 marks)

(c) Design a bandpass filter with a 3-dB bandwith of 8 kHz and a center frequency of 9 kHz. The sampling frequency is 35 kHz. Plot the response of H(z) in the range $0 \le f \le 20$ kHz with step size of 2 kHz.

(Hint: $H(s) = \frac{1}{(s+1)}$, cutoff frequency is 1 rad/s) (8 marks)

Q7 Design an FIR filter using Hamming window to meet the following specifications:

Passband: [1, 10] kHz

Stoppband: [5, 6] kHz

$A_p = 2$ dB

$As = 40$ dB

The sampling frequency, $S = 25$ kHz.

(20 marks)

Euler's Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

Finite Summation Formula

$$\sum_{k=0}^{n} \alpha^{k} = \frac{1-\alpha^{n+1}}{1-\alpha}, \quad \alpha \neq 1$$

Infinite Summation Formula

$$\sum_{k=0}^{\infty} \alpha^{k} = \frac{1}{1-\alpha}, \qquad |\alpha| < 1$$

Table 1 : Properties of the Discrete Fourier Transform (DFT)

Property	**Signal**	**DFT**	**Remarks**
Shift	$x[n - n_o]$	$X_{DFT}[k]e^{-j2\pi k n_o/N}$	No change in magnitude.
Shift	$x[n - 0.5N]$	$(-1)^k X_{DFT}[k]$	Half-period shift for even N.
Modulation	$x[n]e^{j2\pi nk_o/N}$	$X_{DFT}[k - k_o]$	
Modulation	$(-1)^n x[n]$	$X_{DFT}[k - 0.5N]$	Half-period shift for even N.
Folding	$x[-n]$	$X_{DFT}[-k]$	This is circular folding.
Product	$x[n]y[n]$	$\frac{1}{N}X_{DFT}[k] \otimes Y_{DFT}[k]$	The convolution is periodic.
Convolution	$x[n] \otimes y[n]$	$X_{DFT}[k]Y_{DFT}[k]$	The convolution is periodic.
Correlation	$x[n] \otimes \otimes y[n]$	$X_{DFT}[k]Y^{*}_{DFT}[k]$	The correlation is periodic.
Central Ordinates	$x[0] = \frac{1}{N}\sum_{k=0}^{N-1} X_{DFT}[k]$, $\quad X_{DFT}[0] = \sum_{n=0}^{N-1} x[n]$		
Central Ordinates	$x\left[\frac{N}{2}\right] = \frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (N even), $X_{DFT}\left[\frac{N}{2}\right] = \sum_{n=0}^{N-1}(-1)^n x[n]$ (N even)		
Parseval's Relation	$\sum_{n=0}^{N-1}\lvert x[n]\rvert^2 = \frac{1}{N}\sum_{k=0}^{N-1}\lvert X_{DFT}[k]\rvert^2$		

Table 2: Properties of z-transform.

Property	**Signal**	**z-transform**
Linearity	$a_1x_1[n] + a_2x_2[n]$	$a_1X_1(z) + a_2X_2(z)$
Time reversal	$x[-n]$	$X(z^{-1})$
Time shifting	i) $x(n - k)$ ii) $x(n + k)$	i) $z^{-k}X(z)$ ii) $z^{k}X(z)$
Convolution	$x_1(n) * x_2(n)$	$X_1(z)X_2(z)$
Correlation	$r_{x_1x_2}(l) = \sum_{n=-\infty}^{\infty} x_1(n)x_2(n-l)$	$R_{x_1x_2}(z) = X_1(z)X_2(z^{-1})$
Scaling	$a^n x(n)$	$X(a^{-1}z)$
Differentiation	$nx[n]$	$z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$
Time differentiation	$x[n] - x[n-1]$	$X(z)(1 - z^{-1})$
Time integration	$\sum_{k=0}^{\infty} X(k)$	$X(z) = \left(\frac{z}{z-1}\right)$
Initial value theorem	$\lim_{n \to 0} x(n)$	$\lim_{\lvert z \rvert \to \infty} X(z)$
Final value theorem	$\lim_{n \to \infty} x(n)$	$\lim_{\lvert z \rvert \to 1} \left(\frac{z-1}{z}\right) X(z)$

Table 3: Digital-to-Digital Transformations

From	Band Edges	Mapping z →	Parameters
Lowpass to lowpass	Ω_C	$\dfrac{z-\alpha}{1-\alpha z}$	$\alpha = \sin[0.5(\Omega_D - \Omega_C)]/\sin[0.5(\Omega_D + \Omega_C)]$
Lowpass to highpass	Ω_C	$\dfrac{C(z+1)}{z-1}$	$\alpha = -\cos[0.5(\Omega_D + \Omega_C)]/\cos[0.5(\Omega_D - \Omega_C)]$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\dfrac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)/\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = -\cos[0.5(\Omega_2 + \Omega_1)]/\cos[0.5(\Omega_2 - \Omega_1)]$ $A_1 = 2\alpha K/(K+1) \quad A_2 = (K-1)/(K+1)$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\dfrac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)/\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = -\cos[0.5(\Omega_2 + \Omega_1)]/\cos[0.5(\Omega_2 - \Omega_1)]$ $A_1 = 2\alpha/(K+1) \quad A_2 = -(K-1)/(K+1)$

Table 4: Direct Analog-to-Digital Transformations for Bilinear Design

From	Band Edges	Mapping s →	Parameters
Lowpass to lowpass	Ω_C	$\dfrac{z-1}{C(z+1)}$	$C = \tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\dfrac{C(z+1)}{z-1}$	$C = \tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1 < \Omega_0 < \Omega_2$	$\dfrac{z^2 - 2\beta z + 1}{C(z^2 - 1)}$	$C = \tan[0.5(\Omega_2 - \Omega_1)], \beta = \cos\Omega_0$ or $\beta = \dfrac{\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$
Lowpass to bandstop	$\Omega_1 < \Omega_0 < \Omega_2$	$\dfrac{C(z^2 - 1)}{z^2 - 2\beta z + 1}$	$C = \tan[0.5(\Omega_2 - \Omega_1)], \beta = \cos\Omega_0$ or $\beta = \dfrac{\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$

Table 5 : Windows for FIR Filter Design

Note: $I_0(x)$ is the modified Bessel Function of order zero.	
Window	Expression $\omega[n]$, $-0.5(N-1) \le n \le 0.5(N-1)$
Boxcar	1
Cosine	$\cos\left(\frac{n\pi}{N-1}\right)$
Riemann	$\text{sinc}^L\left(\frac{2n}{N-1}\right)$, $L > 0$
Bartlett	$1-\frac{2\lvert n\rvert}{N-1}$
von Hann (Hanning)	$0.5+0.5\cos\left(\frac{2n\pi}{N-1}\right)$
Hamming	$0.54+0.46\cos\left(\frac{2n\pi}{N-1}\right)$
Blackman	$0.42+0.5\cos\left(\frac{2n\pi}{N-1}\right)+0.08\cos\left(\frac{4n\pi}{N-1}\right)$
Kaiser	$\frac{I_0\left(\pi\beta\sqrt{1-4[n/(N-1)]^2}\right)}{I_0(\pi\beta)}$

Table 6 : Characteristics of the windowed spectrum for various windows.

Window	Peak Ripple $\delta p = \delta s$	Passband Attenuation AWP (dB)	Peak Sidelobe Attenuation AWS (dB)	Transition Width $FWS \approx C/N$
Boxcar	0.0897	1.5618	21.7	C = 0.92
Cosine	0.0207	0.3600	33.8	C = 2.10
Riemann	0.0120	0.2087	38.5	C = 2.50
von Hann (Hanning)	0.0063	0.1103	44.0	C = 3.21
Hamming	0.0022	0.0384	53.0	C = 3.47
Blackman	1.71×10^{-4}	2.97×10^{-3}	75.3	C = 5.71