UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2008/2009

SUBJECT NAME	: ELECTROMAGNETIC FIELDS AND WAVES
SUBJECT CODE	: BEE 2263
COURSE	: 2 BEE
EXAMINATION DATE	: APRIL 2009
DURATION	: 3 HOURS
INSTRUCTION	: ANSWER **FIVE (5)** QUESTIONS ONLY.

THIS QUESTION PAPER CONSIST OF **TWELVE (12)** PAGES

Q1 (a) Describe two fundamental laws of electrostatic fields.

(4 marks)

(b) Two finite uniform line charges of 4 nC/m and -2 nC/m lie along the $\hat{x}$ and $\hat{y}$ axis in free space as shown in Figure Q1(b).

(i) Determine electric field intensity, $\vec{E}$ at point P (2, 3, -4).

(10 marks)

(ii) If two identical point charges, Q are placed at (8, 0, 0) and (0, 6, 0) in addition to the two finite uniform line charges, find the value of Q to cause $\vec{E}_z = 0$ at point P (2, 3, -4).

(6 marks)

Q2 (a) The Gauss's Law for static electric field is given by $\oint_s \vec{D} \bullet d\vec{s} = Q_{enc}$ where $\vec{D}$ is the electric flux density and Q_{enc} is the net charge in the closed surface, S. Use Gauss's law to prove that the electric field in a perfect conductor cannot exist and this finding can be applied to build a facility to protect sensitive electronic devices.

(6 marks)

(b) A spherical shell consists of two concentric spheres made up of a very thin conductor sheet. The inner radius, a = 5 m and outer radius, b = 10 m is having surface charge density, ρ_{s1} = 5 nC/m^2 and ρ_{s2} = -1.25 nC/m^2, respectively. The two concentric spheres are separated by polystyrene with relative permittivity, ε_r = 2.55. In addition, a point charge, Q = 2 nC is located at the center of the sphere. The configurations is shown in Figure Q2(b).

(i) Apply Gauss's law to determine the electric flux density, $\vec{D}$ and electric field intensity, $\vec{E}$ in all regions.

(8 marks)

(ii) Find the polarization, $\vec{P}$ in all regions.

(3 marks)

(iii) Determine the electric potential, V between the two conductors sheet.

(3 marks)

Q3 (a) Potential difference is defined as work done to move a unit positive charge from one point to another where the potential at various points in an electric field can be desribed by equipotential surfaces. With an aid of a diagram, briefly explain the equipotential surfaces.

(4 marks)

(b) An electric field, $\vec{E} = 6x^2\hat{x} + 6y\hat{y} + 4\hat{z}$ V/m is expressed in rectangular coordinates by two point charges of Q_1 = 10 μC and Q_2 = -5 μC that are located in free space at A (0, 4, 0) and B (0, -3, 0), respectively. Determine:

(i) The electric potential at the origin due to two point charges.

(4 marks)

(ii) The total amount of work done if Q_2 is moved from A(0, 4, 0) to C(1, 2, 3).

(6 marks)

(c) If the potential field is expressed by equation $V = 2x^2y - 5z$, calculate the following at point D(-4, 3, 6):

(i) Electric flux density, $\vec{D}$.

(3 marks)

(ii) Volume charge density, ρ_v.

(3 marks)

Q4 (a) Express Ampere's circuital law.

(2 marks)

(b) A hollow cylindrical conductor has inner radius, a and outer radius, b and carries current, I along the positive $\hat{z}$-direction. Calculate:

(i) Magnetic field intensity, $\vec{H}$ at $r < a$, $a < r < b$ and $r > b$.

(7 marks)

(ii) Plot the magnitude of $\vec{H}$ against r. Give your comments.

(3 marks)

(c) Determine the magnetic field intensity, $\vec{H}$ at point P(-3, -4, 0) in the field of a 3 A filamentary current directed inward from infinity to the origin in the positive $\hat{z}$-axis and the outward to infinity along the positive $\hat{x}$-axis.

(8 marks)

Q5 (a) Two parallel infinite filamentary current are separated by a distance, d in meter and carrying current, I_1 and I_2 in the same direction along $\hat{z}$ -axis as shown in Figure Q5(a).

(i) Show that the force, $\vec{F}$ exerted on wire 2 due to wire 1 is given as

$$\vec{F} = \frac{\mu_0 I_1 I_2 \ell}{2 \pi d}(-\hat{y}) \quad (\text{N})$$

(8 marks)

(ii) Utilize the formulation of $\vec{F}$ in Q5(a)(i) to explain the potential consequence of designing a high current power system which uses two parallel wires located at a close distance to each other.

(2 marks)

(b) A conducting triangular loop as shown in Figure Q5(b) carrying a current of 2 A is located closely to an infinitely long straight conductor with a current of 5 A. Calculate the total force, $\vec{F}$ on the loop due to infinite long straight conductor.

(10 marks)

Q6 (a) Michael Faraday developed the hypothesis below:

"If a current can produce a magnetic field, then the converse should also be true: a magnetic field should produce a current in a wire".

Recommend an experiment to demonstrate the application of the hypothesis in electrical engineering.

(10 marks)

(b) A conducting bar can slide freely over two conducting rails as shown in Figure Q6 (b). If $\vec{B} = 0.5\hat{z}$ (Wb/m^2), $R = 20\Omega$, $\ell = 10$cm and the rod is moving with a constant velocity of $8\hat{x}$ (m/s), calculate:

(i) The induced emf, V_{emf} in the rod.

(4 marks)

(ii) The current, I through the resistor.

(2 marks)

(iii) The motional force, $\vec{F}_m$ on the rod.

(2 marks)

(iv) The power dissipated, P by the resistor.

(2 marks)

Q7 (a) List and explain briefly the differential form of Maxwell's equations for time-varying electric and magnetic fields.

(4 marks)

(b) A plane wave propagating through a medium with $\varepsilon_r = 8$, $\mu_r = 2$ has $\vec{E} = 0.5\, e^{-z/3} \sin(10^8 t - \beta z)\ \hat{x}$ V/m. Determine:

(i) The direction of wave propagation.

(1 mark)

(ii) The loss tangent, δ.

(2 marks)

(iii) Phase constant, β.

(2 marks)

(iv) The wave impedance, η.

(1 mark)

(v) The wave velocity, u.

(1 mark)

(vi) $\vec{E}$ at z = 2 m, t = 2 ns.

(1 mark)

(vii) Magnetic field intensity, $\vec{H}$.

(2 marks)

(viii) Time average power density carried by the wave.

(2 marks)

(c) The wave in Q7(b) propagates perpendicularly through the medium which has an area of 9 m^2 and thickness 1.5 meter. Calculate the average power loss in dB.

(4 marks)

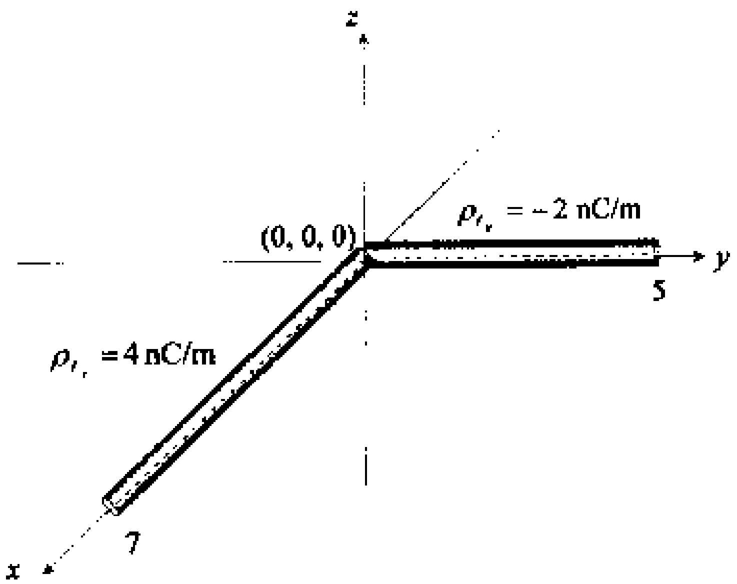

FIGURE Q1(b)

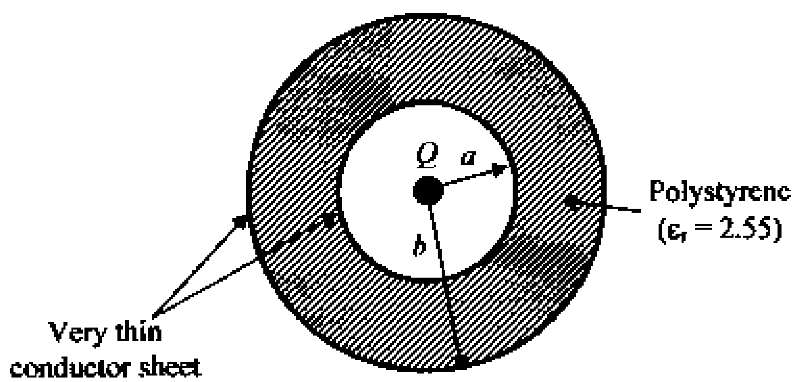

FIGURE Q2(b)

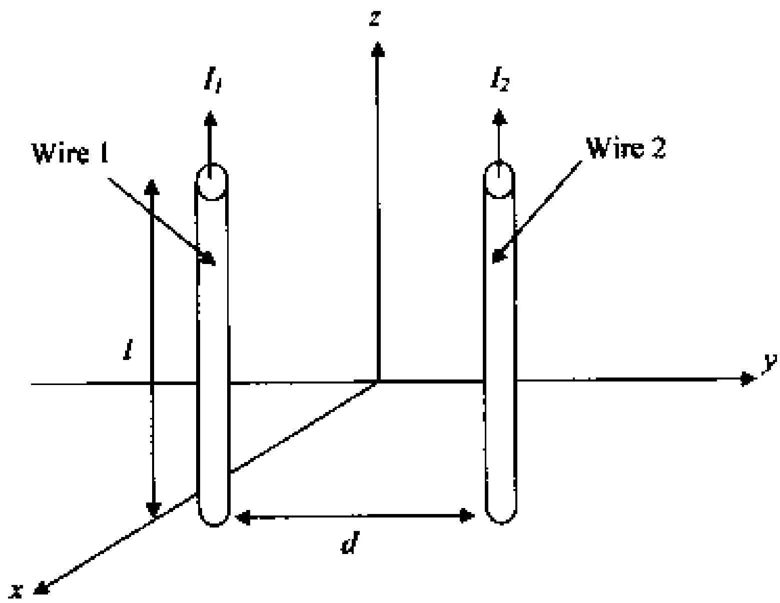

FIGURE Q5(a)

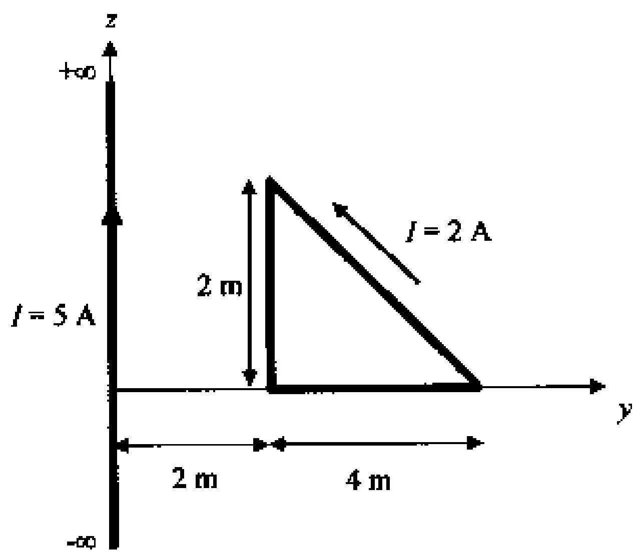

FIGURE Q5(b)

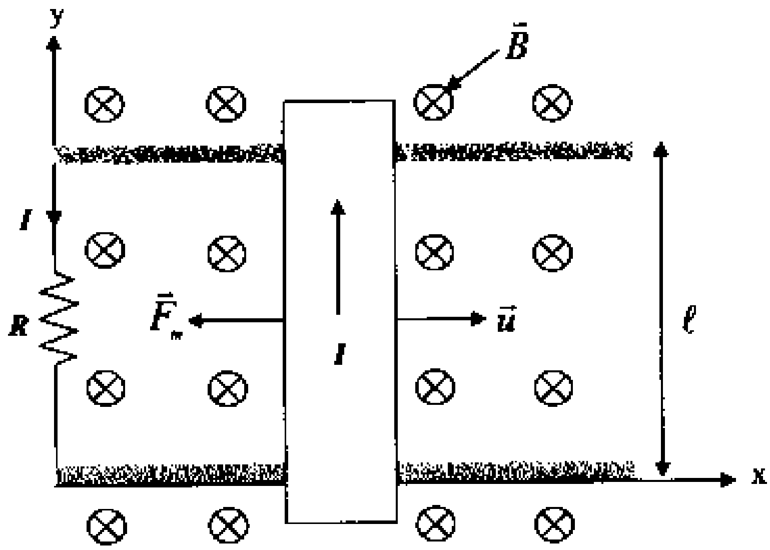

FIGURE Q6(b)

Formula

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$$

$$\nabla f = \frac{\partial f}{\partial R}\hat{\mathbf{R}} + \frac{1}{R}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{R\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence

$$\nabla \bullet \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{r}\left[\frac{\partial (rA_r)}{\partial r}\right] + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \bullet \vec{A} = \frac{1}{R^2}\frac{\partial (R^2 A_R)}{\partial R} + \frac{1}{R\sin\theta}\left[\frac{\partial (A_\theta \sin\theta)}{\partial \theta}\right] + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Curl

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\phi}} + \frac{1}{r}\left(\frac{\partial (rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi}\right)\hat{\mathbf{z}}$$

$$\nabla \times \vec{A} = \frac{1}{R\sin\theta}\left[\frac{\partial (\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{\mathbf{R}} + \frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial (RA_\phi)}{\partial R}\right]\hat{\boldsymbol{\theta}} + \frac{1}{R}\left[\frac{\partial (RA_\theta)}{\partial R} - \frac{\partial A_R}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial f}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial \phi^2}\right)$$

	Cartesian	Cylindrical	Spherical
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\phi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\phi}}$
Magnitude $\vec{A}$	$\sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector, $\overline{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1, y_1, z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1, \phi_1, z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1, \theta_1, \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\bullet\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\bullet\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\overline{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\phi}}$
Differential surface, $\overline{ds}$	$\overline{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\overline{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\overline{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\overline{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\overline{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\phi}}$ $\overline{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\overline{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\overline{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\overline{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\phi}}$
Differential volume, $\overline{dv}$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to Spherical	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi+$ $\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to Cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

FINAL EXAMINATION

SEMESTER/SESSION :	SEMESTER II/2008/2009	COURSE	: 2 BEE
SUBJECT :	ELECTROMAGNETIC FIELDS AND WAVES	SUBJECT CODE	: BEE 2263

$Q = \int \rho_\ell d\ell,$

$Q = \int \rho_s dS,$

$Q = \int \rho_v dv$

$\overline{F}_{12} = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$

$\overline{E} = \dfrac{\overline{F}}{Q},$

$\overline{E} = \dfrac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\overline{E} = \int \dfrac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\overline{E} = \int \dfrac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\overline{E} = \int \dfrac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$

$\overline{D} = \varepsilon\overline{E}$

$\psi_e = \int \overline{D} \bullet d\overline{S}$

$Q_{enc} = \oint_S \overline{D} \bullet d\overline{S}$

$\rho_v = \nabla \bullet \overline{D}$

$V_{AB} = -\int_A^B \overline{E} \bullet d\overline{\ell} = \dfrac{W}{Q}$

$V = \dfrac{Q}{4\pi\varepsilon r}$

$V = \int \dfrac{\rho_\ell d\ell}{4\pi\varepsilon r}$

$\oint \overline{E} \bullet d\overline{\ell} = 0$

$\nabla \times \overline{E} = 0$

$\overline{E} = -\nabla V$

$\nabla^2 V = 0$

$R = \dfrac{\ell}{\sigma S}$

$I = \int \overline{J} \bullet dS$

$d\overline{H} = \dfrac{Id\overline{\ell} \times \overline{R}}{4\pi R^3}$

$Id\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J}dv$

$\oint \overline{H} \bullet d\overline{\ell} = I_{enc} = \int \overline{J}_s dS$

$\nabla \times \overline{H} = \overline{J}$

$\psi_m = \int_S \overline{B} \bullet d\overline{S}$

$\psi_m = \oint \overline{B} \bullet d\overline{S} = 0$

$\psi_m = \oint \overline{A} \bullet d\overline{\ell}$

$\nabla \bullet \overline{B} = 0$

$\overline{B} = \mu\overline{H}$

$\overline{B} = \nabla \times \overline{A}$

$\overline{A} = \int \dfrac{\mu_0 Id\overline{\ell}}{4\pi R}$

$\nabla^2 \overline{A} = -\mu_0 \overline{J}$

$\overline{F} = Q(\overline{E} + \overline{u} \times \overline{B}) = m\dfrac{d\overline{u}}{dt}$

$d\overline{F} = Id\overline{\ell} \times \overline{B}$

$\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$

$\overline{m} = IS\hat{a}_n$

$V_{emf} = -\dfrac{\partial\psi}{\partial t}$

$V_{emf} = -\int \dfrac{\partial\overline{B}}{\partial t} \bullet d\overline{S}$

$V_{emf} = \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$

$I_d = \int J_d . d\overline{S}, J_d = \dfrac{\partial\overline{D}}{\partial t}$

$\gamma = \alpha + j\beta$

$\alpha = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\dfrac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$

$\beta = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\dfrac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$

$\overline{F}_1 = \dfrac{\mu I_1 I_2}{4\pi}\oint_{L1}\oint_{L2}\dfrac{d\overline{\ell}_1 \times (d\overline{\ell}_2 \times \hat{a}_{R_{21}})}{R_{21}^2}$

$|\eta| = \dfrac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\dfrac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$

$\tan 2\theta_\eta = \dfrac{\sigma}{\omega\varepsilon}$

$\tan\theta = \dfrac{\sigma}{\omega\varepsilon} = \dfrac{\overline{J}_s}{\overline{J}_{ds}}$

$\delta = \dfrac{1}{\alpha}$

$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{Fm}^{-1}$

$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$

$\int \dfrac{dx}{(x^2 + c^2)^{3/2}} = \dfrac{x}{c^2(x^2 + c^2)^{1/2}}$

$\int \dfrac{xdx}{(x^2 + c^2)^{3/2}} = \dfrac{-1}{(x^2 + c^2)^{1/2}}$

$\int \dfrac{dx}{(x^2 \pm c^2)^{1/2}} = \ln\left(x + \sqrt{x^2 \pm c^2}\right)$

$\int \dfrac{dx}{(x^2 + c^2)} = \dfrac{1}{c}\tan^{-1}\left(\dfrac{x}{c}\right)$

$\int \dfrac{xdx}{(x^2 + c^2)} = \dfrac{1}{2}\ln(x^2 + c^2)$

$\int \dfrac{xdx}{(x^2 + c^2)^{1/2}} = \sqrt{x^2 + c^2}$