UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	DISCRETE STRUCTURE
COURSE CODE	:	BIT 11003
PROGRAMME CODE	:	BIT
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS B) PLEASE WRITE YOUR ANSWERS IN THIS QUESTION BOOKLET C) CALCULATOR IS NOT ALLOWED

THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

Q1 (a) Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a lion", "x is fierce" and "x drink coffee" respectively. Assume that the domain consists of all creatures. Express the statements in the argument using quantifiers and $P(x)$, $Q(x)$, $R(x)$.

(i) All lions are fierce (1 mark)

Answer:

(ii) Some lions do not drink coffee (1 mark)

Answer:

(iii) Some fierce creatures do not drink coffee (1 mark)

Answer:

(b) Let N be the formula

$$N = (p \rightarrow q) \wedge (q \vee \sim a)$$

Obtain a Disjunctive Normal Form (DNF) and Conjunctive Normal Form (CNF) for N. (7 marks)

Answer:

Answer:

(c) In a certain school, there are 180 pupils in Year 7. One hundred and ten pupils study French, 88 study German and 65 study Indonesian. Forty pupils study both French and German, 38 study German and Indonesian, and 26 study both French and Indonesian, while 19 study German only. Find the number of pupils who study

(i) all three languages (1 mark)

Answer:

(ii) Indonesian only (1 mark)

Answer:

(iii) none of the languages (1 mark)

Answer:

(iv) at least one language (1 mark)

Answer:

(v) either one or two of the three languages. (1 mark)

Answer:

(vi) draw the Venn diagram to illustrate the above situation. (5 marks)

Answer:

(d) If $n(A - B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$. (5 Marks)

Answer:

Q2 (a) **Figure Q2** shows relation between A and B.

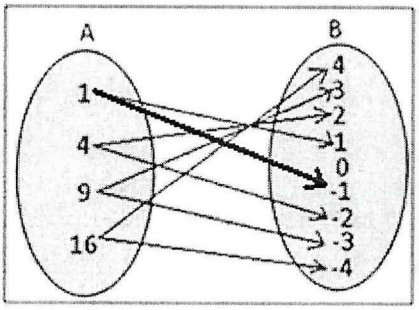

Figure Q2

(i) List the ordered pairs of R. (2 marks)

Answer:

(ii) Determine relationship between A and B and write relation R in set builder form. (3 marks)

Answer:

(iii) Represent relation R in matrix form. (3 marks)

Answer:

(b) If $\left(\frac{x}{3}+1, y-\frac{2}{5}\right)=\left(2.\frac{3}{5}\right)$, find the values of x and y. (2 marks)

(c) Prove by mathematical induction that

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1}n^2 = \frac{(-1)^{n-1}n(n+1)}{2}$$

for every positive integers n.

(10 marks)

Answer:

Q3 (a) The functions f and g are defined by

$f : x \to x^2 - 2x + 3$, $x \in \mathrm{R}$, $0 \le x \le 4$

$g : x \to \lambda x + 1$ where λ is a constant, $x \in \mathrm{R}$

Given that $(g \circ f)(2) = 16$, find the value of λ.

(6 marks)

Answer:

(b) Identify the solution to the recurrence relation

$a_n = a_{n-1} + 6a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$. (6 marks)

Answer:

(c) Let R be a recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 0$ with initial conditions $a_0 = 1$ and $a_1 = 6$.

(i) What is the characteristic polynomial of R. (3 marks)

Answer:

(ii) Examine the general solution of R. (5 marks)

Answer:

Q4 (a) Consider the algorithm shown in **Figure Q3 (a)**. This algorithm calculates the product of matrices $a[\,]$ and $b[\,]$, and stores the result in matrix c[ ]. Determine a time complexity T(n) estimate and the Big O notation for the execution time of the algorithm for matrix multiplication

(10 marks)

```
Procedure mmult(a[1 to n, 1 to n], b[1 to n, 1 to n]:real)
begin
      fori := 1 to n
            for j := 1 to n
            begin
                  c[i, j] := 0
                  for k := 1 to n
                        c[i,j]:=c[i,j] + a[i,k] * b[k,j]
            end
end
```

Figure Q3 (a).

Answer:

(b) Consider the algorithm in **Figure Q3 (b)**.

```
Procedure sum(n: positive integer)
s := 0
for i := 1 to n
  for j := 1 to i
    s := s + j
        returns
```

Figure Q3 (b)

(i) Suppose that procedure sum is started with input $n = 4$. Identify what number is returned by the algorithm? (2 marks)

Answer:

(ii) Evaluate the worst-case time complexity of procedure sum? (3 marks)

Answer:

Q5 (a) For a graph G_2 in **Figure Q5** below, answer the following

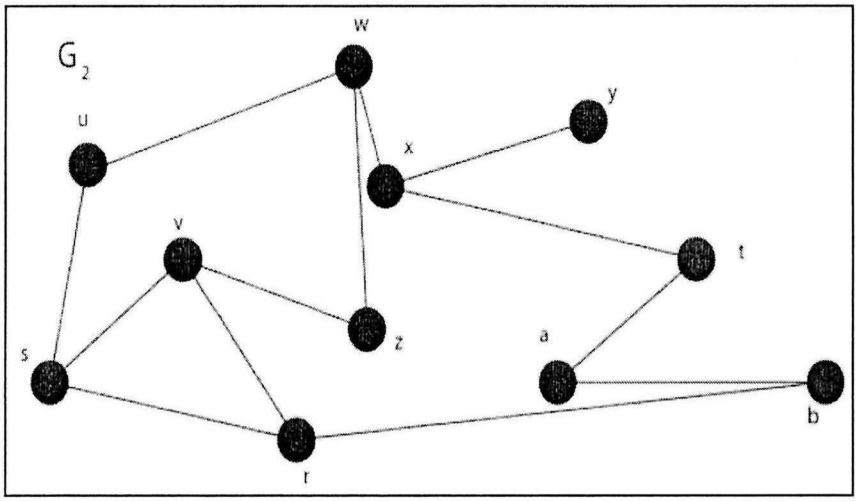

Figure Q5

(i) How many edges and vertices are in G_2. (1 mark)

(ii) List the edges and vertices in G_2. (1 mark)

(iii) List the neighbours of the vertex *v*. (1 mark)

(iv) How many edges are incident with *s*. (1 mark)

(v) Find a walk between *s* and *t*. Is your walk a path? Why or why not? (3 marks)

(vi) Find a cycle in G_2. (2 marks)

Answer:

(i)

(ii)

(iii)

(iv)

(v)

(vi)

(b) It is well-known that in the Netherlands, there is a 2-lane highway from Amsterdam to Breda, another 2-lane highway from Amsterdam to Cappele, a 3-lane highway from Breda to Dordrecht, a 1-lane road from Breda to Ede and another one from Dordrecht to Ede, and a 5-lane superhighway from Cappele to Ede.

(i) Illustrate the situation as weighted graphs.

(6 marks)

Answer:

(ii) Write an adjacency list for G. (5 marks)

Answer:

- END OF QUESTIONS-