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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

**FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020**

COURSE NAME : DISCRETE STRUCTURE
COURSE CODE : BIC 10103
PROGRAMME CODE : BIS / BIP / BIW / BIM
EXAMINATION DATE : DECEMBER 2019 / JANUARY 2020
DURATION : 3 HOURS
INSTRUCTION : A) ANSWER ALL QUESTIONS
B) PLEASE WRITE YOUR
ANSWERS IN THIS QUESTION
BOOKLET
C) CALCULATOR IS NOT
ALLOWED

THIS QUESTION PAPER CONSISTS OF **TWELVE (12) PAGES**

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Q1 (a) Let $P(x)$, $Q(x)$, and $R(x)$ be the statements ‘ x is a lion’, ‘ x is fierce’ and ‘ x drink coffee’ respectively. Assume that the domain consists of all creatures. Express the statements in the argument using quantifiers and $P(x)$, $Q(x)$, $R(x)$.

(i) All lions are fierce (1 mark)

Answer:

(ii) Some lions do not drink coffee (1 mark)

Answer:

(iii) Some fierce creatures do not drink coffee (1 mark)

Answer:

(b) Let N be the formula

$$N = (p \rightarrow q) \wedge (q \vee \sim a)$$

Obtain a Disjunctive Normal Form (DNF) and Conjunctive Normal Form (CNF) for N .

(7 marks)

Answer:

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Answer:

(c) A group of 100 children was asked which of the three films Hook, Batman and Dracula they had seen. Twenty-seven (27) had seen Hook, 38 had seen Batman and 16 had seen Dracula. Eleven children had seen both Hook and Batman, 8 had seen both Hook and Dracula, and 6 had seen Batman and Dracula, while 3 had seen all three films. Find the number of children that had seen

(i) at least one of the three films. (1 mark)

Answer:

(ii) Hook only. (1 mark)

Answer:

(iii) only one of the films. (1 mark)

Answer:

(iv) none of the films. (1 mark)

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- (v) exactly two of the three films. (1 mark)

Answer:

- (vi) draw the Venn diagram to illustrate the above situation. (5 marks)

Answer:

- (d) The following Venn diagram in **Figure Q1** shows set A , B and C .

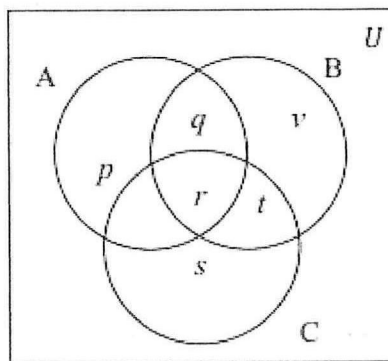


Figure Q1

- (i) Find $(A \cap B \cap C) \cup (B - C)$ (2 marks)

Answer:

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- (ii) Find $(A \cup B') \cap (B \cap C)'$. (3 marks)

- Q2** (a) Suppose that $A = \{3, 4, 5, 6\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. Let R be the relation from A to B defined by $R = \{(a, b) : a \in A, b \in B \text{ and } a \text{ is a multiple of } b\}$.

- (i) Find R . (2 marks)

Answer:

- (ii) Represent relation R in matrix form. (3 marks)

Answer:

- (iii) If A and B are two sets, and $A \times B$ consists of 6 elements. If the three elements of $A \times B$ are $(2, 5) (3, 7) (4, 7)$, compute $A \times B$. (4 marks)

Answer:

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Q3 (a) The function f is defined by

$$f(x) = \frac{x}{3x+1}.$$

(i) Find the value of x that cannot be included in the domain of f .

(1 mark)

Answer:

(ii) Find $f(f(-1))$.

(2 marks)

Answer:

(iii) Compute $f^{-1}(x)$.

(3 marks)

Answer:

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- (iv) If $g(x) = \sqrt{x-4}$, compute $(g \circ f)(x)$. (3 marks)

Answer:

- (b) Identify the solution to the recurrence relation

$$a_n = a_{n-1} + 6a_{n-2} \text{ with } a_0 = 2 \text{ and } a_1 = 7. \quad (5 \text{ marks})$$

Answer:

- (c) Let R be a recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 0$ with initial conditions $a_0 = 1$ and $a_1 = 6$.

- (i) What is the characteristic polynomial of R . (2 marks)

Answer:

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(ii) Examine the general solution of R .

(4 marks)

Answer:

- Q4** (a) Consider the algorithm shown in **Figure Q3 (a)**. This algorithm calculates the product of matrices $a[]$ and $b[]$, and stores the result in matrix $c[]$. Determine a time complexity, $T(n)$ estimate and the Big O notation for the execution time of the algorithm for matrix multiplication

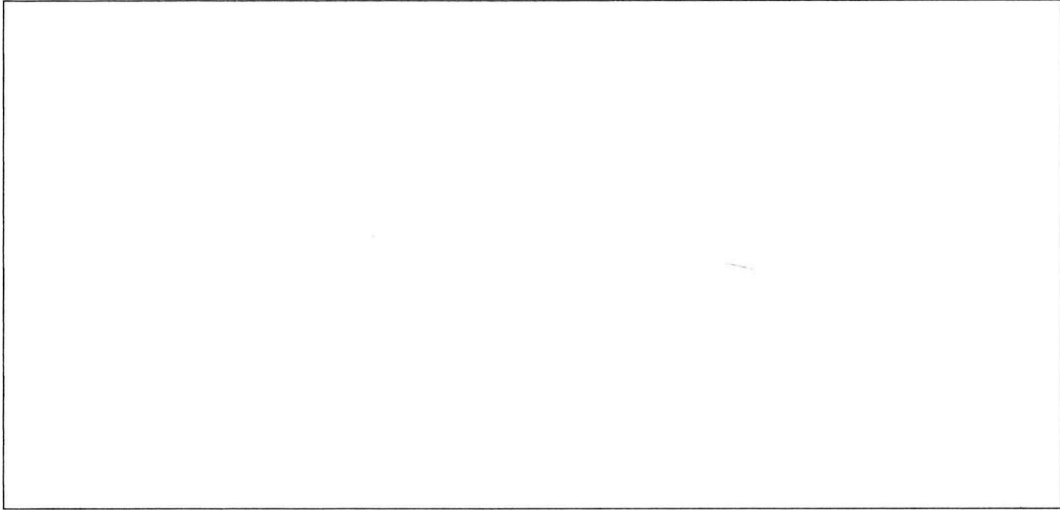
(10 marks)

```
Procedure mmult(a[1 to n, 1 to n], b[1 to n, 1 to n]:real)
begin
  for i := 1 to n
    for j := 1 to n
      begin
        c[i, j] := 0
        for k := 1 to n
          c[i, j] := c[i, j] + a[i, k] * b[k, j]
        end
      end
    end
  end
```

Figure Q3 (a).

Answer:

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- (b) Consider the algorithm in **Figure Q3 (b)**.

```
Procedure sum(n: positive integer)
s := 0
for i := 1 to n
  for j := 1 to i
    s := s + j
  returns
```

Figure Q3 (b)

- (i) Suppose that procedure sum is started with input $n = 4$. Identify what number is returned by the algorithm? (2 marks)

Answer:

- (ii) Evaluate the worst-case time complexity of procedure sum? (3 marks)

Answer:

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Q5 (a) For a graph in **Figure Q5** below,

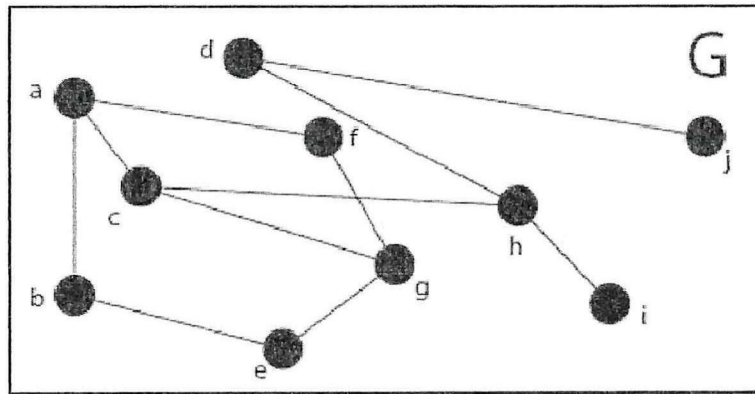


Figure Q5

- (i) List the vertices in G. (1 mark)
- (ii) List the edges in G. (1 mark)
- (iii) List the edges incident with vertex c. (1 mark)
- (iv) List the vertices adjacent to g. (1 mark)
- (v) Find a path from j to b. (2 marks)
- (vi) Find a cycle in G. (2 marks)

Answer:

(i)

(ii)

(iii)

(iv)

(v)

(vi)

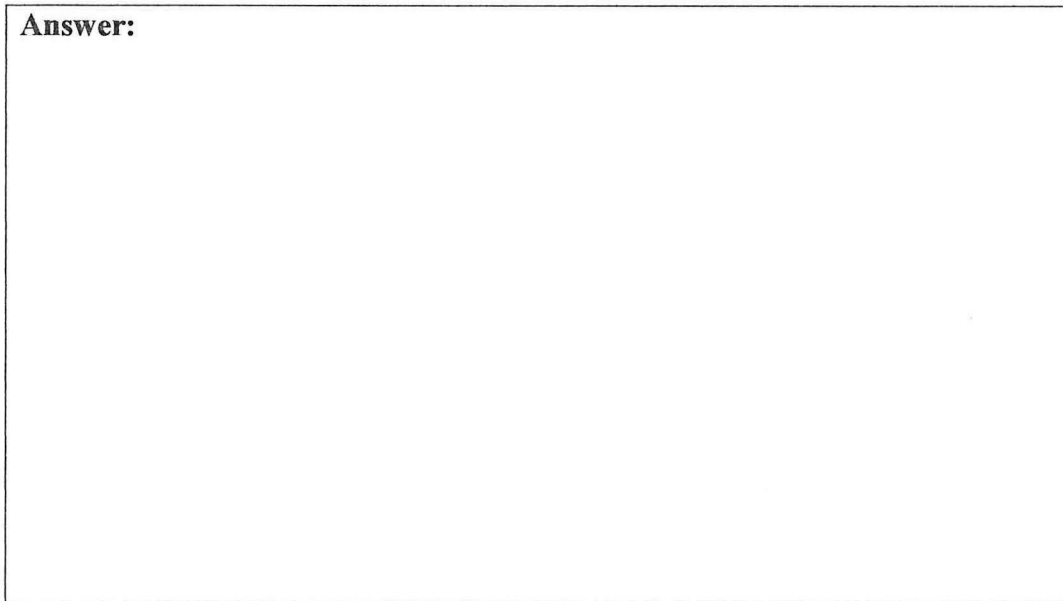
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(b) Wikipedia has five particularly interesting articles: Animal, Burrow, Chile, Desert, and Elephant. According to the report, the Animal article links to none of the others, the Burrow article links to Animal and Chile, the Chile article links to Desert, the Desert article links to Chile, and the Elephant article links to Animal and Desert. Strangely, no relation between Burrow and Elephant seems to be present.

(i) Illustrate the following situation as directed graph.

(6 marks)

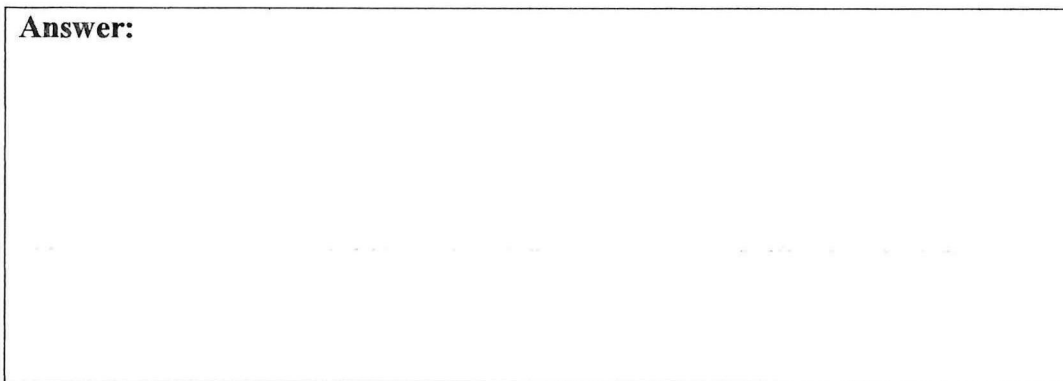
Answer:



(ii) Write an adjacency matrix for G in part (i).

(6 marks)

Answer:



- END OF QUESTIONS -

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