

## UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION **SEMESTER I SESSION 2019/2020**

COURSE NAME

: ALGEBRA

COURSE CODE

: BIC 10303

PROGRAMME CODE :

BIS / BIP / BIW / BIM

EXAMINATION DATE :

DECEMBER 2019 / JANUARY 2020

**DURATION** 

3 HOURS

INSTRUCTION : ANSWER ALL QUESTIONS

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THIS QUESTION PAPER CONSISTS OF FIVE (5) PAGES

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**SECTION A** 

State whether each of the following statement is TRUE or FALSE.

- Q1  $(2+\sqrt{5})$  is the conjugate of  $(5+\sqrt{2})$ .
- Q2 The equation 2x + 7 = 2(x + 5) has one solution.
- Q3 If  $10^y = x$  then  $\log x = y$ .
- Q4  $16x^5 + 3x^3 + 27x$  is a 3<sup>rd</sup> degree of polynomials which is known as cubic.
- Q5  $4\sqrt{k} 9$  is not a polynomial because  $k^{\frac{1}{2}}$  exponent is not a natural number.
- Q6 If a function and its inverse are functions, we say that it's "one-to-one".
- Q7 Let the closed interval [a, b] be the domain of function f. The domain of f(x 3) is given by the open interval (a, b).
- Q8 Let A and B be  $n \times n$  matrices. If A and B commute, then matrices  $A^2$  and B must commute.
- Q9 If A and B are symmetric  $n \times n$  matrices, then the product AB is also symmetric.
- Q10 If the system Ax = b has a unique solution, then A must be a square matrix. (10 marks)

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## **SECTION B**

Answer ALL questions.

Q11 (a) When  $G(x) = 2x^2 + mx + n$  is divided by x - 1 and x + 2, it leaves remainders 5 and 26 respectively. Find the value of m and n.

(4 marks)

- (b) Given that x + 2 is a factor of the polynomial for  $P(x) = ax^3 + bx^2 3x 2$  where a and b are constant.
  - (i) If P(x) is divided by x 3, the remainder is 70, find the values of a and b.

    (4 marks)
  - (ii) Based on your answer in Q11(b)(i), find the remainder when P(x) is divided by 2x + 3.

(4 marks)

(c) Divide  $x^3 + 5$  by x + 2

(4 marks)

- (d) Using proof by A, B and C, show that  $\frac{4}{x-3} + \frac{1-4x}{x^2+2}$  is partial fractions of  $\frac{13x+5}{x^3-3x^2+2x-6}$  (4 marks)
- Q12 Given a function  $f(x) = \sqrt{4-3x}$ 
  - (a) Show that f is a one-to-one function. Justify your answer.

(3 marks)

(b) Find the domain and range of f.

(3 marks)

(c) Determine the inverse function of f and state its domain and range.

(5 marks)

(d) Sketch the graph of f and  $f^{-1}$  on one diagram.

(4 marks)



Q13 If a rock falls from a height of 20 meters on earth, the height, H (in meters) after x seconds is approximately

$$H(x) = 20 - 4.9x^2$$

- (a) Determine the height of the rock when
  - (i) x = 1 second
  - (ii) x = 1.1 seconds
  - (iii) x = 1.2 seconds
  - (iv) x = 1.3 seconds

(8 marks)

- (b) Determine the fall time when the height is
  - (i) 15 meters
  - (ii) 10 meters
  - (iii) 5 meters

(6 marks)

- (c) Based on your answers in Q13(a) and Q13(b), sketch a graph of H in one diagram.

  (3 marks)
- (d) Solve the following equations  $(H \circ H)(x)$

(4 marks)

- Q14 Given that matrix  $A = \begin{pmatrix} 3 & 2x & x \\ 0 & x+1 & 6 \\ 0 & 0 & x \end{pmatrix}$  and |A| = 60
  - (a) Find the values of x.

(3 marks)

(b) Determine the cofactors and the adjoint matrix of A by using the positive value of x obtained in Q14(a).

(4 marks)

(c) Find the inverse matrix of A.



(3 marks)

Q15 If matrix 
$$A = \begin{pmatrix} 3 & -1 & 2 \\ -2 & 4 & 1 \\ 1 & -2 & -1 \end{pmatrix}$$
 and matrix  $B = \begin{pmatrix} 2 & 5 & 9 \\ 1 & 5 & 7 \\ 0 & -5 & -10 \end{pmatrix}$ ,

(a) find the product AB.

(4 marks)

(b) find x, y and z if matrix  $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \\ 5 \end{pmatrix}$  by using the Cramer's Rule.

(7 marks)

- Q16 Two companies A and B are awarding prize monies to their employees for leadership, innovation and punctuality. Company A decided to award a total prize monies of RM8500 for leadership, innovation and punctuality to four, five and two employees respectively. Company B decided to award a total prize monies of RM4550 for leadership, innovation and punctuality to three, two and one employees respectively. The total amount of prize monies awarded to each employee for leadership, innovation and punctuality is RM2150.
  - (a) If RMx, RMy and RMz represent the amount of prize monies awarded to an employee for leadership, innovation and punctuality respectively, write down a system of linear equations in x, y, z to represent the above information.

(3 marks)

- (b) Find the values of x, y and z, by using the Gauss-Jordan elimination method. (7 marks)
- (c) With the same total amount of prize monies spent by companies A and B, determine whether it is possible for company A to award three employees for innovation instead of five employees. Justify your answer.

(3 marks)

-END OF QUESTIONS -

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