Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

COURSE NAME	:	DISCRETE STRUCTURE
COURSE CODE	:	BIT 11003
PROGRAMME CODE	:	BIT
EXAMINATION DATE	:	DECEMBER 2017 / JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS B) PLEASE WRITE YOUR ANSWERS IN THIS QUESTION BOOKLET C) CALCULATOR CANNOT BE USED

THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

Q1 (a) Given, $P(x)$: x is even, $Q(x)$: x is prime number and $R(x,y)$: $x + y$ is even. The variables of x and y represent integers. Write an English sentence for each of the following.

(i) $\forall x \exists y\ R(x,y)$

(2 marks)

(ii) $\neg (\exists x\ P(x))$

(2 marks)

(iii) $\forall x (\neg\ Q(x))$

(2 marks)

(b) State the converse, inverse, and contra-positive of each of the following implications:

i. Maria will find a good job when she learns discrete mathematics

(3 marks)

ii. A necessary condition for this computer program to be correct is that it not produce error messages during translation

(3 marks)

(c) Show that $(p \vee \sim q) \wedge (\sim p \vee \sim r \vee q) \wedge (p \vee \sim p)$ is not a tautology.

(8 marks)

Q2 (a) Prove that $49^n + 16n - 1$ is divisible by 64.

(10 marks)

(b) Use mathematical induction to prove that

$$\sum_{i=1}^{n} i^4 = \frac{n(n+1)\,(2n+1)\,(3n^2+3n-1)}{30}$$

is true for all positive integers n.

(10 marks)

Q3 (a) Let $A = \{1, 2, 4, 5, 7, 8\}$ and $B = \{x | (x \in Z^+) \wedge (x < 10)\}$. Write the element(s) for the following sets.

(i) $A \cup B$

(1 mark)

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(ii) $A \cup \emptyset$

(1 mark)

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(iii) $B - A$

(1 mark)

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(b) Let A and B be sets. The cross product of A and B is the set $A \times B = \{(a, b) | a \in A \wedge b \in B\}$.

Theorem: If $|A| = m$ and $|B| = n$ then $|A \times B| = m \times n$.

Use an example and demonstrate that the above theorem is true.

(3 marks)

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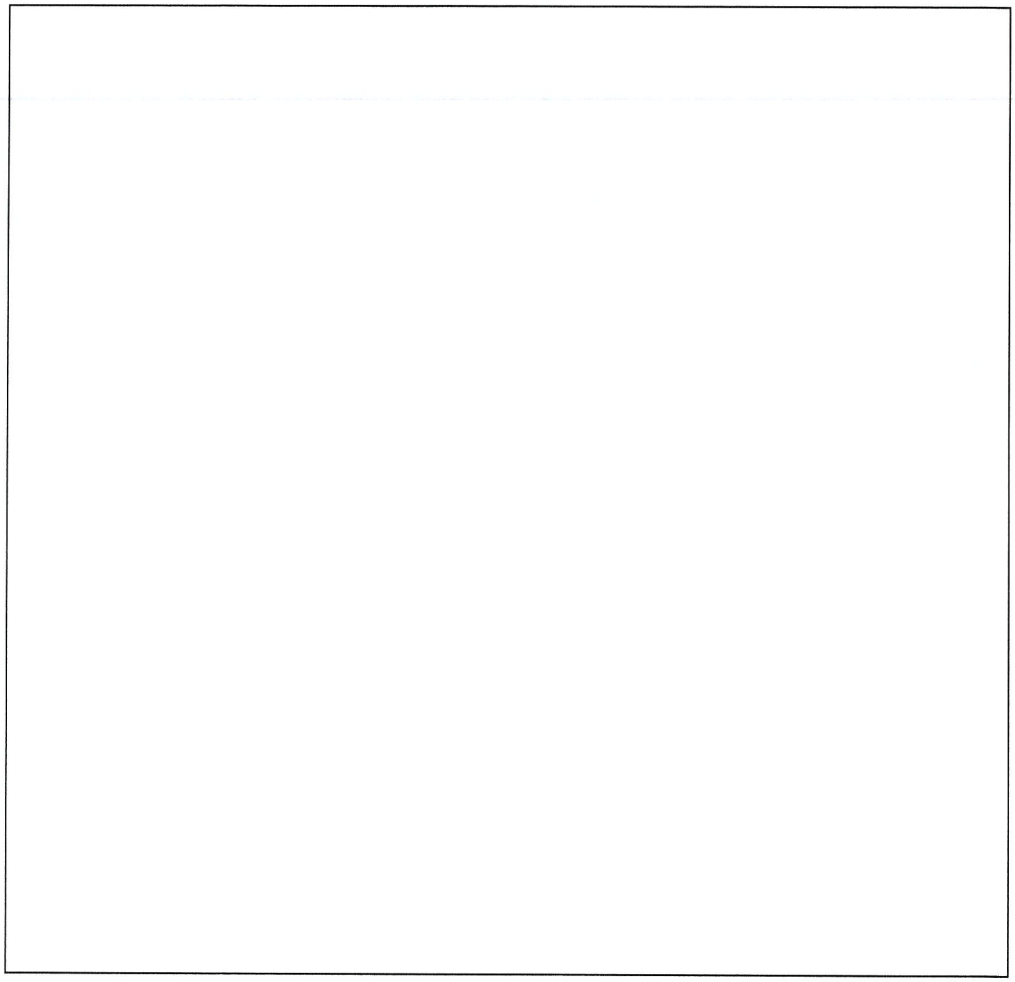
(c) Let $U = \{d, i, s, c, r, e, t, e, m, a, t, h, e, m, a, t, i, c, s\}$ be the universal set. Let $S = \{x \in U \mid x \in \{s, e, c, r, e, t, s\}\}, T = \{x \in U \mid x \in \{t, h, e, m, e, s\}\}$ and $C = \{x \in U \mid x \in \{t, a, c, t, i, c, s\}\}$ be the subsets of the universal set.

(i) Draw a Venn diagram describing U, S, T and C.

(5 marks)



(ii) Write down the elements for the following sets.

$S \cup T$ and $T - C$

(2 marks)

Q4 (a) Identify two ways to represent a relation.

(2 marks)

(b) Let $A = [1, 2, 3, 4, 5\}$ and let R be the relation on A defined as follows:

$R = \{(1, 3), (1, 4), (2, 1), (2, 2), (2, 4), (3, 5), (5, 2), (5, 5)\}$

(i) Write down the matrix representation of R.

(5 marks)

(ii) Draw the graphical representation of R.

(3 marks)

Q5 (a) Estimate worst case, best case and average case of algorithm in **Figure Q5(a).**

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Algorithm:
        k ←1
        found← false
        while (k ≤ n) and (not found) do
        if a_k = x then
                found ←true
        else
                k← k + 1
        endif
        endwhile
        { k > n or found }
        if found then { x found }
                idx ←k
            else
                idx← 0 { x not found }
        end if
```

Figure Q5(a)

(8 marks)

(b) Create an algorithm with only one looping involve using $T(n) = 4n + 2$.

(8 marks)

Q6 (a) Find an Euler circuit from vertex *A* as presented in **Figure Q6(a)**.

(9 marks)

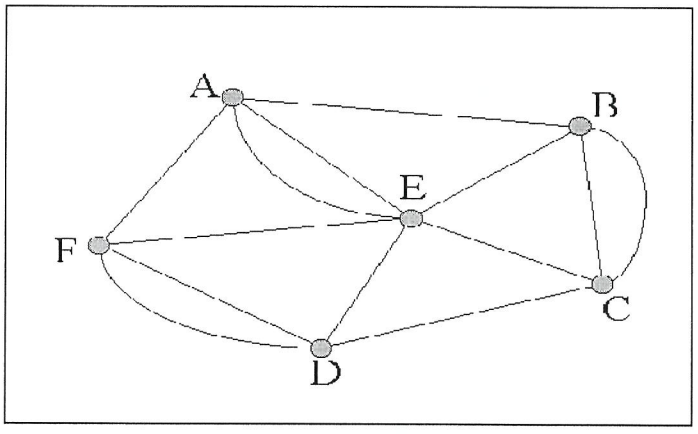

Figure Q6(a)

(b) Answer Q6(b) (i)-(ii), based on **Table 1.** A travelling salesman wants to visit 5 cities exactly once and return to his starting point. Suppose that the salesman wants to visit D, T, K, G, and S and its distances in miles are presented in **Table 1.**

Table 1 : Distances in miles for 5 cities

Cities/Miles	D	T	K	G	S
D	0	58	135	147	98
T	58	0	133	167	142
K	135	133	0	56	137
G	147	167	56	0	113
S	98	142	137	113	0

(i) Draw a connected graph between cities.

(5 marks)

(ii) Find the minimum total distance in his travelling based on *Hamiltonian* circuit approach. List at least 12 different circuits your answer.

(7 marks)

- END OF QUESTION -