Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2013/2014

COURSE NAME	:	DISCRETE STRUCTURE
COURSE CODE	:	BIC10103
PROGRAMME	:	1 BIS/ 1 BIP/ 1 BIW/ 1 BIM
EXAMINATION DATE	:	DECEMBER 2013/JANUARY 2014
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

Q1 (a) Determine the truth value for the following propositions based on scenario in Figure **Q1(a).**

> Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars.

FIGURE Q1(a)

(i) Quixote Media had the largest annual revenue.

(1 mark)

(ii) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.

(1 mark)

(iii) Acme Computer had the largest net profit or Quixote Media had the largest net profit.

(1 mark)

(iv) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.

(1 mark)

(v) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

(1 mark)

(b) Let p and q be the propositions

p : It is below freezing.
q : It is snowing.

Write the following propositions using p , q and logical connectives (including negations).

(i) It is below freezing and snowing.

(1 mark)

(ii) It is below freezing but not snowing.

(1 mark)

(iii) It is not below freezing and it is not snowing.

(1 mark)

(iv) It is either snowing or below freezing (or both).

(1 mark)

(v) If it is below freezing, it is also snowing.

(1 mark)

(vi) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

(1 mark)

(c) State the converse, contrapositive, and inverse for the following conditional statements.

(i) If it snows today, I will ski tomorrow.

(3 marks)

(ii) I come to class whenever there is going to be a quiz.

(3 marks)

(iii) A positive integer is a prime only if it has no divisors other than 1 and itself.

(3 marks)

Q2 (a) Construct truth table for the following logical expressions:

(i) $p \oplus q = (p \vee q) \wedge \neg(p \wedge q)$

(4 marks)

(ii) $p \vee q \wedge [(\neg p) \wedge (\neg q)]$

(4 marks)

(iii) $(p \rightarrow q) \wedge \neg r$

(8 marks)

(b) Prove the following statements, given that
U= {U,N,I,V,E,R,S,I,T,Y} **S**={U,N,I,T,Y} **T**={S,E,R,V}

(i) S **AND** T equal of T **AND** S

(1 mark)

(ii) S **OR** T equal of T **OR** S

(1 mark)

(iii) Whole Compliment of I

(1 mark)

(iv) Double Compliment of T equal T

(1 mark)

Q3 (a) Draw the Venn diagrams for each of these combinations of the sets A,B, and C.

(i) $A \cap (B - C)$

(2 marks)

(ii) $(A \cap B) \cup (A \cap C)$

(2 marks)

(iii) $(A \cap B) \cup (A \cap C)$

(2 marks)

(b) Let $A = \{1, 2, 3, 4\}$ and $B = \{0, 3, 5\}$. Find

(i) $A \cup B$

(1 mark)

(ii) $A \cap B$

(1 mark)

(iii) $A - B$

(1 mark)

(iv) $B - A$

(1 mark)

(c) Let $A = \{a, b, c, d, e, f\}$, $B = \{e, f, g, h, i\}$ and $C = \{x \mid x \text{ is a vowel}\}$.

(i) Draw a Venn diagram consists of set A, B and C.

(2 marks)

(ii) Find $A \cup (B \cap C)$, $(A \cup B) \cap C$, and $(A \cup B) \cap (A \cup C)$. Which of these sets are equal?

(3 marks)

(iii) Find $A \cap (B \cup C)$, $(A \cap B) \cup C$, and $(A \cap B) \cup (A \cap C)$. Which of these sets are equal?

(3 marks)

(iv) Find $A - (B - C)$ and $(A - B) - C$. Prove either these sets are equal or not?

(2 marks)

Q4 (a) Find relation R, Domain(R), Range(R) for the following questions

(i) Let A = {2, 3, 4} and B = {3, 4, 5, 6, 7}, given that aRb if and only if a divides b.

(3 marks)

(ii) Let A = {1, 2, 3, 4}, given that aRb if and only if $a \leq b$.

(3 marks)

(b) Determine whether the relation R on the set of all Web pages is reflexive, symmetric, anti-symmetric, and/or transitive, where $(a, b) \in R$ if and only if

(i) everyone who has visited Web page a has also visited Web page b.

(1 mark)

(ii) there are no common links found on both Web page a and Web page b.

(1 mark)

(iii) there is at least one common link on Web page *a* and Web page *b*.

(1 mark)

(iv) there is a Web page that includes links to both Web page *a* and Web page *b*.

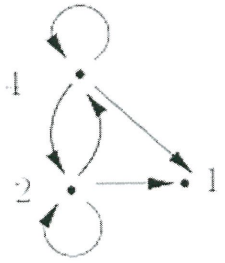
(1 mark)

(c) List the ordered pairs for each of the following relations , (i) to (v).

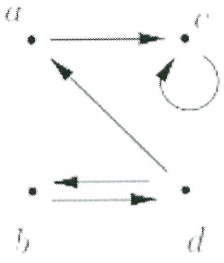
(i)

(ii)

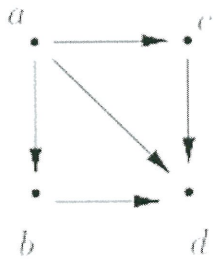
(iii)



(iv)



(v)



(5 marks)

(d) Prepare a table based on answer in **Q4(c)**, to show whether the relations are reflexive, symmetric, anti-symmetric and/or transitive.

(5 marks)

Q5 (a) Let P(n) be the statement that $1^2 + 2^2 + \cdots + n^2 = n(n + 1)(2n + 1)/6$ for the positive integer n.

(i) What is the statement P(1)?

(1 mark)

(ii) Show that P(1) is true, completing the basis step of the proof.

(1 mark)

(iii) Show the inductive hypothesis for P(k+1)

(1 mark)

(iv) Show the early inductive step

(1 mark)

(v) Complete the inductive step answered in **Q5(iv)**,

(5 marks)

(vi) Conclude the inductive steps answered in **Q5(iv)** and **Q5(v)**.

(1 mark)

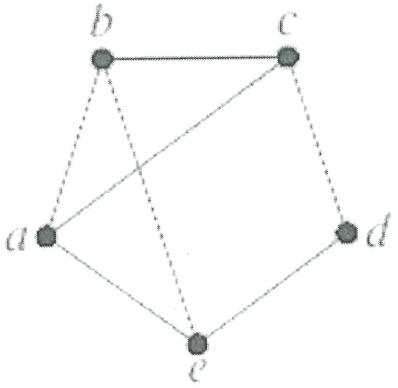
(b) Prove the following formula by induction:

$$2 + 4 + \cdots + 2n = n(n+1)$$

(10 marks)

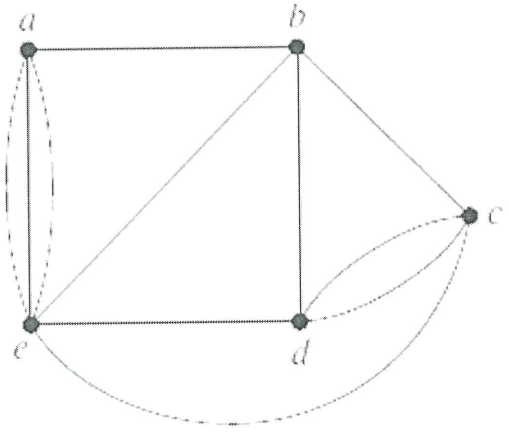
Q6 (a) Determine whether the following directed graph shown below has an Euler circuit or Euler path or none. Construct an Euler circuit or Euler path if one exists.

(i)



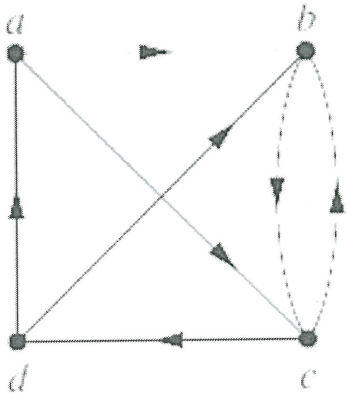
(2 marks)

(ii)



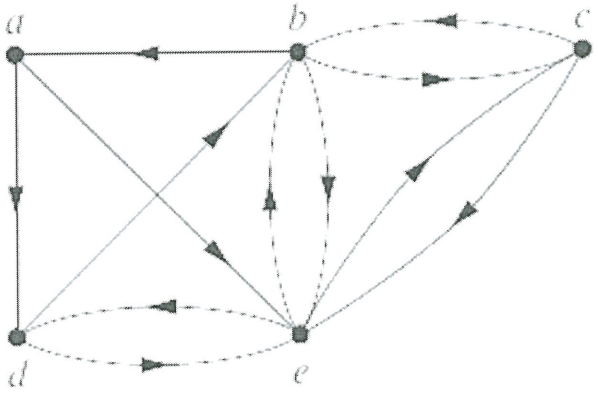
(2 marks)

(iii)



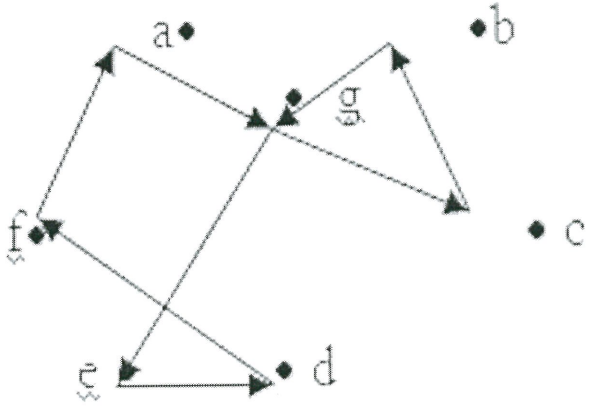
(2 marks)

(iv)



(2 marks)

(v)



(2 marks)

(b) Find the in-degree and out-degree of each vertex in the graph with directed edges in Figure **Q6(b).**

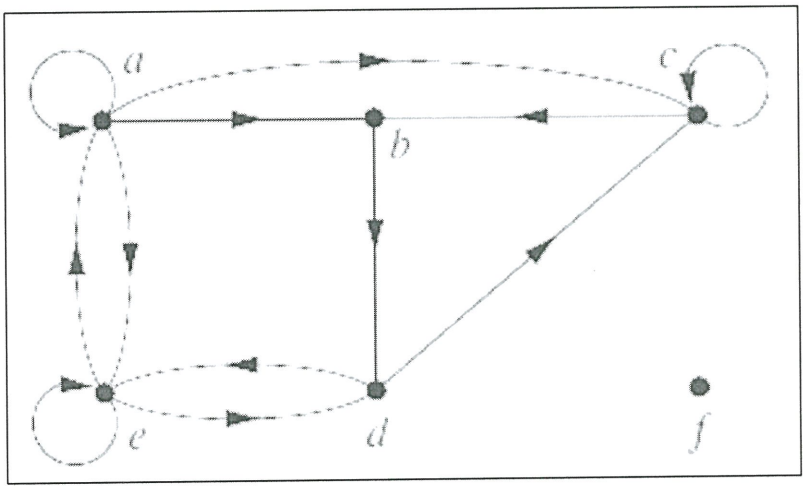


FIGURE Q6(b)

(10 marks)

Q7 (a) Prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots\ldots\ldots\ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

is true for n, k, and k+1 by using mathematical induction methods

(10 marks)

(b) Answer the following questions based on Figure **Q7(b)(i)** and Figure **Q7(b)(ii)** with edge lengths as indicated:

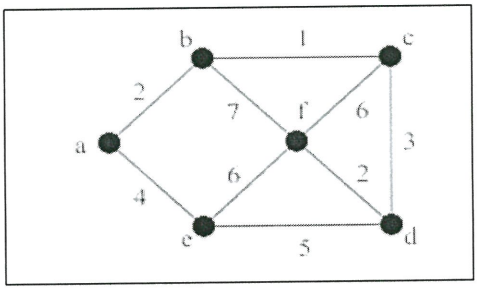


FIGURE Q7(b)(i)

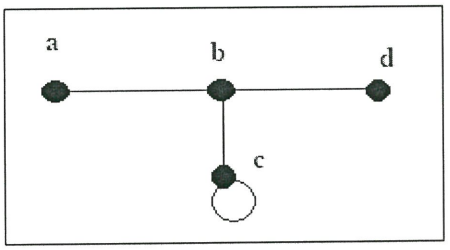


FIGURE Q7(b)(ii)

(i) Find the length of the shortest path from vertex **a** to each of the other vertices from Figure **Q7(b)(i)**.

(3 marks)

(ii) Find the shortest path from vertex **a** to vertex **f** from Figure **Q7(b)(i)**.

(2 marks)

(iii) Find the degree of each vertex from Figure **Q7(b)(ii)**.

(2 marks)

(iv) Sketch a graph representing a binary relation, given that
R := f(0; 0);(1; 1);(2; 2);(0; 3);(1; 3);(2; 3);(3; 3)

(3 marks)

- END OF QUESTION -