Universiti Tun Hussein Onn Malaysia

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER I
## SESSION 2019/2020

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|---|---|---|
| COURSE NAME | : | CIVIL ENGINEERING MATHEMATICS III |
| COURSE CODE | : | BFC 24103 |
| PROGRAMME CODE | : | BFF |
| EXAMINATION DATE | : | DECEMBER 2019/ JANUARY 2020 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWER **ALL** QUESTIONS |

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

**Q1** (a) Show that $\lim_{(x,y)\to(0,0)} \frac{x^3-xy^2}{x^2+y^2} = 0$

(5 marks)

(b) Given $w = x^2 - 2y^2 + z^3, x = \sin t, y = e^t,$ and $z = 3t$. Find $\frac{dw}{dt}$ using chain rule.

(5 marks)

(c) Find $\int_0^4 \int_0^{\sqrt{y}} xe^{y^2}\, dxdy$

(5 marks)

(d) Find the directional derivative of the function $f(x,y,z) = xy\sin z$ at the point $(1, 2, {}^{\pi}/_{2})$ in the direction of the vector $\vec{a} = \vec{\imath} + 2\vec{\jmath} + 2\vec{k}$.

(5 marks)

**Q2** (a) Determine the volume of solid cylinder $x^2 + z^2 = 9$ between the plane $y = 2$ and $y = 4$.

(7 marks)

(b) A lamina has shape of the region in the first quadrant that is bounded by the graphs of $y = \sin x$, $y = \cos x$, between $x = 0$ and $x = \pi/4$. Determine the centre of mass if the density is $y$.

(7 marks)

(c) Determine limit of vector function of the following:

i. $\mathbf{r}(t) = \ell^t\mathbf{i} + \frac{\sin(t)}{t}\mathbf{j}$

ii. $\mathbf{r}(t) = \sin^2(t)\,\mathbf{i} + \tan(t)\,\mathbf{j} + \frac{1}{t}\mathbf{k}$

iii. $\mathbf{r}(t) = \frac{2}{t}\mathbf{i} + \frac{t^3}{2t^3-8}\mathbf{j} + \ell^{-t}\mathbf{k}$

(6 marks)

(d) Determine a unit vector in the direction in which $f(x,y) = \sqrt{x^2+y^2}$ increases most rapidly at point P(5,-2).

(5 marks)

(e) If the temperature at any point in a homogeneous body is given by $T = e^{xy} - xy^2 - x^2yz$, determine the direction of the greatest drop in temperature at the point $(1, -1, 2)$.

(5 marks)

**Q3** (a) The elevation angle at of the top of a tower is found to be $30° \pm 0.5°$ from a point of $300 \pm 0.1$ m from the base. Calculate the height of this tower.

(8 marks)

(b) Sketch the region R enclosed between:

$$y = -x^2 + 4$$
$$y = \frac{x}{2} + 1$$
$$y = -x + 3$$
$$x \geq 0$$

(8 marks)

(c) Given the vector-valued function $\mathbf{r}(t) = 5\cos t\,\mathbf{i} + 5\sin t\;\mathbf{j}$. Calculate its unit tangent vector and principal unit normal vector at $t = {}^{\pi}/_{4}$. Then, sketch the graph of $\mathbf{r}(t)$, $\mathbf{T}(\pi)$ and $\mathbf{N}(\pi)$ in the same axis.

(10 marks)

(d) By applying Green's theorem show that if a region $S$ in the plane has boundary $C$, where $C$ is a piecewise smooth, simple closed curve, then the area of $S$ is given by

$$A(S) = \oint_C (-y\,dx + x\,dy)$$

Use result from above to calculate the area enclosed by the ellipse ${}^{x^2}/_{a^2} + {}^{y^2}/_{b^2} = 1$ under the given parametric equations

$x = a\sin t,$ $y = b\ \sin t,$ $0 \leq t \leq 2\pi$

(14 marks)

**Q4** Given $\iiint_E e^{-x^2-z^2}\,dV$ where $E$ is the region between the two cylinders $x^2 + z^2 = 4$ and $x^2 + z^2 = 9$ with $1 \leq y \leq 5$ and $z \leq 0$. Convert the given integral into cylindrical coordinates and analyse it.

(10 marks)

**- END OF QUESTIONS –**

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**Formulae**

Tangent Plane: $z - z_0 = f_x(x_o, y_0)(x - x_0) + f_y(x_o, y_0)(y - y_0)$

Local Extreme Value: $G(x,y) = f_{xx}(x,y) \times f_{yy}(x,y) - [f_{xy}(x,y)]^2$

| Case | $G(a,b)$ | Result |
|---|---|---|
| 1 | $G(a,b) > 0$<br>$f_{xx}(a,b) < 0$ | $f(x,y)$ has a local maximum value at $(a,b)$ |
| 2 | $G(a,b) > 0$<br>$f_{xx}(a,b) > 0$ | $f(x,y)$ has a local minimum value at $(a,b)$ |
| 3 | $G(a,b) < 0$ | $f(x,y) has\ a\ saddle\ point\ at\ (a,b)$ |
| 4 | $G(a,b) = 0$ | inconclusive |

Polar coordinate: $x = r\cos\theta, y = r\sin\theta, \theta = \tan^{-1}(\frac{y}{x})$ and
$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$

Cylindrical coordinate: $x = r\cos\theta$, , $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)d\ dz\ dr\ d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta, y = \rho\sin\phi\sin\theta, z = \rho\cos\theta, x^2 + y^2 + z^2 = \rho^2, 0 \ll \theta \ll 2\pi, 0 \ll \phi \ll \pi$ and $\iiint f(x,y,z)dV = \iiint f(\rho,\phi,\theta)\rho^2\sin\phi\ d\rho d\phi d\theta$

For lamina

Mass, $m = \iint_R \delta(x,y)\ dA$

Moment of mass: y-axis: $M_y = \iint_R x\delta(x,y)\ dA$ x-axis, $M_x = \iint_R y\delta(x,y)\ dA$

Center of mass, $(\bar{x}, \bar{y}) = (\frac{M_y}{m}, \frac{M_x}{m})$

Centroid for homogenous lamina: $\bar{x} = \frac{1}{area}\iint_R x\ dA$ $\bar{y} = \frac{1}{area}\iint_R y\ dA$

Moment inertia:

Y-axis: $I_y = \iint_R x^2\,\delta(x,y)\ dA$ x-axis: $I_x = \iint_R y^2\,\delta(x,y)\ dA$

Z-axis (or origin): $I_z = I_0 = \iint_R (x^2 + y^2)\,\delta(x,y)\ dA = I_x + I_y$

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For solid

Mass, $m = \iiint_G \delta(x,y)\, dV$

Moment of mass:

yz-plane: $M_{yz} = \iiint_G x\,\delta(x,y,z)\, dV$

xz-plane: $M_{xz} = \iiint_G y\,\delta(x,y,z)\, dV$

xy-plane: $M_{xy} = \iiint_G z\,\delta(x,y,z)\, dV$

Center of gravity, $(\bar{x}, \bar{y}, \bar{z}) = (\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m})$

Moment inertia:

$$I_y = \iiint_G (x^2 + z^2)\,\delta(x,y,z)\, dV$$

$$I_x = \iiint_G (y^2 + z^2)\,\delta(x,y,z)\, dV$$

$$I_z = \iiint_G (x^2 + y^2)\,\delta(x,y,z)\, dV$$

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Directional derivative: $D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot u$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, then the divergence of $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The curl of

$$\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let $C$ is a smooth curve given by $r(t) = x(t)i + y(t)j + z(t)k$, $t$ is parameter, then

The unit tangent vector; $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The binormal vector: $B(t) = T(t) \times N(t)$

The curvature: $K = \frac{\mathbf{T}'(t)}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}' \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

The radius of curvature: $\rho = 1/K$

Green Theorem: $\oint_C M(x,y)\, dx + N(x,y)\, dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} dA$

Gauss Theorem: $\iint_\sigma \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_G \nabla . \mathbf{F}\, dV$

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Stokes Theorem: $\oint_C \mathbf{F}.d\mathbf{r} = \iint_\sigma (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$

Arc length, If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t\epsilon[a, b]$, then the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\|\, dt = \int_a^b \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$$

If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t\epsilon[a, b]$, then the arc length

$$s = \int_a^b \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2 + \left(z'(t)\right)^2}\, dt$$