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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	CIVIL ENGINEERING MATHEMATIC II
COURSE CODE	:	BFC14003
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 (a) Find the Laplace Transform of the following functions

(i) $f(t) = t^2 \sinh 4t$

(3 marks)

(ii) $f(t) = \left(t + t^2 + \frac{1}{6}t^3\right)e^t$

(3 marks)

(b) Determine the solution of the following inverse Laplace Transform

(i) $\mathcal{L}^{-1}\left[\frac{1}{s^3} + \frac{6}{s^4 + 4}\right]$

(2 marks)

(ii) $\mathcal{L}^{-1}\left[\frac{5s}{s^4 - 4}\right]$

(4 marks)

(c) Solve the initial value problem $y' - 5y = -e^{-2t}, y(0) = 3$ by using Laplace Transform

(8 marks)

Q2 (a) Solve a Fourier Series for $f(x) = x,\ -2 < x < 2,\ f(x+4) = f(x)$.

(12 marks)

(b) Find the MacLaurin Series for

(i) $f(x) = e^x$

(4 marks)

(ii) $g(x) = \sin x$

(4 marks)

Q3 (a) Solve $2xy + 6x + (x^2 - 4)y' = 0$ by using separable equation method.

(5 marks)

(b) Evaluate the solution for $xy' + 2y = 4x^2$ by using linear equation method.

(7 marks)

(c) A spring with a *3* kg mass is held stretched *0.6* m beyond its natural length by a force of *20* N. If the spring begins at its equlibrium position but a push gives it an initial velocity of *1.2* m/s, find the position of the mass after *t* seconds.

(8 marks)

Q4 (a) Determine the solution for $xy^2dy = (x^3 + y^3)\,dx$ by using homogenous equation method.

(6 marks)

(b) The temperature of the object is 95°*C*, the ambient temperature is 20 °*C*, and at exactly *20* minutes after the object began to cool, its temperature is 70°*C*.

(i) Develop and equation that relates temperature, T and time, t, given that the general equation of Newton's Law of Cooling is as below

$$\frac{dT}{dt} = -K(T - A)$$

where
K = constant
A = ambient/ room temperature

(12 marks)

(ii) Determine the object's temperature at *45* minutes after the cooling began.

(2 marks)

Q5 (a) Solve $\frac{d^2y}{dx^2} + 2y = \sin(e^x)$ by using method of variation parameters.

(12 marks)

(b) Solve $y'' + y = \sin x$ by using method of undetermined coefficient.

(8 marks)

– END OF QUESTIONS –

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The following information may be useful. The symbols have their usual meaning.

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

Fourier Series

Fourier series expansion of periodic function with period $2L$	Fourier half-range series expansion
$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$	$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$
where	where
$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$	$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$
$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$	$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$
$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$	$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$		

Integration by part

$$\int u\,dv = uv - \int v\,du$$

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Differentiation and Integration

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C$, $n \neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\|+C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\, x]=-\text{cosec x} \cot x$	$\int \text{cosec}\, x \cot x\,dx = -\text{cosec}\, x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=\text{-cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\, x]=\text{-sech}\, x \tanh x$	$\int \text{sech}\, x \tanh x\,dx = -\text{sech}\, x + C$
$\frac{d}{dx}[\text{cosech}\, x]=\text{-cosech}\, x \coth x$	$\int \text{cosech}\, x \coth x\,dx = -\text{cosech}\, x + C$