UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS I
COURSE CODE	:	BFC13903
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2019/ JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

Q1 (a) Find the derivative of $y = 2e^{(\cos x)(\sin(5x))}$

(3 marks)

(b) Calculate $\dfrac{d^2y}{dx^2}$ as a function of t if $x = t - t^2$ and $y = t - t^3$

(7 marks)

(c) Solve $\dfrac{dy}{dx}$ for $x^2(x-y)^2 = x^2 - y^2$ by using implicit differentiation.

(10 marks)

Q2 (a) **FIGURE 2(a)** shows a conical filter. Suppose the liquid is to be cleared by allowing it to drain through a conical filter that is 16 cm high and has a radius of 4 cm at the top. The liquid if forced out of the cone at a constant rate of 2 cm^3/min.

(i) Express the rate of change of the liquid depth.

(6 marks)

(ii) Determine the rate of change of the liquid when the liquid in the cone is 8 cm deep.

(4 marks)

(b) Given $f(x) = x^4 - 6x + 2$. By using second derivative test, estimate the critical number, concavity and relative maximum or minimum of the function.

(6 marks)

(c) Find equation of the tangent line to $x^2 + 3xy + y^5 = 5$ at [1,1].

(4 marks)

Q3 (a) Determine the integrals of the followings:

(i) $\int (3x^2 + 5x^{1/2} + \frac{x^4}{5})dx$

(3 marks)

(ii) $\int_5^{10} (20 + \sec^2 x)\, dx$

(5 marks)

(b) Evaluate the following using suitable integration method:

(i) $\int [x \sin(5 - x^2)] dx$

(5 marks)

(ii) $\int \frac{15x+9}{x+5}\, dx$

(7 marks)

Q4 (a) Calculate the area of the region between the graphs of $f(x) = 2x^2 + 3x$ and $g(x) = 2x$ at [0, 5].

(5 marks)

(b) Determine the area of surface that is generated by revolving the portion of curve $y = \sqrt{4 - x^2}$ between $-1 \leq x \leq 1$ about x- axis.

(5 marks)

(c) Estimate the arc length of the graph of the function $y = \frac{2}{3}(x^2 + 1)^{\frac{3}{2}}$ over the interval [0, 1].

(5 marks)

(d) Estimate the arc length of parametric curve of:

$x(t) = \cos t,\ y(t) = \sin t$ between $0 \leq t \leq \pi$.

(5 marks)

Q5 (a) Integrate the following function;

i) $\int (\frac{9}{1+9x^2})\, dx$

(4 marks)

ii) $\int (\frac{4}{x\sqrt{x^4-4}})\, dx$

(6 marks)

(b) Differentiate $y = cosh^{-1}(x^2 + 1) + sinh^{-1}(coth\, x^2)$ with respect to x.

(10 marks)

– END OF QUESTIONS –

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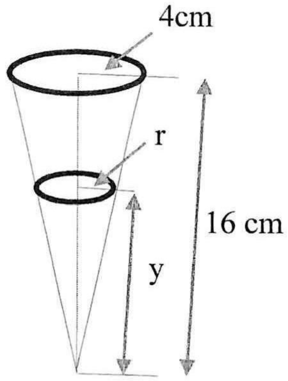

FIGURE Q2(a)

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Formulae

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C, \quad n \neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\, x]=-\text{cosec}\, x \cot x$	$\int \text{cosec}\, x \cot x\,dx = -\text{cosec}\, x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\, x]=-\text{sech}\, x \tanh x$	$\int \text{sech}\, x \tanh x\,dx = -\text{sech}\, x + C$
$\frac{d}{dx}[\text{cosech}\, x]=-\text{cosech}\, x \coth x$	$\int \text{cosech}\, x \coth x\,dx = -\text{cosech}\, x + C$

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Formulae

Trigonometric	Hyperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \csc^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \operatorname{csch}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$	**Inverse Hyperbolic**
Logarithm	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$, any x
$a^x = e^{x \ln a}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$, $x \geq 1$
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1 < x < 1$

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Formulae

Integration of Inverse Functions
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+C$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$
$\int \frac{dx}{\lvert a\rvert\sqrt{a^2-x^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$
$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right)+C, \quad a>0$
$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)+C, \quad x>a$
$\int \frac{dx}{x^2-a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C, & \lvert x\rvert<a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+C, & \lvert x\rvert>a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{x}{a}\right)+C, \quad 0<x<a$
$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\text{cosech}^{-1}\left(\frac{x}{a}\right)+C, \quad 0<x<a$

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Formulae

Differentiation of Inverse Functions	
y	$\frac{dy}{dx}$
$\sin^{-1} u$	$\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad \lvert u\rvert < 1$
$\cos^{-1} u$	$-\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad \lvert u\rvert < 1$
$\tan^{-1} u$	$\frac{1}{1+u^2}\frac{du}{dx}$
$\cot^{-1} u$	$-\frac{1}{1+u^2}\frac{du}{dx}$
$\sec^{-1} u$	$\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}, \quad \lvert u\rvert > 1$
$\text{cosec}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}, \quad \lvert u\rvert > 1$
$\sinh^{-1} u$	$\frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$
$\cosh^{-1} u$	$\frac{1}{\sqrt{u^2-1}}\frac{du}{dx}, \quad \lvert u\rvert > 1$
$\tanh^{-1} u$	$\frac{1}{1-u^2}\frac{du}{dx}, \quad \lvert u\rvert < 1$
$\coth^{-1} u$	$-\frac{1}{1-u^2}\frac{du}{dx}, \quad \lvert u\rvert > 1$
$\text{sech}^{-1} u$	$-\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}, \quad 0 < u < 1$
$\text{cosech}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx}, \quad u \neq 0$

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Formulae

Area between two curves

Case 1- Integrating with respect to *x*: $A = \int_a^b [f(x) - g(x)]dx$

Case 2- Integrating with respect to *y*: $A = \int_c^d [f(y) - g(y)]dy$

Area of surface of revolution

Case 1- Revolving the portion of the curve about *x*-axis: $S = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Case 2- Revolving the portion of the curve about *y*-axis: $S = 2\pi \int_c^d x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve- Revolving the curve about *x*-axis: $S = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Parametric curve- Revolving the curve about *y*-axis: $S = 2\pi \int_c^d x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arc length

x-axis: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

y-axis: $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Curvature

Curvature, $\kappa = \frac{\left|\frac{d^2y}{dx^2}\right|}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

Radius of curvature, $\rho = \frac{1}{\kappa}$

Curvature of parametric curve

Curvature, $\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$

Radius of curvature, $\rho = \frac{1}{\kappa}$