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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2019/2020

COURSE NAME	:	CIVIL ENGINEERING MATHEMATIC IV
COURSE CODE	:	BFC24203
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2019 / JANUARY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Prove that there is a root in the following equations with the given interval by using Intermediate Value Theorem.

(i) $x+\ln x=0$ for [0.2, 1] (3 marks)

(ii) $4x^3+3x^2+2x+11=0$ for [-2, -1] (3 marks)

(b) Given the function $f(x)=\sin 2x+x^3-3$. Approximate root of $f(x)$ by using Newton Raphson method. Iterate until $|f(x_i)|<\varepsilon=0.005$.

(10 marks)

(c) Arrange the data of $f(0)=2$, $f(1)=1$ and $f(4)=4$ in table form. Estimate $f(3)$ by using Lagrange interpolation.

(9 marks)

Q2 (a) **TABLE Q2 (a)** shows the velocity, v of a truck at various times from an intersection during the morning rush hour. Given velocity, $v=x'(t)$. By taking h=2 minutes, approximate velocity of the truck at times $t=4s$ by using all appropriate difference formulas.

(9 marks)

(b) The total force of the water pressure (N) from 0 m to 30 m elevation height of the dam can be expressed as follow:

$$\int_0^{30}\sqrt{\rho g+(50-x)}\ dx$$

where ρ is the density of the water which it is assumed to be a constant of 1000 kg/m^3 while g is acceleration due to gravity of 9.81 m/s^2 and x is the elevation of water surface above the reservoir bottom. Compare the total force of water pressure using 2-points and 3-points Gauss Quadrature. Do the calculation in 3 decimal places using chopping-off technique.

(16 marks)

Q3 (a) Find the smallest eigenvalue (in absolute) for matrix A by shifted power method with initial eigenvector $v^{(0)} = (1 \quad 0 \quad 1)^T$. Analyse computation in three (3) decimal places and use the given vector $v^{(0)}$ as the trail value. Stop the iteration until $|m_{k+1} - m_k| \leq 0.005$.

$$A = \begin{pmatrix} 5 & -2 & 1 \\ -2 & 2 & 0 \\ 2 & -2 & 3 \end{pmatrix}$$

(10 marks)

(b) A lake was polluted by bacteria with the initial concentration of 10^6 parts/m^3. After few weeks the concentration of the bacteria reduces as the fresh water enters the lake. The differential equation that represents the concentration C of the bacteria as a function of time (in weeks) is given by:

$$\frac{dC}{dt} + 0.05C = 0, \qquad C(0) = 10^6$$

(i) By using the Euler's method, evaluate the concentration of the bacteria after 10 weeks. Take a step size of 1 week starting from initial 0 week. Show your answer in four (4) decimal places.

(10 marks)

(ii) Calculate the absolute and relative error if the exact equation for the contamination is given as $C(t) = 10^6 e^{-0.05t}$.

(5 marks)

Q4 Given the heat equation $\frac{\partial u}{\partial t} = 0.75\frac{\partial^2 u}{\partial x^2}$ with the following initial and boundary conditions. Solve the heat equation at first level only by taking $\Delta x = h = 0.5$ and $\Delta t = k = 0.25$ for $0 \leq x \leq 3$.

Boundary conditions: U(1, t) = 100 and U(3, t) = 300

Initial conditions: U(x, 0) = 100(x) for $0 \leq x \leq 3$

(a) List **TWO (2)** persons who invented an implicit scheme of Crank-Nicholson method.

(2 marks)

(b) Draw the general molecule using implicit Crank-Nicholson Method.

(7 marks)

(c) Generate the system of linear equations based on the general molecule from **Q4 (b)**.

(10 marks)

(d) Generate the matrix forms Ax = b based on the equations from **Q4 (c)**.

(6 marks)

– END OF QUESTIONS –

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TABLE Q2 (a)

Time, t (s)	2	4	6	8	10
Distance, x (m)	34.5	165.0	281.5	321.5	553.5

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FORMULAE

Nonlinear equations

Lagrange Interpolating : $L_i = \frac{(x-x_1)(x-x_2)}{(x_i-x_1)(x_i-x_2)} \cdots \frac{(x-x_n)}{(x_i-x_n)}$; $f(x) = \sum_{i=1}^{n} L_i(x) f(x_i)$

Newton-Raphson Method : $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0,1,2 \ldots.$

System of linear equations

Gauss-Seidel Iteration:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}\, x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\, x_j^{(k)}}{a_{ii}}, \forall i = 1,2,3,\ldots,n.$$

Interpolation

Natural Cubic Spline:

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\}, k = 0,1,2,3,\ldots,n-1$$

$$b_k = 6(d_{k+1} - d_k), k = 0,1,2,3,..,n-2,$$

When; $m_0 = 0, m_n = 0,$

$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, k = 0,1,2,3,\ldots,n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6} h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6} h_k\right)(x - x_k) \quad , k = 0,1,2,3,\ldots n-1$$

Numerical Differentiation

2-point forward difference: $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x) - f(x-h)}{h}$

3-point central difference: $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

3-point central difference: $f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

5-point difference formula: $f''(x) \approx \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$

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FORMULAE

Numerical Integration

Simpson $\frac{1}{3}$ Rule : $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\ odd}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\ even}}^{n-2} f_i\right]$

Simpson $\frac{3}{8}$ Rule : $\int_a^b f(x)dx \approx \frac{3}{8}h\,[(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \cdots + f_{n-3}]$

2-point Gauss Quadrature: $\int_a^b g(x)dx = \left[g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)\right]$

3-point Gauss Quadrature: $\int_a^b g(x)dx = \left[\frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)\right]$

Eigen Value

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}Av^{(k)}, k = 0,1,2 \ldots.$

Shifted Power Method :$v^{(k+1)} = \frac{1}{m_{k+1}}A_{shifted}v^{(k)}, k = 0,1,2 \ldots.$

Ordinary Differential Equation

Fourth-order Runge-Kutta Method : $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Partial Differential Equation

Heat Equation : Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2\frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i.j}}{k} = c^2\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Poisson Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i.j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Wave Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2\frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j-1} - 2u_{i.j} + u_{i,j+1}}{k^2} = c^2\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$