UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	CIVIL ENGINEERING MATHEMATIC IV
COURSE CODE	:	BFC24203
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2018/ JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Bathymetry survey done by Noah Surveying Company shows that the total surface area of Tasik Sembrong, Kluang is 8.20 million m^2, whilst calculation from Google Earth image approximate the total surface area of 8.02 million m^2. Estimate the absolute error and percentage of relative error for both measurements.

(5 marks)

(b) Given the function of $f(x) = x^2 - 7$. Approximate $\sqrt{7}$ by using bisection method with $|b - a| = 1$. Iterate until $|f(c_i)| < \varepsilon = 0.005$.

(10 marks)

(c) Show Matrix A in the system of linear equation below is symmetric positive definite by theorem. Subsequently, solve the unknown using Cholesky Method.

$$5x_1 + 2x_2 = 2$$
$$2x_1 + 5x_2 + 2x_3 = 2$$
$$2x_2 + 5x_3 = 8$$

(10 marks)

Q2 (a) Prepare divided difference table using the data given in **TABLE Q2(a).** Then, transform the data from divided difference table obtained into equation of S(x) using natural cubic spline polynomial.

(10 marks)

(b) The altitude of a helicopter at three (3) different height is listed in **TABLE Q2(b).** Compute the rate of climb, $\frac{dh}{dt}$ at t = 0.4 seconds using 2-point forward and 2-point backward finite difference formula. Ensure the answer is appropriate when applying the respective finite difference formula.

(5 marks)

(c) The cross-sectional area of Sungai Melaka was expressed by:

$$\int_0^{30} \sqrt{2 + 3x^3}\,dx$$

As a civil engineer, you have been appointed to design a cross-sectional area of 30 meter width of this river to predict the river's flow rate during rainy or monsoon season. Choose the most accurate Simpson's rule and justify your selection. Divide the domain from 0 meter to 30 meter into 5 equal intervals and determine its absolute error.

(10 marks)

Q3 (a) Given the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}$ with the following initial and boundary conditions:

Boundary conditions: U(0, t) = 0 and U(2, t) = 0

Initial conditions: $U(x, 0) = \begin{cases} 100x, & 0 \leq x \leq 1 \\ 100(2-x) & 1 \leq x \leq 2 \end{cases}$

By using explicit finite-difference method, solve the heat equation at first level only ($t \leq 0.5$) by taking $\Delta x = h = 0.5$ and $\Delta t = k = 0.25$.

(7 marks)

(b) Given that $y' + 4y = 3x^2 + x$, in interval $1 \leq x \leq 2$ with initial condition y(1) = 0. By taking h = 0.2, sketch the diagram and solve the first order initial value problem (IVP) by using fourth-order Runge-Kutta Method (RK4). Do your calculation in 3 decimal places.

(8 marks)

(c) Given that matrix $A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$, estimate the smallest (in absolute value) eigenvalue and its corresponding eigenvector. Let $v^{(0)} = (1 \ 0 \ 1)^T$ and calculate until $|m_{k+1} - m_k| < 0.005$. Do your calculations in 3 decimal places.

(10 marks)

Q4 (a) Given the boundary-value problem $y'' - y' = 12x^2$, for $1 \leq x \leq 2$ with the boundary conditions, $y(1) = 2$ and $y(2) = 17$. By taking h = 0.2, sketch the diagram and derive the system of linear equation in matrix-vector form by finite-difference method without solving the whole system.

(10 marks)

(b) One dimensional bar maintained the temperatures at 1.115°C for both ends. Given the initial temperatures along the horizontal axis of bar as 1.115°C, 1.711°C, 1.825°C, 1.825°C, 1.711°C, 1.115°C. Assume one dimensional heat transfer equation as $\frac{\partial T}{\partial t} = 0.5\frac{\partial^2 T}{\partial x^2}$. Draw the grid for the problem and show all the temperature values in the diagram. Find the temperature of another points at first level only by assuming k = 0.01 and h = 0.2 using Crank Nicholson Implicit method.

(15 marks)

– END OF QUESTIONS –

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TABLE Q2(a)

x	1	2	3	4
f(x)	1	1	0	-1

TABLE Q2(b)

Time, *t*	0.30	0.40	0.55
Height, *h*	450.32	475.25	512.53

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FORMULAE

Nonlinear equations

Lagrange Interpolating : $L_i = \frac{(x-x_1)(x-x_2)}{(x_i-x_1)(x_i-x_2)} \cdots \frac{(x-x_n)}{(x_i-x_n)}$; $f(x) = \sum_{i=1}^{n} L_i(x) f(x_i)$

Newton-Raphson Method : $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0,1,2 \ldots.$

System of linear equations

Gauss-Seidel Iteration:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}\, x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\, x_j^{(k)}}{a_{ii}}, \forall i = 1,2,3,\ldots,n.$$

Interpolation

Natural Cubic Spline:

$$\left.\begin{array}{l} h_k = x_{k+1} - x_k \\ d_k = \frac{f_{k+1} - f_k}{h_k} \end{array}\right\}, k = 0,1,2,3,\ldots,n-1$$

$$b_k = 6(d_{k+1} - d_k), k = 0,1,2,3,..,n-2,$$

When; $m_0 = 0, m_n = 0,$

$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, k = 0,1,2,3,\ldots,n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x - x_k) \quad , k = 0,1,2,3,\ldots n-1$$

Numerical Differentiation

2-point forward difference: $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x) - f(x-h)}{h}$

3-point central difference: $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

3-point central difference: $f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

5-point difference formula: $f''(x) \approx \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$

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Numerical Integration

Simpson $\frac{1}{3}$ Rule : $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\ odd}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\ even}}^{n-2} f_i\right]$

Simpson $\frac{3}{8}$ Rule : $\int_a^b f(x)dx \approx \frac{3}{8}h\,[(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \cdots + f_{n-3}]$

2-point Gauss Quadrature: $\int_a^b g(x)dx = \left[g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)\right]$

3-point Gauss Quadrature: $\int_a^b g(x)dx = \left[\frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)\right]$

Eigen Value

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}Av^{(k)}, k = 0,1,2 \ldots.$

Shifted Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}A_{shifted}v^{(k)}, k = 0,1,2 \ldots.$

Ordinary Differential Equation

Fourth-order Runge-Kutta Method : $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Partial Differential Equation

Heat Equation : Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2\frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Poisson Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Wave Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2\frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

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2-Point Gauss Quadrature

$$\int_{-1}^{1} f(x)dx \approx c_1 f(x_1) + c_2 f(x_2) \approx g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

3-Point Gauss Quadrature

$$\int_{-1}^{1} f(x)dx \approx c_1 f(x_1) + c_2 f(x_2) + c_3 x_3 \approx \frac{5}{9} g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} g(0) + \frac{5}{9} g\left(\sqrt{\frac{3}{5}}\right)$$

Lagrange Interpolating Polynomial

$$L_i(x) = \frac{(x-x_1)}{(x_i-x_1)} \cdot \frac{(x-x_2)}{(x_i-x_2)} \cdots \frac{(x-x_n)}{(x_i-x_n)}$$

$$= \prod_{\substack{j=1 \\ j\neq i}}^{n} \frac{(x-x_i)}{(x_i-x_j)}$$

$$f(x) = \sum_{i=1}^{n} L_i(x) f(x_i)$$