Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2018/2019

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS I
COURSE CODE	:	BFC13903
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JUNE / JULY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF NINE (9) PAGES

Q1 (a) Find the differentiation for the followings:

(i) $y = \sin^5(3x-2)^4$.

(3 marks)

(ii) $y = (\sec x)(-\csc x + \cot x)$.

(4 marks)

(b) Determine the $\frac{dy}{dx}$ of $x^3 + 7y^3 - \sin\frac{x}{y} = 0$ by using implicit differentiation.

(7 marks)

(c) Given that $x = t - \sin t \cos t$ and $y = 4\cos t$. Find $\frac{dy}{dx}$.

(6 marks)

Q2 (a) For a function of $y = \frac{3}{4}x^4 - 3x^3$:

(i) Find all the critical points.

(5 marks)

(ii) Determine the intervals where the function is increasing, decreasing, concave up and concave down. Find the inflection point, if any.

(9 marks)

(b) A police helicopter is flying at 200 kilometers per hour at a constant altitude of 1 km above a straight road as shown in **Figure Q2**. The pilot uses radar to determine that an oncoming car is at a distance of exactly 2 kilometers from the helicopter, and that this distance is decreasing at 250 kilometers per hour. Find the speed of the car.

(6 marks)

Q3 (a) Determine the integrals of the followings:

(i) $\int \left(5x^3 + 2\sqrt[3]{x} - \frac{3}{x^2} \right) dx$.

(3 marks)

(ii) $\int_0^{\pi} (2 + \sin x) dx$.

(5 marks)

(b) Evaluate the following using suitable integration method:

(i) $\int x^3 \sin(x^4) dx$.

(5 marks)

(ii) $\int \frac{x-3}{3x^2 + 2x - 5}$.

(7 marks)

Q4 (a) Find the arc length of curve $x = r\cos t$ and $y = r\sin t$ within $0 \le t \le 2\pi$.

(5 marks)

(b) Determine the area of the region enclosed by $y = 5$ and $y = x^2 - 4$.

(5 marks)

(c) Evaluate the area of surface that is generated from the curve $y = x^2$ from (1,1) to (2,4) that is revolved about y-axis.

(10 marks)

Q5 (a) Find $f^{-1}(x)$ and show that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ for $y = \frac{x+3}{x-2}$.

(6 marks)

(b) Determine the derivative, $\frac{dy}{dx}$ of $y = \sec^{-1}(-2x+5) + \tan^{-1}(\ln x)$.

(8 marks)

(c) Evaluate the integral of $\int \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx$.

(6 marks)

– END OF QUESTIONS –

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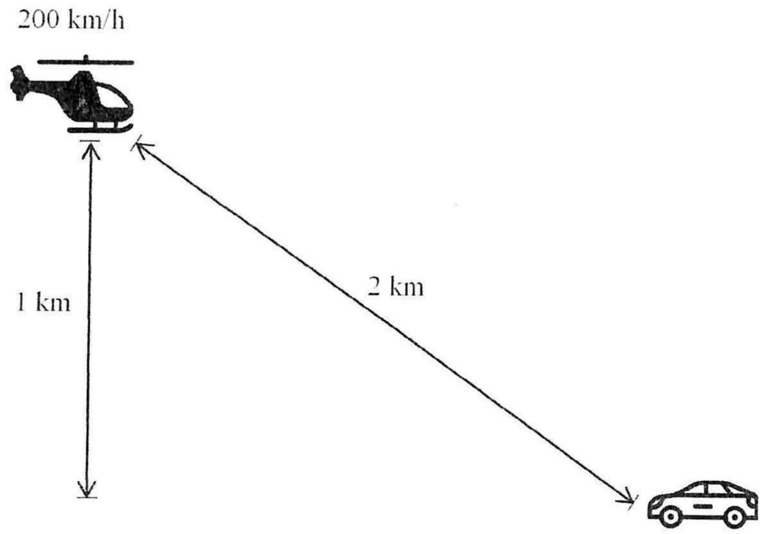

FIGURE Q2

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Formulae

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C \qquad n \neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\, x]=-\text{cosec}\, x \cot x$	$\int \text{cosec}\, x \cot x\,dx = -\text{cosec}\, x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\, x]=-\text{sech}\, x \tanh x$	$\int \text{sech}\, x \tanh x\,dx = -\text{sech}\, x + C$
$\frac{d}{dx}[\text{cosech}\, x]=-\text{cosech}\, x \coth x$	$\int \text{cosech}\, x \coth x\,dx = -\text{cosech}\, x + C$

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Formulae

Trigonometric	Hyperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \csc^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \operatorname{sec h}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \operatorname{csc h}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$	
	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$, any x
$a^x = e^{x \ln a}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$, $x \geq 1$
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1 < x < 1$

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Formulae

Integration of Inverse Functions
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{\lvert a\rvert\sqrt{a^2-x^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C, \quad a > 0$
$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C, \quad x > a$
$\int \frac{dx}{x^2-a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert > a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < x < a$
$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\operatorname{cosech}^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < x < a$

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Formulae

Differentiation of Inverse Functions	
y	$\dfrac{dy}{dx}$
$\sin^{-1} u$	$\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad \lvert u\rvert<1$
$\cos^{-1} u$	$-\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad \lvert u\rvert<1$
$\tan^{-1} u$	$\dfrac{1}{1+u^2}\dfrac{du}{dx}$
$\cot^{-1} u$	$-\dfrac{1}{1+u^2}\dfrac{du}{dx}$
$\sec^{-1} u$	$\dfrac{1}{\lvert u\rvert\sqrt{u^2-1}}\dfrac{du}{dx}, \quad \lvert u\rvert>1$
$\text{cosec}^{-1} u$	$-\dfrac{1}{\lvert u\rvert\sqrt{u^2-1}}\dfrac{du}{dx}, \quad \lvert u\rvert>1$
$\sinh^{-1} u$	$\dfrac{1}{\sqrt{u^2+1}}\dfrac{du}{dx}$
$\cosh^{-1} u$	$\dfrac{1}{\sqrt{u^2-1}}\dfrac{du}{dx}, \quad \lvert u\rvert>1$
$\tanh^{-1} u$	$\dfrac{1}{1-u^2}\dfrac{du}{dx}, \quad \lvert u\rvert<1$
$\coth^{-1} u$	$-\dfrac{1}{1-u^2}\dfrac{du}{dx}, \quad \lvert u\rvert>1$
$\text{sech}^{-1} u$	$-\dfrac{1}{u\sqrt{1-u^2}}\dfrac{du}{dx}, \quad 0<u<1$
$\text{cosech}^{-1} u$	$-\dfrac{1}{\lvert u\rvert\sqrt{1+u^2}}\dfrac{du}{dx}, \quad u\neq 0$

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Formulae

Area between two curves

Case 1- Integrating with respect to *x*: $A = \int_a^b [f(x) - g(x)]dx$

Case 2- Integrating with respect to *y*: $A = \int_c^d [f(y) - g(y)]dy$

Area of surface of revolution

Case 1- Revolving the portion of the curve about *x*-axis: $S = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Case 2- Revolving the portion of the curve about *y*-axis: $S = 2\pi \int_c^d x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve- Revolving the curve about *x*-axis: $S = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Parametric curve- Revolving the curve about *y*-axis: $S = 2\pi \int_c^d x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arc length

x-axis: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

y-axis: $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Curvature

Curvature, $\mathrm{K} = \dfrac{\left|\frac{d^2y}{dx^2}\right|}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{\mathrm{K}}$

Curvature of parametric curve

Curvature, $\mathrm{K} = \dfrac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{\mathrm{K}}$