Universiti Tun Hussein Onn Malaysia

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2018/2019

| | | |
|---|---|---|
| COURSE NAME | : | CIVIL ENGINEERING MATHEMATIC III |
| COURSE CODE | : | BFC24103 |
| PROGRAMME CODE | : | BFF |
| EXAMINATION DATE | : | JUNE / JULY 2019 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWER **ALL** QUESTIONS |

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

**Q1** (a) A vector function is defined by $\boldsymbol{r}(t) = \cos t\, \boldsymbol{i} + \sin t\, \boldsymbol{j} + 2t\, \boldsymbol{k}$. Determine the following parameter:

i. Differentiation of $\boldsymbol{r}(t)$ (1 mark)

ii. Second differentiation of $\boldsymbol{r}(t)$ (2 marks)

iii. Dot product of (i) and (ii) (2 marks)

iv. Cross product of (i) and (ii) (3 marks)

(b) Given the vector function $\boldsymbol{r}(t) = \sqrt{t}\, \boldsymbol{i} + (2t + 4)\boldsymbol{j}$. Plot the graph for $0 \le t \le 2$ and unit tangent. Find the unit tangent vector when $t = 1$.

(8 marks)

(c) In a building, water is directly supplied to fixtures from service pipe. The position vector of the water in the pipe is given by $\boldsymbol{r}(t) = \cos t\, \boldsymbol{i} + \sin t\, \boldsymbol{j} + t^3\, \boldsymbol{k}$. Find the velocity and acceleration vectors of the water flow. Then, calculate the speed of the water flow at 2 second.

(4 marks)

**Q2** (a) Use the Divergence Theorem and cylindrical coordinates to compute the outward flux of vector field $\boldsymbol{F}(x, y) = x^3\, \boldsymbol{i} + y^3\, \boldsymbol{j} + z^2\, \boldsymbol{k}$ across the surface of the region that is enclosed by the circular paraboloid $z = 4 - x^2 - y^2$ and plane $z = 0$.

(11 marks)

(b) Evaluate $\oint \mathrm{F} \bullet \mathrm{dr}$ for the vector $\boldsymbol{F}(x, y) = xz\, \boldsymbol{i} + xy^2 \boldsymbol{j} + 3\, xz\, \boldsymbol{k}$ and the space curve C which is intersection of the plane $x + z = 3$ and the cylinder $x^2 + y^2 = 4$, in the counter clockwise direction when viewed from positive axis.

(9 marks)

**Q3** (a) The height of a tree increases at a rate of 2 m per year and the radius of its timber increases at 0.1 m per years. Determine the rate of the timber volume increasing when the height is 20 m and the radius is 1.5 m (Assume the tree is circular cylinder).

(8 marks)

(b) Given $w = 3xy^2z^3; y = 3x^2 + 2, z = \sqrt{x-1}$, find $\frac{\partial w}{\partial x}$ using chain rule.

(5 marks)

(c) Evaluate the $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ of function

$$z^2 + z\sin(xy) = 0$$

(7 marks)

**Q4** (a) Calculate the area of regions enclosed by the curve $y = \sqrt{x}$, the line $y = x$, $y = 1$and $y = 2$ using double integrals

(4 marks)

(b) Evaluate the following integral

$$\iiint\limits_G z\, dV$$

Where $G$ is the tetrahedron in the first octant bounded by $x + y + z = 4$

(9 marks)

(c) Examine the volume of solid bounded above by sphere $\rho = 4$, and below by the cone $\emptyset = \frac{\pi}{3}$ using spherical coordinates

(7 marks)

**Q5** (a) Determine the unit tangent vector and normal vector for curve $x = e^t \cos t$, $y = e^t \sin t$, $z = e^t$ at $t = 0$.

(8 marks)

(b) Find the position vector satisfying the following condition:-

$$a(t) = -32j, v(0) = 600\sqrt{3}i + 600j$$

(8 marks)

(c) Find the velocity and acceleration for function

$$R(\theta) = (\sin\theta)i + (\cos\theta)j$$

(4 marks)

– END OF QUESTIONS –

**FINAL EXAMINATION**

SEMESTER/SESSION : SEM II / 2018/2019 PROGRAMME CODE : 4 BFF
COURSE NAME : CIVIL ENGINEERING MATHEMATIC III COURSE CODE : BFC24103

*The following information may be useful. The symbols have their usual meaning.*

**Formulae**

**Implicit Partial Differentiation:**

$$\frac{\partial z}{\partial x}=-\frac{f_x(x,y,z)}{f_z(x,y,z)} \quad or \quad \frac{\partial z}{\partial y}=-\frac{f_y(x,y,z)}{f_z(x,y,z)}$$

**Small Increment, Estimating Value:**

**Total differential/approximate change**, $\partial z=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$
**Exact change**, $dz=f(x_1,y_1)-f(x_0,y_0)$
**Approximate value**, $z=f(x_0,y_0)+dz$
**Exact value**, $z=f(x_1,y_1)$

**Error**: $|dz|=\left|\frac{\partial z}{\partial x}dx\right|+\left|\frac{\partial z}{\partial y}dy\right|$ and **Relative error**: $\left|\frac{dz}{z}\right|=\left|\frac{\partial z}{\partial x}\frac{dx}{z}\right|+\left|\frac{\partial z}{\partial y}\frac{dy}{z}\right|$

**Polar coordinate:** $x=r\cos\theta$, $y=r\sin\theta$, $x^2+y^2=r^2$, and $\iint_R f(x,y)dA=\iint_R f(r,\theta)\,r\,dr\,d\theta$

**Cylindrical coordinate:** $x=r\cos\theta$, $y=r\sin\theta$, $z=z$, $\iiint_G f(x,y,z)dV=\iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$

**Spherical coordinate:** $x=\rho\cos\theta\sin\emptyset$, $y=\rho\sin\theta\sin\emptyset$, $z=\rho\cos\emptyset$, $x^2+y^2+z^2=\rho^2$,
$0\le\theta\le 2\pi$, $0\le\phi\le\pi$ and $\iiint_G f(x,y,z)dV=\iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi\,d\rho\,d\phi\,d$

**Directional derivative**: $D_{\mathbf{u}}f(x,y)=(f_x\mathbf{i}+f_y\mathbf{j})\cdot\mathbf{u}$

Let $\mathbf{F}(x,y,z)=M\,\mathbf{i}+N\,\mathbf{j}+P\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F}=\nabla\cdot\mathbf{F}=\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}+\frac{\partial P}{\partial z}$

the **curl** of $\mathbf{F}=\nabla\times\mathbf{F}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ M & N & P\end{vmatrix}=\left(\frac{\partial P}{\partial y}-\frac{\partial N}{\partial z}\right)\mathbf{i}-\left(\frac{\partial P}{\partial x}-\frac{\partial M}{\partial z}\right)\mathbf{j}+\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)\mathbf{k}$

Let $C$ is a smooth curve given by $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}$, $t$ is parameter, then

| | |
|---|---|
| **the unit tangent vector**: | $\mathbf{T}(t)=\frac{\mathbf{r}'(t)}{\lVert\mathbf{r}'(t)\rVert}$ |
| **the unit normal vector**: | $\mathbf{N}(t)=\frac{\mathbf{T}'(t)}{\lVert\mathbf{T}'(t)\rVert}$ |
| **the binormal vector**: | $\mathbf{B}(t)=\mathbf{T}(t)\times\mathbf{N}(t)$ |
| **the curvature**: | $\kappa=\frac{\lVert\mathbf{T}'(t)\rVert}{\lVert\mathbf{r}'(t)\rVert}=\frac{\lVert\mathbf{r}'(t)\times\mathbf{r}''(t)\rVert}{\lVert\mathbf{r}'(t)\rVert^3}$ |
| **the radius of curvature**: | $\rho=1/\kappa$ |

**FINAL EXAMINATION**

SEMESTER/SESSION : SEM II / 2018/2019 PROGRAMME CODE : 4 BFF
COURSE NAME : CIVIL ENGINEERING MATHEMATIC III COURSE CODE : BFC24103

**Green Theorem:** $\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

**Gauss Theorem:** $\iint_S \mathbf{F} \bullet \mathbf{n}\,dS = \iiint_G \nabla \bullet \mathbf{F}\,dV$

**Stokes' Theorem:** $\oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\,dS$

**Arc length**

If $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\|\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

If $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$

**Tangent Plane**

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

**Extreme of two variable functions**

$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - (f_{xy}(x,y))^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then $f$ has local maximum at $(a,b)$

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then $f$ has local minimum at $(a,b)$

Case3: If $G(a,b) < 0$ then $f$ has a saddle point at $(a,b)$

Case4: If $G(a,b) = 0$ then no conclusion can be made.

**In 2-D: Lamina**

**Mass:** $m = \iint_R \delta(x,y)\,dA$, where $\delta(x,y)$ is a density of lamina.

**Moment of mass:** (i) about $y$-axis, $M_y = \iint_R x\delta(x,y)\,dA$, (ii) about $x$-axis, $M_x = \iint_R y\delta(x,y)\,dA$

**Centre of mass,** $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$

**Moment inertia:** (i) $I_y = \iint_R x^2\delta(x,y)\,dA$, (ii) $I_x = \iint_R y^2\delta(x,y)\,dA$, (iii) $I_o = \iint_R (x^2 + y^2)\delta(x,y)\,dA$

**FINAL EXAMINATION**

SEMESTER/SESSION : SEM II / 2018/2019 PROGRAMME CODE : 4 BFF
COURSE NAME : CIVIL ENGINEERING MATHEMATIC III COURSE CODE : BFC24103

**In 3-D: Solid**

**Mass**, $m = \iiint_G \delta(x,y,z)\,dV$. If $\delta(x,y,z) = c$, $c$ is a constant, then $m = \iiint_G dA$ is volume.

**Moment of mass**

(i) about $yz$-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

(ii) about $xz$-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

(iii) about $xy$-pane, $M_{xy} = \iiint z\delta(x,y,z)\,dV$

---

**Centre of gravity**, $(\bar{x},\bar{y},\bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$

**Moment inertia**

(i) about $x$-axis: $I_x = \iiint_G \left(y^2 + z^2\right)\delta(x,y,z)\,dV$

(ii) about $y$-axis: $I_y = \iiint_G \left(x^2 + z^2\right)\delta(x,y,z)\,dV$

(iii) about $z$-axis: $I_z = \iiint_G \left(x^2 + y^2\right)\delta(x,y,z)\,dV$