UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS III
COURSE CODE	:	BFC 24103
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2018 / JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **SECTION A** AND **THREE (3)** QUESTIONS IN **SECTION B.**

THIS EXAMINATION PAPER CONSISTS OF **EIGHT (8)** PAGES

SECTION A

Q1 (a) The position acceleration of particle is given as

$$\mathbf{a}(t) = 2\,\mathbf{i} + 6t\,\mathbf{j} + 12t^2\,\mathbf{k}$$

and initial velocity is $\mathbf{v}(0) = \mathbf{i}$ and position is $\mathbf{r}(0) = \mathbf{j} - \mathbf{k}$. Calculate the velocity and position vector of particle when $t = 2$.

(10 marks)

(b) Find the unit tangent and normal vector of the vector function

$$\mathbf{r}(t) = \langle t^2, \sin t - t\cos t, \cos t + t\sin t\rangle \qquad , t > 0$$

(10 marks)

Q2 (a) Verify Green's theorem for;

$$\oint_C (2x - y)dx + xy\,dy$$

where C is the boundary of the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

(10 marks)

(b) Evaluate $\oint_C \mathbf{F} \cdot dr$ by using Stokes' theorem for the vector $\mathbf{F} = xz\,\mathbf{i} + xy\,\mathbf{j} + 3yz\,\mathbf{k}$ and the curve C which is the perimeter of a closed triangle in the first octant with vertices at (3, 0, 0), (0, 3, 0), and (0, 0, 6) in the counterclockwise direction, when view from the positive z-axis.

(10 marks)

SECTION B

Q3 (a) Determine whether the function $f(x,y)$ exist and continuous at the point (4, 0).

$$f(x,y)=\begin{cases}\dfrac{xy-4y}{\sqrt{x}-2}, & if\ (x,y)\neq(4,0)\\ 0, & if\ (x,y)=(4,0)\end{cases}$$

(4 marks)

(b) Find f_x and f_y for $ye^{2xz}-5ye^{yz}=-10ze^{2xy}+1$, if $z=f(x,y)$ is implicitly defined as a function of x and y.

(10 marks)

(c) Use the differential dz to approximate the change in $z=\sqrt{25-x^2-y^2}$ as (x,y) move from the point (2, 2) to the point (1.95, 2.05). Compare this approximation change with the exact change in z.

(6 marks)

Q4 (a) Given that $f(x,y)=2x^2+xy-y^2$. Find the partial derivatives f_x and f_y.

(4 marks)

(b) A hot air balloon is rising straight up from a field and tracked by a range finder 150 m from the lift-off point as shown in **Figure Q4(b)**. The range finder angle is $\pi/4$, the angle is increasing at the rate of 0.14 rad/min. Predict how fast the balloon is rising at that moment.

(7 marks)

(c) The radius of a right circular cylinder is measured with an error of at most 4%, and the height is measured with and error of at most 8%. Approximate the maximum possible percentage error in the volume V calculated from these measurements.

(9 marks)

Q5 (a) Evaluate the triple integral

$$\int_0^1\int_0^{x^2}\int_0^{x+y}(x-2y+z)\,dxdydz$$

(6 marks)

(b) Consider an object which is bounded above by the inverted paraboloid $z=16-x^2-y^2$ and below by the xy-plane. Suppose that the density of the object is given by $\delta(x,y,z)=8+x+y$. Analyse the mass of the object by using cylindrical coordinates.

(7 marks)

(c) Find the area bounded by $y=2x^2$ and $y=1+x^2$ using double integral.

(7 marks)

Q6 (a) Given that a lamina with density function $\delta(x,y)=y$ is bounded by $y=\sin x$, $y=0$, $x=0$ and $x=\pi$. Estimate the centre of mass of the lamina.

(6 marks)

(b) Analyze the volume of the solid within the cylinder $x^2+y^2=4$ and between the planes $z=2$ and $y+z=5$ using cylindrical coordinates.

(7 marks)

(c) Calculate the volume of the solid bounded above by $\rho=5$ and below by cone $\phi=\frac{\pi}{4}$ using spherical coordinates

(7 marks)

- END OF QUESTIONS –

FINAL EXAMINATION

SEMESTER / SESSION : SEM I/ 2018/2019 PROGRAMME :BFF
COURSE : CIVIL ENGINEERING MATHEMATICS III CODE :BFC 24103

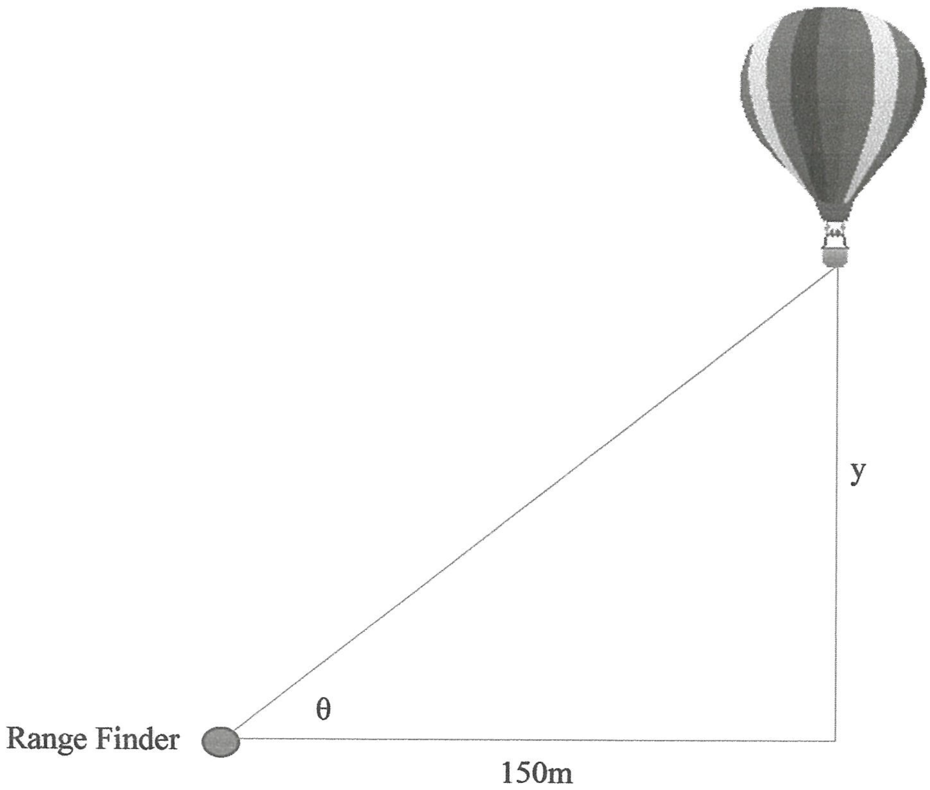

FIGURE Q4(b)

FINAL EXAMINATION

SEMESTER / SESSION : SEM I/ 2018/2019 PROGRAMME :BFF
COURSE : CIVIL ENGINEERING MATHEMATICS III CODE :BFC 24103

Formulae

Implicit Partial Differentiation:

$$\frac{\partial z}{\partial x} = -\frac{f_x(x,y,z)}{f_z(x,y,z)} \quad or \quad \frac{\partial z}{\partial y} = -\frac{f_y(x,y,z)}{f_z(x,y,z)}$$

Small Increment, Estimating Value:

Total differential/approximate change, $\partial z = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$
Exact change, $dz = f(x_1, y_1) - f(x_0, y_0)$
Approximate value, $z = f(x_0, y_0) + dz$
Exact value, $z = f(x_1, y_1)$

Error: $|dz| = \left|\frac{\partial z}{\partial x}dx\right| + \left|\frac{\partial z}{\partial y}dy\right|$ and **Relative error:** $\left|\frac{dz}{z}\right| = \left|\frac{\partial z}{\partial x}\frac{dx}{z}\right| + \left|\frac{\partial z}{\partial y}\frac{dy}{z}\right|$

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $x^2 + y^2 = r^2$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate: $x = \rho\cos\theta\sin\emptyset$, $y = \rho\sin\theta\sin\emptyset$, $z = \rho\cos\emptyset$, $x^2 + y^2 + z^2 = \rho^2$,
$0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$ and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

Directional derivative: $D_{\mathbf{u}} f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot \mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

the **curl** of $\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

the curvature: $\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I/ 2018/2019 PROGRAMME :BFF
COURSE : CIVIL ENGINEERING MATHEMATICS III CODE :BFC 24103

Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$

Gauss Theorem: $\iint_S \mathbf{F} \bullet \mathbf{n}\,dS = \iiint_G \nabla \bullet \mathbf{F}\,dV$

Stokes' Theorem: $\oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\,dS$

Arc length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\|\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$

Tangent Plane

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Extreme of two variable functions

$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - (f_{xy}(x,y))^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m = \iint_R \delta(x,y)\,dA$, where $\delta(x,y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x,y)\,dA$, (ii) about x-axis, $M_x = \iint_R y\delta(x,y)\,dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right)$

Moment inertia: (i) $I_y = \iint_R x^2 \delta(x,y)\,dA$, (ii) $I_x = \iint_R y^2 \delta(x,y)\,dA$, (iii) $I_o = \iint_R (x^2 + y^2)\delta(x,y)\,dA$

In 3-D: Solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$. If $\delta(x,y,z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x,y,z)\,dV$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I/ 2018/2019 PROGRAMME :BFF
COURSE : CIVIL ENGINEERING MATHEMATICS III CODE :BFC 24103

Centre of gravity, $(\bar{x},\bar{y},\bar{z})=\left(\dfrac{M_{yz}}{m},\dfrac{M_{xz}}{m},\dfrac{M_{xy}}{m}\right)$

Moment inertia

(i) about x-axis: $I_x=\iiint\limits_G \left(y^2+z^2\right)\delta(x,y,z)\,dV$

(ii) about y-axis: $I_y=\iiint\limits_G \left(x^2+z^2\right)\delta(x,y,z)\,dV$

(iii) about z-axis: $I_z=\iiint\limits_G \left(x^2+y^2\right)\delta(x,y,z)\,dV$