Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS II
COURSE CODE	:	BFC 14003
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2018 / JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **TWO (2)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

PART A

Q1 (a) Identify the Laplace transform for each given function.

(i) $f(t) = 2t^4 - e^{-4t}$

(ii) $g(t) = \cosh^2 2t$

(4 marks)

(b) Determine the solution for the equation below.

$$L^{-1}\left(\frac{a}{s\left(s^2+a^2\right)}\right)$$

(6 marks)

(b) Solve the given differential equation by using Laplace Transform method.

$$t\frac{d^2y}{dt^2} - \frac{dy}{dt} = -t; \quad y(0) = 0$$

(10 marks)

Q2 (a) Determine the interval and radius of convergence for the following power series.

$$\sum_{n=0}^{\infty} \frac{6^n}{n}(4x-1)^{n-1}$$

(10 marks)

(a) Construct a Taylor series for a function $f(x) = e^{-6x}$ with x = - 4.

(10 marks)

Q3 A periodic function is defined by

$$f(x) = \begin{cases} 1 + \dfrac{2x}{\pi}, & -\pi \le x \le \pi \\ 1 - \dfrac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$$

(a) Calculate the Fourier coefficient, a_0 and a_n.

(8 marks)

(b) Determine the Fourier series.

(5 marks)

(c) Sketch the graph of over $-\pi < x < \pi$.

(4 marks)

(d) Determine whether the function is even, odd or neither. Justify the answer.

(3 marks)

PART B

Q4 (a) Determine the particular solution for the following differential equation by using linear method. Given that y = 10 when x is 4.

$$(x-2)\frac{dy}{dx}-y=(x-2)^3$$

(9 marks)

(b) One tonne of bituminous premix was placed in a room with temperature 10°C. The temperature of the bituminous premix dropped from 90°C to 30°C in 30 minutes.

(i) Define the cooling model according to Newton's law.

(2 marks)

(ii) Determine the temperature of the bituminous premix at 20 minutes.

(9 marks)

Q5 (a) Prove the following equation.

$$\mathcal{L}\left\{\frac{1}{2}\sin 2t - t\cos 2t\right\} = \frac{8}{(s^2+4)^2}$$

(4 marks)

(b) A damped forced oscillation is given by

$$y'' + 2y' + 5y = e^{-t}\sin 2t$$

which satisfies the initial conditions $y(0) = 0$ and $y'(0) = 1$.

(i) Determine y (t) by using Laplace transform.

(4 marks)

(ii) Assume that t is small and $t_n \rightarrow 0$ for $n \geq 3$ is negligible, show that $y(t) \approx t - t^2$.

(12 marks)

Q6 (a) A non-homogenous second order differential equation is defined by $y'' - y' - 2y = e^{3x}$. Determine the general solution for the equation by using variation of parameters method.

(10 marks)

(b) A spring with a mass of 2 kg has a natural length of 0.5 m. A force of 25.6 N is required to maintain it stretched to a length of 0.7 m. If the spring stretched to a length of 0.7 m and then released with initial velocity (v) = 0, calculate the position of the mass at any time, t.

(10 marks)

- END OF QUESTIONS –

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W}\,dx, \quad u_2 = \int \frac{y_1 f(x)}{W}\,dx$	$y_p = u_1 y_1 + u_2 y_2$

Fourier Series

Fourier series expansion of periodic function with period $2L$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

Fourier half-range series expansion

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt, \quad s > 0.$

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Trigonometric and Hyperbolic Identities

Trigonometric	Hiperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \operatorname{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \operatorname{cosech}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$	

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Differentiation and Integration

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C$, $n \neq -1$
$\frac{d}{dx}[\ln\lvert x\rvert]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\lvert x\rvert + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\mathrm{cosec}^2 x$	$\int \mathrm{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\mathrm{cosec}\, x]=-\mathrm{cosec}\, x \cot x$	$\int \mathrm{cosec}\, x \cot x\,dx = -\mathrm{cosec}\, x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\mathrm{sech}^2 x$	$\int \mathrm{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\mathrm{cosech}^2 x$	$\int \mathrm{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\mathrm{sech}\, x]=-\mathrm{sech}\, x \tanh x$	$\int \mathrm{sech}\, x \tanh x\,dx = -\mathrm{sech}\, x + C$
$\frac{d}{dx}[\mathrm{cosech}\, x]=-\mathrm{cosech}\, x \coth x$	$\int \mathrm{cosech}\, x \coth x\,dx = -\mathrm{cosech}\, x + C$