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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2018/2019

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS 1
COURSE CODE	:	BFC 13903
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2018 /JANUARY 2019
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

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Q1 (a) Determine the following limits:

i) $\lim_{x \to 1} \frac{x^2 - 2x + 1}{x^2 - 3x + 2}$

(3 marks)

ii) $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$

(3 marks)

(b) Prove that the function below is equal to 3/8.

$$\lim_{x \to 0} \frac{x \sin 3x}{\sin 2x \sin 4x}$$

(3 marks)

(c) The function $f(x)$ is defined by:

$$f(x) = \begin{cases} 2x^3 + x + 10, & x \le -2 \\ a(x+2) + b, & -2 < x \le 3 \\ x^2 - 5, & x > 3 \end{cases}$$

Find the value of constants 'a' and 'b' that will make the function $f(x)$ continuous everywhere. With this value of 'a' and 'b', plot the graph of function $f(x)$.

(11 marks)

Q2 (a) Find the derivatives of the following functions:

i) $y = \sqrt[3]{\cos \sqrt[3]{10x - 5}}$

(4 marks)

ii) $y = \sin\left(e^{3x}\right) + \ln\left(1 - 2x^2\right)^3$

(4 marks)

(b) By using implicit differentiation method, find $\frac{dy}{dx}$ for

$\cos^2 x + \cos^2 y = \cos(2x + 2y)$.

(6 marks)

(c) If $y = x^2 - 6x$ and $x = \sqrt{2t^2 + 4}$, find $\frac{dy}{dt}$ when $t = \sqrt{16}$ and $x = \sqrt{5}$.

(6 marks)

Q3 (a) At t seconds after lift-off, the height of a rocket is $5t^2$ m. Calculate how fast the rocket has climbed 10 seconds after the lift-off.

(2 marks)

(b) Find the gradients of tangents drawn to the circle $x^2 + y^2 - 2x - 2y = 3$ at $x = 2$.

(10 marks)

(c) Determine the maximum and minimum points of the following graph.

$$y = \frac{x^3}{3} - x^2 - 8x + 1.$$

(8 marks)

Q4 (a) Evaluate the followings:

i) $\int \sqrt{x}(x+2)\,dx$

(2 marks)

ii) $\int \cos x - \sin 2x\,dx$

(2 marks)

(b) Evaluate the following integral by using integration by substitution technique.

$$\int_0^{\sqrt{3}} x^3 \sqrt{1+x^2}\,dx$$

(8 marks)

(c) Solve the following using integration by partial fractions method.

$$\int \frac{x^3 - 2x^2 + 2x - 7}{(x-3)(x+1)}\,dx$$

(8 marks)

Q5 (a) The area of the region is enclosed by the curve $y = x(4-x)$ and the line $y = 2x - 3$.

i) Sketch the area of bounded region.

(3 marks)

ii) Determine the coordinates of the points of intersection of the curves and the line.

(3 marks)

iii) Find the area of bounded region.

(4 marks)

(b) Find the radius of curvature of $x = t^2 + 5$, $y = 2t - 1$ at $t = 2$.

(10 marks)

Q6 (a) Solve for the following expression:

$\tan\left(\sin^{-1}(-1/\sqrt{2})\right)$.

(2 marks)

(b) Show that if $y = \cos^{-1} x$, then $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$. Hence, evaluate the differential coefficient of $y = \cos^{-1}(1-8x^3)$.

(9 marks)

(c) Evaluate $\int \sinh^{-1}(x)dx$.

(9 marks)

- END OF QUESTIONS –

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Formulae

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0, \quad k \text{ constant}$	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C, \quad n \neq -1$
$\frac{d}{dx}[\ln\lvert x\rvert]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\lvert x\rvert + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\, x]=-\text{cosec}\, x \cot x$	$\int \text{cosec}\, x \cot x\,dx = -\text{cosec}\, x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\, x]=-\text{sech}\, x \tanh x$	$\int \text{sech}\, x \tanh x\,dx = -\text{sech}\, x + C$
$\frac{d}{dx}[\text{cosech}\, x]=-\text{cosech}\, x \coth x$	$\int \text{cosech}\, x \coth x\,dx = -\text{cosech}\, x + C$

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Formulae

Trigonometric	Hiperbolic
$\cos^2 x+\sin^2 x=1$	$\sinh x=\dfrac{e^x-e^{-x}}{2}$
$1+\tan^2 x=\sec^2 x$	$\cosh x=\dfrac{e^x+e^{-x}}{2}$
$\cot^2 x+1=\operatorname{cosec}^2 x$	$\cosh^2 x-\sinh^2 x=1$
$\sin 2x=2\sin x\cos x$	$1-\tanh^2 x=\operatorname{sech}^2 x$
$\cos 2x=\cos^2 x-\sin^2 x$	$\coth^2 x-1=\operatorname{cosech}^2 x$
$\cos 2x=2\cos^2 x-1$	$\sinh 2x=2\sinh x\cosh x$
$\cos 2x=1-2\sin^2 x$	$\cosh 2x=\cosh^2 x+\sinh^2 x$
$\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$	$\cosh 2x=2\cosh^2 x-1$
$\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$	$\cosh 2x=1+2\sinh^2 x$
$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$	$\tanh 2x=\dfrac{2\tanh x}{1+\tanh^2 x}$
$\tan(x\pm y)=\dfrac{\tan x\pm\tan y}{1\mp\tan x\tan y}$	$\sinh(x\pm y)=\sinh x\cosh y\pm\cosh x\sinh y$
$2\sin x\cos y=\sin(x+y)+\sin(x-y)$	$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$
$2\sin x\sin y=-\cos(x+y)+\cos(x-y)$	$\tanh(x\pm y)=\dfrac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$
$2\cos x\cos y=\cos(x+y)+\cos(x-y)$	
Logarithm	**Inverse Hiperbolic**
$a^x=e^{x\ln a}$	$\sinh^{-1} x=\ln\left(x+\sqrt{x^2+1}\right)$, any x.
$\log_a x=\dfrac{\log_b x}{\log_b a}$	$\cosh^{-1} x=\ln\left(x+\sqrt{x^2-1}\right)$, $x\ge 1$
	$\tanh^{-1} x=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1<x<1$

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Formulae

Integration of Inverse Functions
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+C.$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C.$
$\int \frac{dx}{\|a\|\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C.$
$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right)+C, \quad a>0.$
$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)+C, \quad x>a$
$\int \frac{dx}{x^2-a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C, & \|x\|<a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+C, & \|x\|>a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{x}{a}\right)+C, \quad 0<x<a.$
$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\text{cosech}^{-1}\left(\frac{x}{a}\right)+C, \quad 0<x<a.$

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Formulae

Differentiation of Inverse Functions	
y	$\frac{dy}{dx}$
$\sin^{-1} u$	$\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}, \quad \lvert u \rvert < 1.$
$\cos^{-1} u$	$-\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}, \quad \lvert u \rvert < 1.$
$\tan^{-1} u$	$\frac{1}{1+u^2} \cdot \frac{du}{dx}$
$\cot^{-1} u$	$-\frac{1}{1+u^2} \cdot \frac{du}{dx}$
$\sec^{-1} u$	$\frac{1}{\lvert u \rvert \sqrt{u^2-1}} \cdot \frac{du}{dx}, \quad \lvert u \rvert > 1.$
$\mathrm{cosec}^{-1} u$	$-\frac{1}{\lvert u \rvert \sqrt{u^2-1}} \cdot \frac{du}{dx}, \quad \lvert u \rvert > 1.$
$\sinh^{-1} u$	$\frac{1}{\sqrt{u^2+1}} \cdot \frac{du}{dx}$
$\cosh^{-1} u$	$\frac{1}{\sqrt{u^2-1}} \cdot \frac{du}{dx}, \quad \lvert u \rvert > 1.$
$\tanh^{-1} u$	$\frac{1}{1-u^2} \cdot \frac{du}{dx}, \quad \lvert u \rvert < 1.$
$\coth^{-1} u$	$-\frac{1}{1-u^2} \cdot \frac{du}{dx}, \quad \lvert u \rvert > 1.$
$\mathrm{sech}^{-1} u$	$-\frac{1}{u\sqrt{1-u^2}} \cdot \frac{du}{dx}, \quad 0 < u < 1.$
$\mathrm{cosech}^{-1} u$	$-\frac{1}{\lvert u \rvert \sqrt{1+u^2}} \cdot \frac{du}{dx}, \quad u \neq 0.$

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Formulae

Area between two curves

Case 1- Integrating with respect to x: $A = \int_a^b [f(x) - g(x)]dx$

Case 2- Integrating with respect to y: $A = \int_c^d [f(y) - g(y)]dy$

Area of surface of revolution

Case 1- Revolving the portion of the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

Case 2- Revolving the portion of the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve- Revolving the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$

Parametric curve- Revolving the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$

Arc length

x-axis: $L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

y-axis: $L = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$

Curvature

Curvature, $\mathrm{K} = \dfrac{\left|\frac{d^2y}{dx^2}\right|}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{\mathrm{K}}$

Curvature of parametric curve

Curvature, $\mathrm{K} = \dfrac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{\mathrm{K}}$