CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2017/2018

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS IV
COURSE CODE	:	BFC24203
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2017 / JANUARY 2018
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

TERBUKA

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

CONFIDENTIAL

Q1 (a) The function of the graph is $f(x) = \sin 2x + x^2 - 2$.

(i) Draw the graph over the interval $0 \leq x \leq 5$.

(4 marks)

(ii) Show that function $f(x) = \sin 2x + x^2 - 2$ has at least a root using Intermediate Value Theorem. Then calculate the positive root of $f(x)$ by using Newton Raphson Method. Give your answer in four (4) decimal places. Iterate it until $|f(xi|< \varepsilon= 0.0005$. Justify your answer by using a calculator.

(9 marks)

(b) Given $f(x) = e^x$

(i) Complete the following table:

x	0	0.25	0.75	1.0
$f(x) = e^x$				

(4 marks)

(ii) Referring to the table in **Q1b(i)**, find the $P_3(0.5)$ by using Newton's divided difference method.

(6 marks)

(iii) Alliy claims that the above data can be used to estimate f(0.2) and f(0.65). Justify your answer with calculations.

(2 marks)

Q2 (a) **Table 1** shows the velocity, v of an object at various points in time, t.

Table 1

Time, t (minute)	1.3	1.4	1.5	1.6	1.7	1.8
Velocity, v (m/min)	1.418	1.715	2.037	2.336	2.746	3.099

(i) Detect the velocity at t=1.5 minutes using two (2) point and three (3) point forward difference formula with h=0.1 minute.

(6 marks)

(ii) Given that velocity, $v = x''(t)$. By taking h=0.2 minute, estimate the velocity at time t=1.6 by using three (3) point central difference formula.

(4 marks)

(b) Calculate the approximate value for $\int_{-1}^{1} \frac{1}{\sqrt{x^2+1}} dx$ using the 3/8 Simpson's rule and 1/3 Simpson's rule with h=0.2. For each case, find the more accurate rules. Do the calculations in 3 decimal places. [Hint : $\int_{-1}^{1} \frac{1}{\sqrt{x^2+1}} dx = 1.570796$]

(15 marks)

Q3 (a) Given the matrix and solve by using initial eigenvector as $\mathbf{v}^{(0)} = (1 \quad 1 \quad 1)^T$ for both calculation and do your computation in 3 decimal places.

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

(i) Compute the dominant eigenvalue, $\lambda_{Largest}$ of A and its associated eigenvector v_1.

(7 marks)

(ii) Then, find the smallest eigenvalue, $\lambda_{Smallest}$ of A.

(8 marks)

(b) The initial value problem $y' = 4e^{0.8x} - 0.5y$, with $y(0) = 2$, has an unique solution of $y(4) = 75.339$. Generate the solution at $x = 4$ using the fourth order Runge-Kutta method with the step size $h = 1$ and compute the percentage of relative error.

(10 marks)

Q4 Given the heat equation; $\frac{\partial u}{\partial t} = 0.5 \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$, with boundary conditions $u(0,t) = 20e^{-t}$ and $u(1,t) = 60e^{-2t}$. While initial condition is $u(x,0) = 20 + 40x$ for $0 \le x \le 1$. By using implicit Crank-Nicolson method, solve the heat equation at first level only by taking $\Delta x = h = 0.25$ and $\Delta t = k = 0.1$. Solve the matrix by using Gauss method.

(25 marks)

- END OF QUESTIONS-

TERBUKA

FINAL EXAMINATION

SEMESTER/SESSION : SEM I / 2017/2018 PROGRAM CODE: BFF
COURSE NAME : CIVIL ENGINEERING MATHEMATICS IV COURSE CODE : BFC 24203

FORMULAE

Nonlinear equations

Newton-Raphson Method : $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0,1,2 \ldots.$

System of linear equations

Gauss-Seidel Iteration:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}\, x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\, x_j^{(k)}}{a_{ii}}, \forall i = 1,2,3, \ldots, n.$$

Interpolation

Natural Cubic Spline:

$$\left.\begin{matrix} h_k = x_{k+1} - x_k \\ d_k = \frac{f_{k+1} - f_k}{h_k} \end{matrix}\right\}, k = 0,1,2,3, \ldots, n-1$$

$$b_k = 6(d_{k+1} - d_k), k = 0,1,2,3, \ldots, n-2,$$

$$m_0 = 0,$$

$$m_n = 0,$$

$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, k = 0,1,2,3, \ldots, n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6} h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6} h_k\right)(x - x_k) \quad , k = 0,1,2,3, \ldots n-1$$

TERBUKA

Numerical Differentiation

2-point forward difference: $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x) - f(x-h)}{h}$

3-point central difference: $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

3-point central difference: $f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

5-point difference formula: $f''(x) \approx \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$

FINAL EXAMINATION

SEMESTER/SESSION : SEM I / 2017/2018 PROGRAM CODE: BFF
COURSE NAME : CIVIL ENGINEERING MATHEMATICS IV COURSE CODE : BFC 24203

Numerical Integration

Simpson $\frac{1}{3}$ Rule : $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\,odd}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\,even}}^{n-2} f_i\right]$

Simpson $\frac{3}{8}$ Rule : $\int_a^b f(x)dx \approx \frac{3}{8}h\,[(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \cdots + f_{n-3}]$

Eigen Value

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, k = 0,1,2 \ldots.$

Shifted Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}} A_{shifted} v^{(k)}, k = 0,1,2 \ldots.$

Ordinary Differential Equation

Fourth-order Runge-Kutta Method : $y_{i+1} = y_i + \frac{1}{4}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Partial Differential Equation

Heat Equation : Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i.j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Or $u_{i,j+1} = ru_{i-1,j} + (1-2r)u_{i,j} + ru_{i+1,j}, r = \frac{kc^2}{h^2}$

Wave Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j-1} - 2u_{i.j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$