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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESI 2016/2017

COURSE NAME	:	CIVIL ENGINEERING MATHEMATIC IV
COURSE CODE	:	BFC24203
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) Given the function of $f(x) = x \cos x - 1$.

(i) Construct the graph over the interval $-4 \leq x \leq 4$.

(5 marks)

(ii) Analyse the smallest negative root of f(x) by using Newton Raphson Method with three decimal places. Iterate until $|f(x)| < \varepsilon = 0.005$.

(8 marks)

(b) Given the system linear algebraic equation in matrix form;

$$\begin{pmatrix} 2m & 1 & 0 \\ 3m & 2 & 1 \\ 0 & 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$$

(i) Change the matrix form into the linear algebraic equation.

(3 marks)

(ii) State the methods of solving linear algebraic equation in engineering problems.

(3 marks)

(iii) Analyse the unknown variables of m,y and z if x = -2.35 using Gauss Elimination Method,

(6 marks)

Q2 (a) Given the set of data in **Table 1**.

(i) Construct the graph based on the data given.

(4 marks)

(ii) Generate the Newton Divided Difference Polynomial Table.

(5 marks)

(iii) Adam claims that the above data can be used to estimate f(4.5). Do you agree with him? Justify your answer.

(2 marks)

(b) **Figure 1** shows the data of velocity versus time.

(i) Convert the data of graph into the table form.

(3 marks)

(ii) Using the appropriate difference formulas given in Formulae, analyse the first derivatives of f(x) at x = 1.5 with step size of h = 0.5 on two point backward difference and three point forward difference. Approximate value of f(x) can be obtained from the graph given. Determine the relative error (in percentage) for each difference formulas in two (2) decimal places with rounding off technique. Given the true value of equation f(x) → $y = 0.75x^4 - 6.1667x^3 + 14.25x^2 - 6.8333x + 2$.

(7 marks)

(iii) Your clients enquire the data of acceleration based on the data given. Generate the new data.

(4 marks)

Q3 (a) Analyse the approximate value for $\int_1^5 \sqrt{1+x^2}\,dx$ using trapezoidal rule. Given subintervals is eight (8). Analyse too the relative error in percentage. Examine the calculations in two (2) decimal places with chopping off technique.

(12 marks)

(b) Suppose you are an engineer in the consultant company and you are assigned to design the roof of the building that used the concept of large parabolic arch. The total length of the arc roof is given by;

$$L = \int_0^{30} 1.08x^2 + 7.6dx$$

Generate the total length of the arch by using Simpson's 1/3 Rule. Divide the domain from x = 0 meter to x = 30 meter into 6 equally spaced intervals.

(8 marks)

(c) Draw a diagram ordinate for linear boundary value problem of $y''-xy'+4y = 8x,\ 1 \le x \le 5$. Take $y'_i \approx \frac{y_{i+1}-y_{i-1}}{2h}$, $y''_i \approx \frac{y_{i+1}-2y_i+y_{i-1}}{h^2}$ and $\Delta x = h = 1.0$. Using finite difference method, ensure the finite different form without substitution.

(5 marks)

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Q4 (a) Given the wave equation $\frac{\partial^2 u}{\partial t^2} = 2\frac{\partial^2 u}{\partial x^2}$, with the interval of $0 \le x \le 1$ and $0 \le t \le 1.0$. The boundary conditions of the given wave problem are $u(0,t) = u(1,t) = t$ for $0 \le t \le 1.0$. The initial condition is set as $u(x,0) = 2\sin(\pi x)$, $\frac{\partial u(x,0)}{\partial t}\sin(2\pi x)$ for interval of $0 \le x \le 1.0$. Take $h = \Delta x = 0.2$ and $k = \Delta t = 0.5$.

(i) Draw the grid for the problem and show all the boundary and initial values in the diagram.

(2 marks)

(ii) Solve the wave equation by using the finite difference method.

(13 marks)

(b) Consider the heat flow equation $\frac{d^2R}{dx^2} = 0$, for $0 \le x \le 1$, on a steel beam consisting of four equal size elements. Analyse all the shape functions (interpolating functions) and their derivatives of the given heat problem.

(10 marks)

- END OF QUESTIONS-

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TABLE 1 : Velocity of the particles

Times (sec), x_i	0	1	2	3	4
$f(x_i)$	0	1	8	27	64

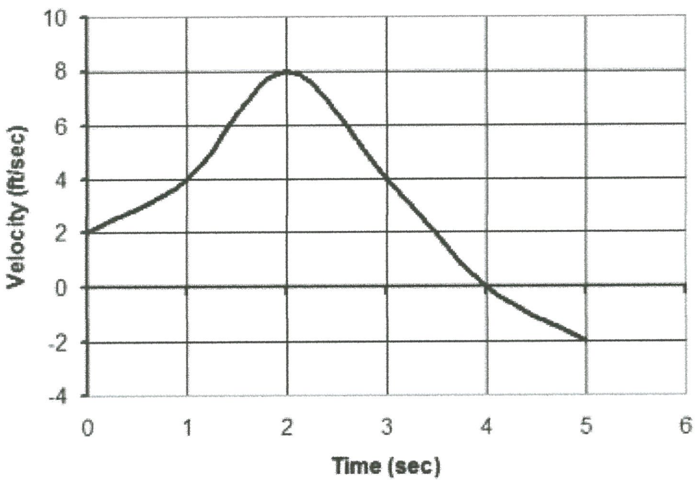

Figure 1 : Velocity versus times

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FORMULAE

Nonlinear equations

Newton-Raphson method : $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$, $i = 0, 1, 2, \ldots$

System of linear equations

Gauss-Seidel iteration : $x_i^{(k+1)} = \dfrac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}$, $\forall i = 1, 2, 3, \ldots, n$.

Interpolation
Natural Cubic Spline:

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\}, \quad k = 0, 1, 2, 3, \ldots, n-1,$$

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$b_k = 6(d_{k+1} - d_k)$, $k = 0, 1, 2, 3, \ldots, n-2$.

$$m_0 = 0,$$
$$m_n = 0,$$
$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, \qquad k = 0, 1, 2, 3, \ldots, n-2.$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6} h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6} h_k\right)(x - x_k)$$

$k = 0, 1, 2, 3, \ldots, n-1$.

Numerical Differentiation

2-point forward difference: $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \dfrac{f(x) - f(x-h)}{h}$

3-point central difference: $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \dfrac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \dfrac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \dfrac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

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FORMULAE

Numerical Integration

Simpson $\frac{1}{3}$ Rule: $\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i \right]$

Simpson $\frac{3}{8}$ rule : $\int_a^b f(x)\,dx \approx \frac{3}{8}h\begin{bmatrix} (f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) \\ + 2(f_3 + f_6 + \cdots + f_{n-3}) \end{bmatrix}$

Eigen value

Power Method: $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \qquad k = 0, 1, 2, \ldots..$

Ordinary Differential Equation

Fourth-order Runge-Kutta Method: $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\qquad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\qquad k_4 = hf(x_i + h, y_i + k_3)$

Partial Differential Equation

Heat Equation: Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Poisson Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

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FORMULAE

Finite Element Method

Heat flow problem in 1 dimension for $a \le x \le b$

$N(x) = \lfloor N_1(x) \;\; N_2(x) \;\; \cdots \;\; N_n(x) \rfloor$

$N_m(x) = \lfloor N_m^e(x) \rfloor$ is global shaped function for element e at node m

$T = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix}$ is the temperature vector at node

$$\mathbf{K}T = \mathbf{f}_b - \mathbf{f}_L$$

where

stiffness matrix, $\mathbf{K} = \int_a^b \mathbf{B}^T Ak\mathbf{B}\, dx$ or

$K_{ij} = \int_a^b \frac{dN_i}{dx} Ak \frac{dN_j}{dx}\, dx$ is a square matrix with dimension $n \times n$,

boundary vector, $\mathbf{f}_b = -\lfloor \mathbf{N}^T Aq \rfloor_a^b$ have the dimension $n \times 1$,

load vector, $\mathbf{f}_L = -\int_a^b \mathbf{N}_i Q(x)\, dx$ have the dimension $n \times 1$.

2-Point Gauss Quadrature

$$\int_{-1}^{1} f(x)dx \approx c_1 f(x_1) + c_2 f(x_2) \approx g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

3-Point Gauss Quadrature

$$\int_{-1}^{1} f(x)dx \approx c_1 f(x_1) + c_2 f(x_2) + c_3 x_3 \approx \frac{5}{9} g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} g(0) + \frac{5}{9} g\left(\sqrt{\frac{3}{5}}\right)$$