UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

TERBUKA

COURSE NAME	:	CIVIL ENGINEERING MATHEMATIC III
COURSE CODE	:	BFC24103
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **SECTION A, THREE (3)** QUESTIONS IN **SECTION B**

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

SECTION A

Q1 (a) The position vector of a particle after a time t is given as,

$$\boldsymbol{r}(\boldsymbol{t}) = \cos t\, \boldsymbol{i} + \sin t\, \boldsymbol{j} + t^3\, \boldsymbol{k}$$

Simulate the velocity, speed and acceleration of the particle when $t = 2$

(10 marks)

(b) For a helix $\boldsymbol{r}(\boldsymbol{t}) = 3\cos t\, \boldsymbol{i} + 3\sin t\, \boldsymbol{j} + 4t\, \boldsymbol{k}$, Find

(i) The unit tangent vector, T

(ii) The principal unit normal vector N

(iii) The curvature, κ

(iv) The radius of curvature, ϱ.

(10 marks)

Q2 (a) Solve $\oint_c (y^2 + \cos x)dx + (x - \tan^{-1} y)dy$, where C is the boundary of the region between $y = 4 - x^2$ and $y = 0$ using Green's Theorem.

(8 marks)

(b) Calculate the flux of the vector field $\mathbf{F}(x, y, z) = xy^2\mathbf{i} + yz^2\mathbf{j} + zx^2\mathbf{k}$ across surface by bounded $x^2 + y^2 = 4;\ z = 0$ and $z = 5$ using Divergence Theorem.

(12 marks)

SECTION B

Q3 (a) Compute the partial derivative of f_x and f_y for $f(x, y) = 3x^2y - 2xy + 5y^2$ using the first principle

(6 marks)

(b) Given $f(x,y) = \dfrac{3x^2}{x+4y}$. Validate that $f_{xy} = f_{yx}$.

(7 marks)

(c) Calculate the differential dz to approximate the change in $z = \sqrt{6 - x^2 - y^2}$ as (x, y) moves from point (0.3, 0.5) to the point (0.72, 0.66).

(7 marks)

Q4 (a) A civil engineer is designing a vertical circular cone tank for the use at residential area. Water is taken out from reservoir at the rate of 104 cm^3/min. By the same pace the water is then put into the tank with increasing rate of radius 2 cm/min. If 30 cm^3 of water is taken out and the radius of the tank is 6 cm, predict the increasing rate of height of water at that time.

(9 marks)

(b) Describe how rapidly will the water level inside a vertical cylindrical tank drop if we pump the fluid out at the rate of 3000 L/min, and the tank's radius at 1 m and 10 m respectively.

(6 marks)

(c) A hot air balloon is rising straight up from a field and tracked by a range finder 150 m from the liftoff point as shown in **Figure 1**. The range finder angle is $\pi/4$, the angle is increasing at the rate of 0.14 rad/min. Predict how fast the balloon is rising at that moment.

(5 marks)

Q5 (a) Solve $\iint_R \sqrt{x^2 + y^2}\, dA$, where R is the region inside the circle $(x-1)^2 + y^2 = 1$ in the first quadrant using polar coordinates.

(7 marks)

(b) Consider an object which is bounded above by the inverted paraboloid $z = 16 - x^2 - y^2$ and below by the xy-plane. Suppose that the density of the object is given by $\delta(x, y, z) = 8 + x + y$. Analyse the mass of the object by using cylindrical coordinates.

(7 marks)

(c) Interpret $\displaystyle\int_{-\sqrt{2}}^{0} \int_{0}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\, dz\, dx\, dy$ using spherical coordinates.

(6 marks)

Q6 (a) Analyze the volume of the solid within the cylinder $x^2 + y^2 = 4$ and between the planes $z = 2$ and $y + z = 5$ using cylindrical coordinates.

(7 marks)

(b) Calculate the volume of the solid bounded above by $\rho = 5$ and below by cone $\phi = \dfrac{\pi}{4}$ using spherical coordinates.

(7 marks)

(c) Given that a lamina with density function $\delta(x, y) = y$ is bounded by $y = \sin x,\ y = 0,\ x = 0$ and $x = \pi$. Estimate the centre of mass of the lamina.

(6 marks)

- END OF QUESTIONS –

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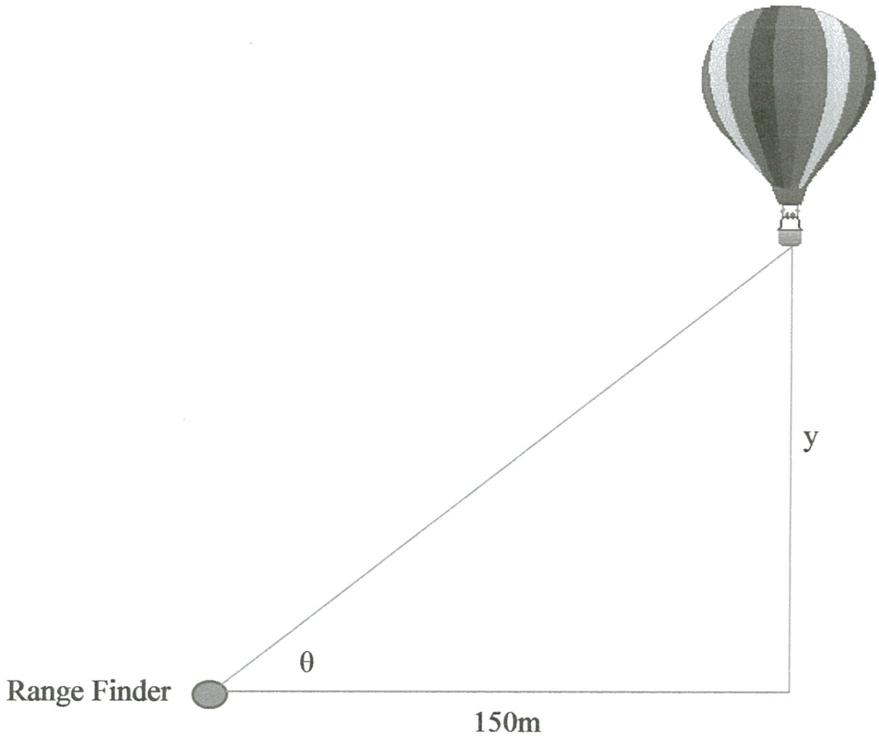

FIGURE 1

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Formulae

Implicit Partial Differentiation:

$$\frac{\partial z}{\partial x} = -\frac{f_x(x,y,z)}{f_z(x,y,z)} \text{ or } \frac{\partial z}{\partial y} = -\frac{f_y(x,y,z)}{f_z(x,y,z)}$$

Small Increment, Estimating Value:

Total differential/approximate change, $\partial z = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$
Exact change, $dz = f(x_1, y_1) - f(x_0, y_0)$
Approximate value, $z = f(x_0, y_0) + dz$
Exact value, $z = f(x_1, y_1)$

Error: $|dz| = \left|\frac{\partial z}{\partial x}dx\right| + \left|\frac{\partial z}{\partial y}dy\right|$ and **Relative error:** $\left|\frac{dz}{z}\right| = \left|\frac{\partial z}{\partial x}\frac{dx}{z}\right| + \left|\frac{\partial z}{\partial y}\frac{dy}{z}\right|$

Polar coordinate: $x = r\cos\theta,\ y = r\sin\theta,\ x^2 + y^2 = r^2$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate: $x = r\cos\theta,\ y = r\sin\theta,\ z = z,\ \iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\, r\, dz\, dr\, d\theta$

Spherical coordinate: $x = \rho\cos\theta\sin\emptyset,\ y = \rho\sin\theta\sin\emptyset, z = \rho\cos\emptyset,\ x^2 + y^2 + z^2 = \rho^2,$
$0 \le \theta \le 2\pi,\ 0 \le \phi \le \pi$ and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

Directional derivative: $D_{\mathbf{u}} f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\,\mathbf{i} + N\,\mathbf{j} + P\,\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

the **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

the unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

the curvature: $\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho = 1/\kappa$

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Green Theorem: $\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$

Gauss Theorem: $\iint_S \mathbf{F} \bullet \mathbf{n}\,dS = \iiint_G \nabla \bullet \mathbf{F}\,dV$

Stokes' Theorem: $\oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\,dS$

Arc length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \|\mathbf{r}'(t)\|\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t \in [a,b]$, then the **arc length** $s = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$

Tangent Plane

$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

Extreme of two variable functions

$G(x,y) = f_{xx}(x,y) f_{yy}(x,y) - (f_{xy}(x,y))^2$

Case1: If $G(a,b) > 0$ and $f_{xx}(x,y) < 0$ then f has local maximum at (a,b)

Case2: If $G(a,b) > 0$ and $f_{xx}(x,y) > 0$ then f has local minimum at (a,b)

Case3: If $G(a,b) < 0$ then f has a saddle point at (a,b)

Case4: If $G(a,b) = 0$ then no conclusion can be made.

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In 2-D: Lamina

Mass: $m = \iint_R \delta(x,y)\,dA$, where $\delta(x,y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y = \iint_R x\delta(x,y)\,dA$, (ii) about x-axis, $M_x = \iint_R y\delta(x,y)\,dA$

Centre of mass, $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right)$

Moment inertia: (i) $I_y = \iint_R x^2 \delta(x,y)\,dA$, (ii) $I_x = \iint_R y^2 \delta(x,y)\,dA$, (iii) $I_o = \iint_R (x^2 + y^2)\delta(x,y)\,dA$

In 3-D: Solid

Mass, $m = \iiint_G \delta(x,y,z)\,dV$. If $\delta(x,y,z) = c$, c is a constant, then $m = \iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz} = \iiint_G x\delta(x,y,z)\,dV$

(ii) about xz-plane, $M_{xz} = \iiint_G y\delta(x,y,z)\,dV$

(iii) about xy-pane, $M_{xy} = \iiint z\delta(x,y,z)\,dV$

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Centre of gravity, $(\bar{x},\bar{y},\bar{z})=\left(\frac{M_{yz}}{m},\frac{M_{xz}}{m},\frac{M_{xy}}{m}\right)$

Moment inertia

(i) about x-axis: $I_x=\iiint_G (y^2+z^2)\delta(x,y,z)\,dV$

(ii) about y-axis: $I_y=\iiint_G (x^2+z^2)\delta(x,y,z)\,dV$

(iii) about z-axis: $I_z=\iiint_G (x^2+y^2)\delta(x,y,z)\,dV$