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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

**FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017**

TERBUKA

COURSE NAME : CIVIL ENGINEERING MATHEMATIC II
COURSE CODE : BFC14003 / BWM10203
PROGRAMME CODE : BFF
EXAMINATION DATE : DECEMBER 2016 / JANUARY 2017
DURATION : 3 HOURS
INSTRUCTION : ANSWER ALL QUESTIONS IN SECTION A, AND ANY TWO (2) QUESTIONS IN SECTION B.

THIS QUESTION PAPER CONSISTS OF EIGHT (8) PAGES

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SECTION A

Q1 (a) By using formula $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$, find the laplace transform of $f(t) = t$ (5 marks)

(b) By using linear property of inverse laplace transform, find the inverse of
$$\frac{4s + 7}{s^2 + 9}$$
 (5 marks)

(c) By using laplace transform method, solve the initial value problem
$$y'' - 10y' + 9y = 5t; y(0) = 1; y'(0) = 2$$
 (10 marks)

Q2 (a) Use one of the Maclaurin Series derived to determine the Maclaurin Series for
$$f(x) = \sin 4x$$
 (5 marks)

(b) By using differentiation of power series, find
$$\frac{d}{dx}(\sin 4x)$$
 (3 marks)

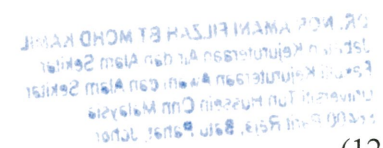
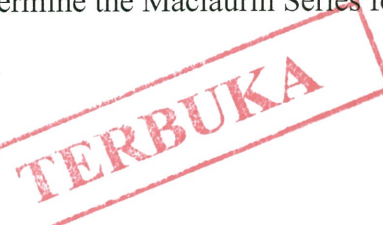
(c) Determine whether the following series converges or diverges. If converges, find the sum.

i)
$$\sum_{k=1}^{\infty} \frac{1}{3^k + 1}$$

ii)
$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{2} + \frac{1}{\sqrt{5}}$$

iii)
$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{5n + 6}$$

iv)
$$\sum_{n=1}^{\infty} 9^{-n+2} 4^{n+1}$$



(12 marks)

Q3 A periodic function $f(x)$ is defined by

$$f(x) = \begin{cases} -2, & -\pi < x < -\frac{\pi}{2} \\ x, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 2, & \frac{\pi}{2} < x < \pi \end{cases}$$

$$f(x) = f(x + 2\pi)$$

which it can be written as a Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

- (a) Sketch the graph of $f(x)$ over $-2\pi \leq x \leq 2\pi$. (3 marks)
- (b) Determine whether the function is even, odd or neither. (1 mark)
- (c) For $n = 1, 2, 3, \dots$, show that
 - (i) $a_0 = 0$.
 - (ii) $a_n = 0$.(7 marks)
- (d) Calculate the Fourier coefficient, b_n . (5 marks)
- (e) Find the Fourier series. (4 marks)



DR. NOR AMANI FILZAH BT MOHD RAMLI
 Pengerusi Kaji Seliaan Akademi Alam Baitul
 Falsafah, Universiti Awam dan Islam, Seremban
 70300 Seremban, Negeri Sembilan, Malaysia
 E-mail: noramani@uam.edu.my, noramani@uam.edu.my

SECTION B

- Q4** (a) Form an ordinary differential equation by eliminating the constants A and B from the equation:

$$y = Ax^3 + \frac{B}{x^2} - 6x$$

(5 marks)

- (b) Given $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$. Show that $y = x^{-\frac{1}{2}}$ is a solution.

(5 marks)

- (c) In a murder investigation, a dead body was found by a detective at exactly 8 am. Being alert, the detective also measured the body temperature and found it to be 20°C. Two hours later, the detective measured the body temperature again and found it to be 15°C. If the room temperature is 27°C, and assuming that the body temperature of the person before death was 37°C, determine the time the body reach 18°C?

(10 marks)

- Q5** (a) By using substitution $z = xy$, solve $xy' + y = 2x\sqrt{1 - (xy)^2}$ by using separable method.

(10 marks)

- (b) A population of worms in a region will grow at a rate that is proportional to the current population. In the absence of any outside factors, the population will double in two weeks time. On any given day, there is a net migration into the area of 5 worms and 6 are eaten by the birds and 4 die of natural causes. If there are initially 50 insects in the area, determine the population after four weeks?

(10 marks)

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- Q6** (a) Interpret $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = \sin 2x$ by using the method of undetermined coefficient.

(10 marks)

- (b) A 3 kg of iron ball is attached to spring and will stretch the spring 392 mm by itself. There is no damping in the system and a forcing function of the form $F(t) = 10 \cos(\omega t)$ is attached to the ball and the system will experience resonance. If the ball is initially displaced 20 cm downward from its equilibrium position and given a velocity of 10 cm/s upward, find the displacement at any time t .

(10 marks)

- END OF QUESTIONS-

UNIVERSITI TEKNOLOGI MALAYSIA
 FACULTY OF ENGINEERING
 DEPARTMENT OF MECHANICAL ENGINEERING
 41060 SEREMBAN, NEGERI SEMBILAN, MALAYSIA
 TEL: 06-733 3111 FAX: 06-733 3112
 E-MAIL: info@utms.edu.my

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1x} + Be^{m_2x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x} (A \cos \beta x + B \sin \beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \dots + A_1 x + A_0$	$x^r (B_n x^n + \dots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r (Pe^{\alpha x})$
$C \cos \beta x$ or $C \sin \beta x$	$x^r (p \cos \beta x + q \sin \beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

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Fourier Series

Fourier series expansion of periodic function with period $2L$	Fourier half-range series expansion
$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$ <p>where</p> $a_0 = \frac{1}{L} \int_{-L}^L f(x) dx$ $a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx$ $b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx$	$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$ <p>where</p> $a_0 = \frac{2}{L} \int_0^L f(x) dx$ $a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx$ $b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n, n = 1, 2, 3, \dots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2 + a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2 + a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2 - a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2 - a^2}$	$\dot{y}(t)$	$sY(s) - y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s) - sy(0) - \dot{y}(0)$
$t^n f(t), n = 1, 2, 3, \dots$	$(-1)^n \frac{d^n F(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt, \quad s > 0.$



JIMAN GHOM TG HASJIT INAMI ROM JI
 usi4e2 m6A m6A m6A m6A m6A m6A
 tal4e2 m6A m6A m6A m6A m6A m6A
 k4y4eM m6A m6A m6A m6A m6A m6A
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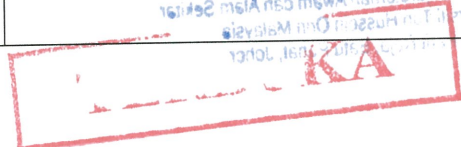
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Trigonometric and Hyperbolic Identities

Trigonometric	Hiperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \frac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \frac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \operatorname{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2 \sin x \cos x$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \operatorname{cosech}^2 x$
$\cos 2x = 2 \cos^2 x - 1$	$\sinh 2x = 2 \sinh x \cosh x$
$\cos 2x = 1 - 2 \sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$	$\cosh 2x = 2 \cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2 \sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2 \sin x \cos y = \sin(x + y) + \sin(x - y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2 \sin x \sin y = -\cos(x + y) + \cos(x - y)$	$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2 \cos x \cos y = \cos(x + y) + \cos(x - y)$	



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Differentiation and Integration

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k] = 0, \quad k \text{ constant}$	$\int k \, dx = kx + C$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}[\ln x] = \frac{1}{x}$	$\int \frac{dx}{x} = \ln x + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x \, dx = -\cos x + C$
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x \, dx = \sin x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$
$\frac{d}{dx}[\cot x] = -\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x \, dx = -\cot x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + C$
$\frac{d}{dx}[\operatorname{cosec} x] = -\operatorname{cosec} x \cot x$	$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$
$\frac{d}{dx}[e^x] = e^x$	$\int e^x \, dx = e^x + C$
$\frac{d}{dx}[\cosh x] = \sinh x$	$\int \sinh x \, dx = \cosh x + C$
$\frac{d}{dx}[\sinh x] = \cosh x$	$\int \cosh x \, dx = \sinh x + C$
$\frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x$	$\int \operatorname{sech}^2 x \, dx = \tanh x + C$
$\frac{d}{dx}[\operatorname{coth} x] = -\operatorname{cosech}^2 x$	$\int \operatorname{cosech}^2 x \, dx = -\operatorname{coth} x + C$
$\frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x$	$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$
$\frac{d}{dx}[\operatorname{cosech} x] = -\operatorname{cosech} x \operatorname{coth} x$	$\int \operatorname{cosech} x \operatorname{coth} x \, dx = -\operatorname{cosech} x + C$

