CONFIDENTIAL

Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2016/2017

TERBUKA

COURSE NAME	:	CIVIL ENGINEERING MATHEMATIC II
COURSE CODE	:	BFC14003 / BWM10203
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	DECEMBER 2016 / JANUARY 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **SECTION A**, AND ANY **TWO (2)** QUESTIONS IN **SECTION B**.

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

CONFIDENTIAL

SECTION A

Q1 (a) By using formula $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt$, find the laplace transform of $f(t) = t$

(5 marks)

(b) By using linear property of inverse laplace transform, find the inverse of

$$\frac{4s+7}{s^2+9}$$

(5 marks)

(c) By using laplace transform method, solve the initial value problem

$$y'' - 10y' + 9y = 5t; y(0) = 1;\ y'(0) = 2$$

(10 marks)

Q2 (a) Use one of the Maclaurin Series derived to determine the Maclaurin Series for

$$f(x) = \sin 4x$$

(5 marks)

(b) By using differentiation of power series, find

$$\frac{d}{dx}(\sin 4x)$$

(3 marks)

(c) Determine whether the following series converges or diverges. If converges, find the sum.

i) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{3^k + 1}$

ii) $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{2} + \frac{1}{\sqrt{5}}$

iii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{5n+6}$

iv) $\displaystyle\sum_{n=1}^{\infty} 9^{-n+2} 4^{n+1}$

(12 marks)

Q3 A periodic function $f(x)$ is defined by

$$f(x) = \begin{cases} -2, & -\pi < x < -\dfrac{\pi}{2} \\ x, & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ 2, & \dfrac{\pi}{2} < x < \pi \end{cases}$$

$$f(x) = f(x + 2\pi)$$

which it can be written as a Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

(a) Sketch the graph of $f(x)$ over $-2\pi \le x \le 2\pi$.

(3 marks)

(b) Determine whether the function is even, odd or neither.

(1 mark)

(c) For $n = 1, 2, 3, \ldots$, show that

(i) $a_0 = 0$.

(ii) $a_n = 0$.

(7 marks)

(d) Calculate the Fourier coefficient, b_n.

(5 marks)

(e) Find the Fourier series.

(4 marks)

SECTION B

Q4 (a) Form an ordinary differential equation by eliminating the constants A and B from the equation:

$$y = Ax^3 + \frac{B}{x^2} - 6x$$

(5 marks)

(b) Given $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$. Show that $y = x^{-\frac{1}{2}}$ is a solution.

(5 marks)

(c) In a murder investigation, a dead body was found by a detective at exactly 8 am. Being alert, the detective also measured the body temperature and found it to be 20°C. Two hours later, the detective measured the body temperature again and found it to be 15°C. If the room temperature is 27°C, and assuming that the body temperature of the person before death was 37°C, determine the time the body reach 18°C?

(10 marks)

Q5 (a) By using substitution $z = xy$, solve $xy' + y = 2x\sqrt{1-(xy)^2}$ by using separable method.

(10 marks)

(b) A population of worms in a region will grow at a rate that is proportional to the current population. In the absence of any outside factors, the population will double in two weeks time. On any given day, there is a net migration into the area of 5 worms and 6 are eaten by the birds and 4 die of natural causes. If there are initially 50 insects in the area, determine the population after four weeks?

(10 marks)

Q6 (a) Interpret $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = \sin 2x$ by using the method of undetermined coefficient.

(10 marks)

(b) A 3 kg of iron ball is attached to spring and will stretch the spring 392 mm by itself. There is no damping in the system and a forcing function of the form $F(t) = 10\cos(\omega t)$ is attached to the ball and the system will experience resonance. If the ball is initially displaced 20 cm downward from its equilibrium position and given a velocity of 10 cm/s upward, find the displacement at any time t.

(10 marks)

- END OF QUESTIONS-

FINAL EXAMINATION

SEMESTER / SESSION: SEM I / 2016/2017 PROGRAMME CODE: BFF
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II COURSE CODE: BFC 14003 / BWM 10203

Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W}\,dx, \quad u_2 = \int \frac{y_1 f(x)}{W}\,dx$	$y_p = u_1 y_1 + u_2 y_2$

TERBUKA

Fourier Series

Fourier series expansion of periodic function with period $2L$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

Fourier half-range series expansion

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)\,dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

FINAL EXAMINATION

SEMESTER / SESSION: SEM I / 2016/2017 PROGRAMME CODE: BFF
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II COURSE CODE: BFC 14003 / BWM 10203

Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1, 2, 3, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt, \quad s>0.$

TERBUKA

FINAL EXAMINATION

SEMESTER / SESSION: SEM I / 2016/2017 PROGRAMME CODE: BFF
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II COURSE CODE: BFC 14003 / BWM 10203

Trigonometric and Hyperbolic Identities

Trigonometric	Hiperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \operatorname{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \operatorname{cosech}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x + y) + \sin(x - y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x + y) + \cos(x - y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x + y) + \cos(x - y)$	

FINAL EXAMINATION

SEMESTER / SESSION: SEM I / 2016/2017
COURSE NAME: CIVIL ENGINEERING MATHEMATICS II

PROGRAMME CODE: BFF
COURSE CODE: BFC 14003 / BWM 10203

Differentiation and Integration

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C$, $n \neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\,x]=-\text{cosec}\,x \cot x$	$\int \text{cosec}\,x \cot x\,dx = -\text{cosec}\,x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\,x]=-\text{sech}\,x \tanh x$	$\int \text{sech}\,x \tanh x\,dx = -\text{sech}\,x + C$
$\frac{d}{dx}[\text{cosech}\,x]=-\text{cosech}\,x \coth x$	$\int \text{cosech}\,x \coth x\,dx = -\text{cosech}\,x + C$