UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2016/2017

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS III
COURSE CODE	:	BFC 24103
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JUNE 2017
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ONE (1)** QUESTION IN **SECTION A** AND **ALL** QUESTIONS IN **SECTION B**.

THIS QUESTION PAPER CONSISTS OF **SEVEN** (7) PAGES

SECTION A: ANSWER ONE (1) QUESTION ONLY

Q1 (a) For a helix, $\boldsymbol{r}(t) = 3\cos t\,\boldsymbol{i} + 3\sin t\,\boldsymbol{j} + 4t\,\boldsymbol{k}$. Find;

(i) The unit tangent vector, T

(ii) The principal unit normal vector N

(iii) The curvature, κ

(iv) The radius of curvature, ϱ

(12 marks)

(b) Find the unit tangent and principal unit normal vectors to the curve determined by $\boldsymbol{r}(\boldsymbol{t}) = \langle \sin 2t, \cos 2t, t \rangle$.

(8 marks)

Q2 (a) Compute the line integral $\int_C (4xz + 2y)dx$, where C is the line segment from (2,1,0) to (4,0,2).

(10 marks)

(b) Evaluate the line integral $\oint_C (e^x + 6xy)dx + (8x^2 + siny^2)dy$, where C is the positively-oriented boundary of the region bounded by the circles of radius 1 and 3, centered at the origin and lying in the first quadrant.

(10 marks)

SECTION B

Q3 (a) Let $u = x^2y^3e^{xy}$ where $u = s^2 + t^2, y = 2st, z = s\ln t$, Determine $\frac{\partial u}{\partial s}$.

(6 marks)

(b) Given, $f(x,y) = \frac{xy}{x^2+y^2}$. Prove that $f_{xy} = f_{yx}$.

(7 marks)

(c) Formulate the equation of the tangent plane to $z = x^2 \cos(\pi y) - \frac{6}{xy}$ at (2,-1).

(7 marks)

Q4 (a) A solid is in the shape of a half right circular cone as shown in **Figure Q4(a)**. Given that the upper radius x decreases at the rate of 2 mm per minute, the lower radius y increases at the rate of 3 mm per minute, and the height z decreases at the rate of 4 mm per minute., at what rate is the volume V changing at the instant with the upper radius is 10 mm, the lower radius is 12 mm, and the height is 18 mm. With x, y, z given as, $V = \frac{1}{3}\pi z(x^2 + xy + y^2)$.

(12 marks)

(b) Calculate on how rapidly will the water level inside a vertical cylindrical tank drop if we pump the fluid out at the rate of 3000 L/min, and the tanks's radius are at 1 m and 10 m respectively.

(8 marks)

Q5 (a) Calculate the area of the regions enclosed by $y = x^2$and $y = x + 2$ using double integral.

(8 marks)

(b) The region 'R' is a triangle and it is located on the xy-plane. If the given planes is $4x + 2y + z = 4, x = 0, y = 0$ and $z = 0$, visualize and calculate the volume of the tetrahedron region bounded.

(12 marks)

Q6 (a) Find the center of mass of the lamina in the shape of the region bounded by the graphs of $y = x^2$and $y = 4$, having mass density given by $\rho(x,y) = 1 + 2y + 6x^2$.

(12 marks)

(b) Analyse the volume of the solid within the cylinder $x^2 + y^2 = 4$ and between the planes $z = 2$ and $y + z = 5$ using cylindrical coordiates.

(8 marks)

- END OF QUESTIONS –

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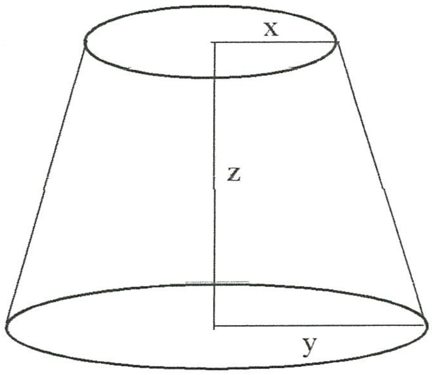

FIGURE Q4(a)

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Formulae

Implicit Partial Differentiation:

$$\frac{\partial z}{\partial x} = -\frac{f_x(x,y,z)}{f_z(x,y,z)} \quad or \quad \frac{\partial z}{\partial y} = -\frac{f_y(x,y,z)}{f_z(x,y,z)}$$

Small Increment, Estimating Value:

Total differential/approximate change, $\partial z = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

Exact change, $dz = f(x_1, y_1) - f(x_0, y_0)$

Approximate value, $z = f(x_0, y_0) + dz$

Exact value, $z = f(x_1, y_1)$

Error: $|dz| = \left|\frac{\partial z}{\partial x}dx\right| + \left|\frac{\partial z}{\partial y}dy\right|$ and **Relative error**: $\left|\frac{dz}{z}\right| = \left|\frac{\partial z}{\partial x}\frac{dx}{z}\right| + \left|\frac{\partial z}{\partial y}\frac{dy}{z}\right|$

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $x^2 + y^2 = r^2$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\,r\,dr\,d\theta$

Cylindrical coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$

Spherical coordinate: $x = \rho\cos\theta\sin\emptyset$, $y = \rho\sin\theta\sin\emptyset$, $z = \rho\cos\emptyset$, $x^2 + y^2 + z^2 = \rho^2$,
$0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$ and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

Directional derivative: $D_{\mathbf{u}}f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, then

the **divergence** of $\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

the **curl** of $\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$

Let C is a smooth curve given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, t is parameter, then

the unit tangent vector: $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

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the unit normal vector: $\mathbf{N}(t)=\dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

the binormal vector: $\mathbf{B}(t)=\mathbf{T}(t)\times\mathbf{N}(t)$

the curvature: $\kappa=\dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}=\dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

the radius of curvature: $\rho=1/\kappa$

Green Theorem: $\oint\limits_C M\,dx+N\,dy=\iint\limits_R\left(\dfrac{\partial N}{\partial x}-\dfrac{\partial M}{\partial y}\right)dA$

Gauss Theorem: $\iint\limits_S \mathbf{F}\bullet\mathbf{n}\,dS=\iiint\limits_G \nabla\bullet\mathbf{F}\,dV$

Stokes' Theorem: $\oint\limits_C \mathbf{F}\bullet d\mathbf{r}=\iint\limits_S(\nabla\times\mathbf{F})\bullet\mathbf{n}\,dS$

Arc length

If $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}$, $t\in[a,b]$, then the **arc length** $s=\int_a^b\|\mathbf{r}'(t)\|dt=\int_a^b\sqrt{[x'(t)]^2+[y'(t)]^2}\,dt$

If $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}$, $t\in[a,b]$, then the **arc length** $s=\int_a^b\sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\,dt$

Tangent Plane

$z-z_0=f_x(x_0,y_0)(x-x_0)+f_y(x_0,y_0)(y-y_0)$

Extreme of two variable functions

$G(x,y)=f_{xx}(x,y)f_{yy}(x,y)-\left(f_{xy}(x,y)\right)^2$

Case1: If $G(a,b)>0$ and $f_{xx}(x,y)<0$ then f has local maximum at (a,b)

Case2: If $G(a,b)>0$ and $f_{xx}(x,y)>0$ then f has local minimum at (a,b)

Case3: If $G(a,b)<0$ then f has a saddle point at (a,b)

Case4: If $G(a,b)=0$ then no conclusion can be made.

In 2-D: Lamina

Mass: $m=\iint\limits_R\delta(x,y)dA$, where $\delta(x,y)$ is a density of lamina.

Moment of mass: (i) about y-axis, $M_y=\iint\limits_R x\delta(x,y)dA$, (ii) about x-axis, $M_x=\iint\limits_R y\delta(x,y)dA$

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Centre of mass, $(\bar{x},\bar{y})=\left(\dfrac{M_y}{m},\dfrac{M_x}{m}\right)$

Moment inertia: (i) $I_y=\iint_R x^2\delta(x,y)dA$, (ii) $I_x=\iint_R y^2\delta(x,y)dA$, (iii) $I_o=\iint_R\left(x^2+y^2\right)\delta(x,y)dA$

In 3-D: Solid

Mass, $m=\iiint_G \delta(x,y,z)dV$. If $\delta(x,y,z)=c$, c is a constant, then $m=\iiint_G dA$ is volume.

Moment of mass

(i) about yz-plane, $M_{yz}=\iiint_G x\delta(x,y,z)dV$

(ii) about xz-plane, $M_{xz}=\iiint_G y\delta(x,y,z)dV$

(iii) about xy-pane, $M_{xy}=\iiint_G z\delta(x,y,z)dV$

Centre of gravity, $(\bar{x},\bar{y},\bar{z})=\left(\dfrac{M_{yz}}{m},\dfrac{M_{xz}}{m},\dfrac{M_{xy}}{m}\right)$

Moment inertia

(i) about x-axis: $I_x=\iiint_G\left(y^2+z^2\right)\delta(x,y,z)dV$

(ii) about y-axis: $I_y=\iiint_G\left(x^2+z^2\right)\delta(x,y,z)dV$

(iii) about z-axis: $I_z=\iiint_G\left(x^2+y^2\right)\delta(x,y,z)dV$